A continuación, se adjunta un listado de ejercicios que se proponen al lector. Observará que no se indica técnica alguna solicitada para el desarrollo de los mismos, y que además no se han respetado normas relativas a niveles de dificultad, ni a las técnicas mismas. Como siempre, se adjuntaran las soluciones cuyos desarrollos pueden diferir de los aquí presentados. No importa, eso es posible; además una consulta con su profesor aclarará cualquier discrepancia.

Encontrar:

1.- $\int t^3 e^{\operatorname{sen} t^4} \cos t^4 \, dt$

2.- $\int \frac{\theta \, d\theta}{(1+\theta)^2}$

3.- $\int \frac{\theta e^{\theta} \, d\theta}{(1+\theta)^2}$

4.- $\int e^{\tau g 3\theta} \sec^2 3\theta \, d\theta$

5.- $\int \frac{x \, dx}{\sqrt[3]{ax+b}}$

6.- $\int \sqrt{\frac{x^2-1}{x+1}}$

7.- $\int \frac{dx}{(2-x)\sqrt{1-x}}$

8.- $\int e^{2-x} \, dx$

9.- $\int \frac{e^x \, dx}{ae^x - b}$

10.- $\int \frac{(t+1)\,dt}{t^2+2t-5}$

11.- $\int \sec \frac{\varphi}{2} \, d\varphi$

12.- $\int \tau g \theta \, d\theta$

13.- $\int \frac{\eta^2}{a} \operatorname{sen} \frac{\eta}{b} \, d\eta$

14.- $\int \varphi \sec^2 \varphi \, d\varphi$

15.- $\int \frac{dx}{5^x}$

16.- $\int \sec^2 (1-x) \, dx$

17.- $\int \frac{x \, dx}{\sqrt{16-x^4}}$

18.- $\int \frac{dy}{\sqrt{1+\sqrt{1+y}}}$

19.- $\int \frac{dx}{\sqrt{x+4}-\sqrt{x+3}}$

20.- $\int \cos ec\theta \, d\theta$

21.- $\int t(1-t^2)^{1/2} \, dt$

22.- $\int t(1-t^2)^{1/2} \operatorname{arcsen} t \, dt$

23.- $\int \frac{1+\cos 2x}{\operatorname{sen}^2 2x} \, dx$

24.- $\int \frac{x^2+1}{x^3-x} \, dx$

25.- $\int \frac{e^x \, dx}{\sqrt{9-e^{2x}}}$

26.- $\int \frac{dx}{(x-1)^3}$

27.- $\int \frac{(3x+4)\,dx}{\sqrt{2x+x^2}}$

28.- $\int \frac{ds}{\sqrt{4-s^2}}$

29.- $\int \frac{dx}{x^2\sqrt{x^2+e}}$

30.- $\int \frac{x \, dx}{\sqrt{1+x}}$

31.- $\int \frac{y^2 dy}{\sqrt{y+1}}$

32.- $\int \frac{y^3 dy}{\sqrt{y^2-1}}$

33.- $\int \frac{d\theta}{1+2\cos\theta}$

34.- $\int \frac{t^4-t^3+4t^2-2t+1}{t^3+1}dt$

35.- $\int \frac{d\varphi}{\ell\eta e}$

36.- $\int x(10+8x^2)^9 dx$

37.- $\int \frac{dx}{\sqrt{(16+x^2)^3}}$

38.- $\int \frac{x^3 dx}{\sqrt{x^2+4}}$

39.- $\int \frac{x^3 dx}{\sqrt{16-x^2}}$

40.- $\int a(x^2+1)^{1/2} dy$

41.- $\int \frac{dx}{(\sqrt{6-x^2})^3}$

42.- $\int \frac{dx}{x(3+\ell\eta x)}$

43.- $\int \frac{e^x}{16+e^{2x}}dx$

44.- $\int \cos\sqrt{1-x}\,dx$

45.- $\int \frac{x^3 dx}{\sqrt{x-1}}$

46.- $\int \frac{2y^5-7y^4+7y^3-19y^2+7y-6}{(y-1)^2(y^2+1)^2}dy$

47.- $\int \operatorname{sen}\sqrt{x+1}\,dx$

48.- $\int \frac{9x^2+7x-6}{x^3-x}dx$

49.- $\int \frac{5w^3-5w^2+2w-1}{w^4+w^2}dw$

50.- $\int \frac{3dx}{1+2x}$

51.- $\int \frac{(1-x)^2 dx}{x}$

