FUNCIONES DE LOS MÚLTIPLOS DE UN ÁNGULO

Ángulo doble

$\operatorname{sen} 2\alpha = 2\operatorname{sen}\alpha\cos\alpha$

$\cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha$

$\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$

Ángulo triple

$\operatorname{sen} 3\alpha = 3\operatorname{sen}\alpha - 4\operatorname{sen}^3\alpha$

$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$

$\tan 3\alpha = \dfrac{3\tan\alpha}{1-3\tan^2\alpha}$

FUNCIONES TRIGONOMÉTRICAS INVERSAS

$$\operatorname{arc\,sen} x = \arccos\sqrt{1-x^2} = \frac{\pi}{2} - \arccos x$$

$$\operatorname{arc\,cos} x = \operatorname{arcsen}\sqrt{1-x^2} = \frac{\pi}{2} - \operatorname{arcsen} x$$

$$\arctan x = \operatorname{arcsen}\frac{x}{\sqrt{1+x^2}} = \frac{\pi}{2} - \operatorname{arctg} x$$

$$\operatorname{arcsen} x + \operatorname{arcsen} y = \operatorname{arcsen}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$$

$$\operatorname{arcsen} x - \operatorname{arcsen} y = \operatorname{arcsen}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$$

$$\arccos x + \arccos y = \arccos\left[xy - \sqrt{(1-x^2)(1-y^2)}\right]$$

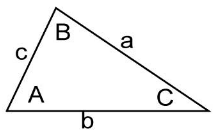
$$\arccos x - \arccos y = \arccos\left[xy + \sqrt{(1-x^2)(1-y^2)}\right]$$

$$\arctan x + \arctan y = \frac{x+y}{1-xy} \qquad \arctan x - \arctan y = \frac{x-y}{1+xy}$$

FÓRMULAS DE BRIGGS

Para las tangentes de los ángulos mitad, se dividen las expresiones análogas miembro a miembro. Para el ángulo entero se utilizan las fórmulas que dan las razones de un ángulo en función del coseno del ángulo doble. Estas fórmulas ya se han tratado anteriormente.

$$\operatorname{sen}\frac{A}{2} = \sqrt{\frac{(p-b)(p-c)}{bc}}$$

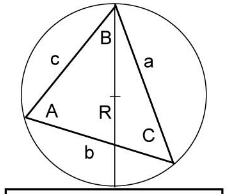


$$\cos\frac{B}{2} = \sqrt{\frac{p(p-b)}{ac}}$$

$$\operatorname{sen}\frac{C}{2} = \sqrt{\frac{(p-b)(p-a)}{ab}} \qquad \cos\frac{C}{2} = \sqrt{\frac{p(p-c)}{ab}}$$

$$\cos\frac{A}{2} = \sqrt{\frac{p(p-c)}{bc}} \qquad p = \frac{a+b+c}{2}$$

TEOREMAS IMPORTANTES:



Ley de los senos:

$$\frac{a}{\operatorname{sen} A} = \frac{b}{\operatorname{sen} B} = \frac{c}{\operatorname{sen} C} = 2R$$

Ley de los cosenos:

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

Ley de las tangentes

$$\frac{a+b}{a-b} = \frac{\operatorname{sen} A + \operatorname{sen} B}{\operatorname{sen} A - \operatorname{sen} B} = \frac{\tan\frac{A+B}{2}}{\tan\frac{A-B}{2}}$$

AREA DEL TRIÁNGULO

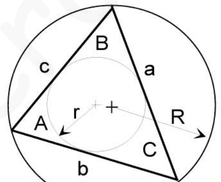
$$S = \frac{1}{2}ab\operatorname{sen} C = \frac{1}{2}cb\operatorname{sen} A = \frac{1}{2}ac\operatorname{sen} B$$

(Fórmula de Herón)

$$S = \sqrt{p(p-a)(p-b)(p-c)}$$

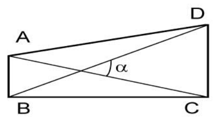
$$S = pr = \frac{abc}{4R} = \frac{p}{2R}$$

$$p = \frac{a+b+c}{2}$$



AREA DE UN CUADRILÁTERO

$$S = \frac{\overline{AC}\,\overline{BD}}{2}\operatorname{sen}\alpha$$



II.- FUNCIONES HIPERBÓLICAS

FÓRMULAS BÁSICAS

$$\operatorname{Sh}\alpha = \frac{e^\alpha - e^{-\alpha}}{2} \qquad \operatorname{Th}\alpha = \frac{\operatorname{Sh}\alpha}{\operatorname{Ch}\alpha} = \frac{e^\alpha - e^{-\alpha}}{e^\alpha + e^{-\alpha}}$$

$$\operatorname{Ch}\alpha = \frac{e^\alpha + e^{-\alpha}}{2} \qquad \operatorname{Ch}\alpha - \operatorname{Sh}\alpha = e^{-\alpha}$$

$$\operatorname{Sh}\alpha + \operatorname{Ch}\alpha = e^\alpha \qquad \operatorname{Ch}^2\alpha - \operatorname{Sh}^2\alpha = 1$$