52.- $\int \frac{xe^{-2x^2}}{2}dx$

53.- $\int e^{2t}\cos(e^t)dt$

54.- $\int \sqrt{x}(x^{3/2}-4)^3 dx$

55.- $\int \frac{\operatorname{sen} x e^{\sec x}}{\cos^2 x}dx$

56.- $\int \frac{ds}{s^{1/3}(1+s^{2/3})}$

57.- $\int \frac{1}{z^3}\left(\frac{1-z^2}{z^2}\right)^{10} dz$

58.- $\int \frac{x\ell\eta(1+x^2)}{1+x^2}dx$

59.- $\int \frac{\operatorname{co}\tau g x dx}{\ell\eta|\operatorname{sen} x|}$

60.- $\int \frac{ax^2-bx+c}{ax^2+bx-c}dx$

61.- $\int \frac{dx}{\cos^2 5x}$

62.- $\int \frac{dx}{12-7x}$

63.- $\int \tau g 16x dx$

64.- $\int \tau g 4\theta \sec^2 4\theta d\theta$

65.- $\int \frac{xdx}{\sqrt{x-5}}$

66.- $\int \frac{7t-2}{\sqrt{7-2t^2}}dt$

67.- $\int (1+x)\cos\sqrt{x}\,dx$

68.- $\int \frac{dx}{x(\sqrt{1+x}-1)}$

69.- $\int \frac{dx}{\operatorname{co}\tau g 6x}$

70.- $\int \operatorname{co}\tau g(2x-4)dx$

71.- $\int (e^t-e^{-2t})^2 dt$

72.- $\int \frac{(x+1)dx}{(x+2)^2(x+3)}$

73.- $\int (\operatorname{co}\tau g e^x)e^x dx$

74.- $\int \frac{\operatorname{sen}\theta+\theta}{\cos\theta+1}d\theta$

75.- $\int \frac{\operatorname{arc}\tau g x dx}{(1+x^2)^{3/2}}$

76.- $\int x\operatorname{co}\tau g(x^2/5)dx$

77.- $\int x\sqrt{4x^2-2}\,dx$

78.- $\int \frac{(x^2+9)^{1/2}dx}{x^4}$

79.- $\int x^2\operatorname{sen}^5 x^3\cos x^3 dx$

80.- $\int \frac{xdx}{\sqrt{5x^2+7}}$

81.- $\int \frac{x^3 dx}{x^2-x-6}$

82.- $\int \operatorname{sen} 2\theta e^{\operatorname{sen}^2\theta} d\theta$

83.- $\int \frac{dx}{e^x-9e^{-x}}$

84.- $\int \frac{dw}{1+\cos w}$

85.- $\int e^{\left(\frac{1-\text{sen}^2 x/2}{3}\right)^2}(\cos^3 x/2 \; \text{sen}\, x/2)dx$

86.- $\int \frac{x^3 dx}{\sqrt{19-x^2}}$

87.- $\int \frac{\text{sen}\,\varphi d\varphi}{\cos^{1/2}\varphi}$

88.- $\int (\sec\varphi + \tau g\varphi)^2 d\varphi$

89.- $\int \frac{dt}{t(4+\ell\eta^2 t)^{1/2}}$

90.- $\int a^{\theta} b^{2\theta} c^{3\theta} d\theta$

91.- $\int \text{sen}^{1/2}\varphi \cos^3\varphi d\varphi$

92.- $\int \frac{\sec^2\theta d\theta}{9+\tau g^2\theta}$

93.- $\int \frac{dx}{\sqrt{e^{2x}-16}}$

94.- $\int (e^{2s}-1)(e^{2s}+1)ds$

95.- $\int \frac{dx}{5x^2+8x+5}$

96.- $\int \frac{x^3+1}{x^3-x}dx$

97.- $\int (\text{arcsen}\sqrt{1-x^2})^0 dx$

98.- $\int \frac{3dy}{1+\sqrt{y}}$

99.- $\int x(1+x)^{1/5}dx$

100.- $\int \frac{d\varphi}{a^2\,\text{sen}^2\varphi + b^2\cos^2\varphi}$

101.- $\int \frac{tdt}{(2t+1)^{1/2}}$

102.- $\int \frac{s\ell\eta|s|ds}{(1-s^2)^{1/2}}$

103.- $\int (2\cos\alpha\,\text{sen}\,\alpha - \text{sen}\,2\alpha)d\alpha$

104.- $\int t^4 \ell\eta^2 t dt$

105.- $\int u^2(1+v)^{11}dx$

106.- $\int \frac{(\varphi+\text{sen}\,3\varphi)d\varphi}{3\varphi^2-2\cos 3\varphi}$