FUNCIONES DEL ÁNGULO SUMA/DIFERENCIA

$$\operatorname{Sh}(\alpha+\beta) = \operatorname{Sh}\alpha\operatorname{Ch}\beta + \operatorname{Sh}\beta\operatorname{Ch}\alpha \qquad \operatorname{Sh}(\alpha-\beta) = \operatorname{Sh}\alpha\operatorname{Ch}\beta - \operatorname{Sh}\beta\operatorname{Ch}\alpha$$

$$\operatorname{Ch}(\alpha+\beta) = \operatorname{Ch}\alpha\operatorname{Ch}\beta + \operatorname{Sh}\beta\operatorname{Sh}\alpha \qquad \operatorname{Ch}(\alpha-\beta) = \operatorname{Ch}\alpha\operatorname{Ch}\beta - \operatorname{Sh}\alpha\operatorname{Sh}\beta$$

$$\operatorname{Th}(\alpha+\beta) = \frac{\operatorname{Th}\alpha + \operatorname{Th}\beta}{1+\operatorname{Th}\alpha\operatorname{Th}\beta} \qquad \operatorname{Th}(\alpha-\beta) = \frac{\operatorname{Th}\alpha - \operatorname{Th}\beta}{1-\operatorname{Th}\alpha\operatorname{Th}\beta}$$

FUNCIONES DEL ÁNGULO DOBLE/MITAD

$$\operatorname{Sh}2\alpha = 2\operatorname{Sh}\alpha\operatorname{Ch}\alpha \qquad \operatorname{Ch}2\alpha = \operatorname{Sh}^2\alpha + \operatorname{Ch}^2\alpha \qquad \operatorname{Th}2\alpha = \frac{2\operatorname{Sh}\alpha\operatorname{Ch}\alpha}{\operatorname{Sh}^2\alpha + \operatorname{Ch}^2\alpha}$$

$$\operatorname{Sh}\frac{\alpha}{2} = \sqrt{\frac{1}{2}(\operatorname{Ch}\alpha - 1)} \qquad \operatorname{Ch}\frac{\alpha}{2} = \sqrt{\frac{1}{2}(\operatorname{Ch}\alpha + 1)} \qquad \operatorname{Th}\frac{\alpha}{2} = \sqrt{\frac{\operatorname{Ch}\alpha - 1}{\operatorname{Ch}\alpha + 1}}$$

TRANSFORMACION DE PRODUCTOS A SUMAS

$$\operatorname{Sh}\alpha\operatorname{Sh}\beta = \frac{1}{2}\left[\operatorname{Ch}(\alpha+\beta) - \operatorname{Ch}(\alpha-\beta)\right] \qquad \operatorname{Ch}\alpha\operatorname{Ch}\beta = \frac{1}{2}\left[\operatorname{Ch}(\alpha+\beta) + \operatorname{Ch}(\alpha-\beta)\right]$$

$$\operatorname{Sh}\alpha\operatorname{Ch}\beta = \frac{1}{2}\left[\operatorname{Sh}(\alpha+\beta) + \operatorname{Sh}(\alpha-\beta)\right] \qquad (\operatorname{Ch}\alpha \pm \operatorname{Sh}\alpha)^n = \operatorname{Ch} n\alpha \pm \operatorname{Sh} n\alpha$$

FUNCIONES HIPERBÓLICAS INVERSAS

$$\operatorname{ArgSh} x = \ln\left(x + \sqrt{x^2+1}\right) \qquad \operatorname{ArgCh} x = \ln\left(x + \sqrt{x^2-1}\right)$$

$$\operatorname{ArgTh} x = \frac{1}{2}\ln\frac{1+x}{1-x} \qquad \operatorname{ArgSh} x = \operatorname{ArgCh}\sqrt{x^2+1} = \operatorname{ArgTh}\frac{x}{\sqrt{x^2+1}}$$

$$\operatorname{ArgCth} x = \frac{1}{2}\ln\frac{x+1}{x-1} \qquad \operatorname{ArgCh} x = \operatorname{ArgSh}\sqrt{x^2-1} = \operatorname{ArgTh}\frac{\sqrt{x^2-1}}{x}$$

$$\operatorname{ArgTh} x = \operatorname{ArgSh}\frac{x}{\sqrt{1-x^2}} = \operatorname{ArgCh}\frac{x}{\sqrt{1-x^2}} = \operatorname{ArgCth}\frac{1}{x}$$

RELACIONES ENTRE FUNCIONES CIRCULARES E HIPERBÓLICAS

$$\operatorname{sen} x = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right) \qquad \cos x = \frac{1}{2}\left(e^{ix} + e^{-ix}\right)$$

$\operatorname{Sh} ix = i\operatorname{sen} x$ $\qquad$ $\operatorname{sen} ix = i\operatorname{Sh} x$ $\qquad$ $e^{ix} = \cos x + i\operatorname{sen} x$

$\operatorname{Ch} ix = \cos x$ $\qquad$ $\cos ix = \operatorname{Ch} x$ $\qquad$ $e^{-ix} = \cos x - i\operatorname{sen} x$

$\operatorname{Th} ix = i\tan x$ $\qquad$ $\tan ix = i\operatorname{Th} x$ $\qquad$ $\operatorname{arcsen} ix = i\operatorname{Arg\,Sh} x$

$\operatorname{sen}(x+iy) = \operatorname{sen} x\operatorname{Ch} y + i\cos x\operatorname{Sh} y$ $\qquad$ $\arccos ix = -i\operatorname{Arg\,Ch} x$

$\cos(x+iy) = \cos x\operatorname{Ch} y - i\operatorname{sen} x\operatorname{Sh} y$ $\qquad$ $\arctan ix = i\operatorname{Arg\,Th} x = \dfrac{1}{2}i\ln\dfrac{1+x}{1-x}$