107.- $\int \frac{(y^{1/2}+1)dy}{y^{1/2}(y+1)}$

108.- $\int \frac{ds}{s^3(s^2-4)^{1/2}}$

109.- $\int \sqrt{u}(1+u^2)^2 du$

110.- $\int \frac{(x^3+x^2)dx}{x^2+x-2}$

111- $\int adb$

112.- $\int \frac{dx}{\sqrt{x^2-2x-8}}$

113.- $\int \frac{(x+1)dx}{\sqrt{2x-x^2}}$

114.- $\int f(x)f'(x)dx$

115.- $\int \frac{x^3+7x^2-5x+5}{x^2+2x-3}dx$

116.- $\int e^{\ell\eta|1+x+x^2|}dx$

117.- $\int \frac{(x-1)dx}{\sqrt{x^2-4x+3}}$

118.- $\int \frac{xdx}{\sqrt{x^2+4x+5}}$

119.- $\int \frac{4dx}{x^3+4x}$

120.- $\int \frac{\text{co}\,\tau g x dx}{\ell\eta|\text{sen}\,x|}$

121.- $\int \ell\eta \exp\sqrt{x-1}dx$

122.- $\int \frac{\sqrt{1+x^3}}{x}dx$

123.- $\int \sqrt{\frac{x-1}{x+1}}\frac{1}{x}dx$

124.- $\int \frac{\text{sen}\,xdx}{1+\text{sen}\,x+\cos x}$

125.- $\int \frac{dx}{3+2\cos x}$

126.- $\int \frac{xdx}{\sqrt{x^2-2x+5}}$

127.- $\int \frac{(1+\text{sen}\,x)dx}{\text{sen}\,x(2+\cos x)}$

128.- $\int \frac{dx}{x^4+4}$

RESPUESTAS

1.- $\int t^3 e^{\text{sen}\,t^4}\cos t^4 dt$

Solución.- Sea: $u = \text{sen}\,t^4, du = (\cos t^4)4t^3 dt$; luego:

$$\int t^3 e^{\text{sen}\,t^4}\cos t^4 dt = \frac{1}{4}\int 4t^3 e^{\text{sen}\,t^4}\cos t^4 dt = \frac{1}{4}\int e^u du = \frac{1}{4}e^u + c = \frac{1}{4}e^{\text{sen}\,t^4} + c$$

2.- $\int \frac{\theta d\theta}{(1+\theta)^2}$

Solución.-

$$\int \frac{\theta d\theta}{(1+\theta)^2} = \int \frac{A d\theta}{1+\theta} + \int \frac{B d\theta}{(1+\theta)^2} (*)$$

$\frac{\theta}{(1+\theta)^2} = \frac{A}{1+\theta} + \frac{B}{(1+\theta)^2} \Rightarrow \theta = A(1+\theta) + B \Rightarrow \theta = A\theta + (A+B)$, de donde:

$A = 1, B = -1$, entonces: $(*) \int \frac{\theta d\theta}{(1+\theta)^2} = \int \frac{d\theta}{1+\theta} - \int \frac{d\theta}{(1+\theta)^2} = \ell\eta\left|1+\theta\right| + \frac{1}{1+\theta} + c$

3.- $\int \frac{\theta e^\theta d\theta}{(1+\theta)^2}$

Solución.-

Sea: $u = e^\theta \qquad dv = \frac{\theta d\theta}{(1+\theta)^2}$

$du = e^\theta d\theta \qquad v = \ell\eta\left|1+\theta\right| + \frac{1}{1+\theta}$

$$\int \frac{\theta e^\theta d\theta}{(1+\theta)^2} = e^\theta \ell\eta\left|1+\theta\right| + \frac{e^\theta}{1+\theta} - \int (\ell\eta\left|1+\theta\right| + \frac{1}{1+\theta}) e^\theta d\theta$$

$= e^\theta \ell\eta\left|1+\theta\right| + \frac{e^\theta}{1+\theta} - \int e^\theta \ell\eta\left|1+\theta\right| d\theta - \int \frac{e^\theta d\theta}{1+\theta} (*)$, resolviendo por partes la segunda integral se tiene: $u = e^\theta \qquad dv = \frac{\theta d\theta}{1+\theta}$

$du = e^\theta d\theta \qquad v = \ell\eta\left|1+\theta\right|$

Luego: $\int \frac{e^\theta d\theta}{1+\theta} = e^\theta \ell\eta\left|1+\theta\right| - \int e^\theta \ell\eta\left|1+\theta\right| d\theta$, esto es:

$$(*) = \cancel{e^\theta \ell\eta\left|1+\theta\right|} + \frac{e^\theta}{1+\theta} \cancel{- \int e^\theta \ell\eta\left|1+\theta\right| d\theta} \cancel{- e^\theta \ell\eta\left|1+\theta\right|} \cancel{+ \int e^\theta \ell\eta\left|1+\theta\right| d\theta}$$

$$= \frac{e^\theta}{1+\theta}$$

4.- $\int e^{\tau g 3\theta} \sec^2 3\theta d\theta$

Solución.- Sea: $u = \tau g 3\theta, du = 3\sec^2 3\theta d\theta$

$$\int e^{\tau g 3\theta} \sec^2 3\theta d\theta = \frac{1}{3}\int e^u du = \frac{1}{3} e^u + c = \frac{e^{\tau g 3\theta}}{3} + c$$

5.- $\int \frac{x dx}{\sqrt[3]{ax+b}}$

Solución.- Sea: $ax + b = t^3 \Rightarrow x = \frac{t^3 - b}{a}, dx = \frac{3t^2}{a} dt$

$$\int \frac{xdx}{\sqrt[3]{ax+b}} = \int \frac{\left(\frac{t^3-b}{a}\right)\frac{3t^2}{a}dt}{t} = \int \frac{3t(t^3-b)}{a^2}dt = \frac{3}{a^2}\int (t^4-bt)dt = \frac{3}{a^2}\left(\frac{t^5}{5}-\frac{bt^2}{2}\right)+c$$

$$= \frac{3t^5}{5a^2} - \frac{3bt^2}{2a^2} + c = \frac{3(ax+b)^{5/3}}{5a^2} - \frac{3b(ax+b)^{2/3}}{2a^2} + c$$

$$= \frac{3(ax+b)\sqrt[3]{(ax+b)^2}}{5a^2} - \frac{3b\sqrt[3]{(ax+b)^2}}{2a^2} + c$$

6.- $\int \sqrt{\frac{x^2-1}{x+1}}dx$

Solución.-

$$\int \sqrt{\frac{x^2-1}{x+1}}dx = \int \sqrt{\frac{\cancel{(x+1)}(x-1)}{\cancel{x+1}}} = \int (x-1)^{1/2}dx = \frac{(x-1)^{3/2}}{3/2} + c = \frac{2(x-1)^{3/2}}{3} + c$$

$$= \frac{2(x-1)\sqrt{x-1}}{3} + c$$

7.- $\int \frac{dx}{(2-x)\sqrt{1-x}}$

Solución.- Sea: $1-x=t^2 \Rightarrow x = 1-t^2, dx = -2tdt$

$$\int \frac{dx}{(2-x)\sqrt{1-x}} = \int \frac{-2tdt}{\left[2-(1-t^2)\right]t} = -2\int \frac{dt}{1+t^2} = -2\operatorname{arc}\tau g t + c = -2\operatorname{arc}\tau g\sqrt{1-x} + c$$

8.- $\int e^{2-x}dx$

Solución.- Sea: $u = 2-x, du = -dx$

$$\int e^{2-x}dx = -\int e^u du = -e^u + c = -e^{2-x} + c$$

9.- $\int \frac{e^x dx}{ae^x - b}$

Solución.- Sea: $u = ae^x - b, du = ae^x dx$

$$\int \frac{e^x dx}{ae^x-b} = \frac{1}{a}\int \frac{du}{u} = \frac{1}{a}\ell\eta|u| + c = \frac{1}{a}\ell\eta\left|ae^x - b\right| + c$$

10.- $\int \frac{(t+1)dt}{t^2+2t-5}$

Solución.- Sea: $u = t^2+2t-5, du = 2(t+1)dt$

$$\int \frac{(t+1)dt}{t^2+2t-5} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ell\eta|u| + c = \frac{1}{2}\ell\eta\left|t^2+2t-5\right| + c$$

11.- $\int \sec\frac{\varphi}{2}d\varphi$

Solución.- Sea: $u = \sec\frac{\varphi}{2} + \tau g\frac{\varphi}{2}, du = \frac{1}{2}(\sec\frac{\varphi}{2}\tau g\frac{\varphi}{2} + \sec^2\frac{\varphi}{2})d\varphi$

$$\int \sec\frac{\varphi}{2}d\varphi = \int \frac{\sec\varphi/2\,(\sec\varphi/2+\tau g\,\varphi/2)}{\sec\varphi/2+\tau g\,\varphi/2}d\varphi = \int \frac{\sec^2\varphi/2+\sec\varphi/2\,\tau g\,\varphi/2}{\sec\varphi/2+\tau g\,\varphi/2}d\varphi$$

$$= 2\int \frac{du}{u} = 2\ell\eta|u| + c = 2\ell\eta\left|\sec\varphi/2+\tau g\,\varphi/2\right| + c$$

12.- $\int \tau g\theta d\theta$

Solución.- Sea: $u = \cos\theta, du = -\operatorname{sen}\theta d\theta$

$$\int \tau g\theta d\theta = \int \frac{\operatorname{sen}\theta}{\cos\theta}d\theta = -\int \frac{du}{u} = -\ell\eta|u| + c = -\ell\eta|\cos\theta| + c = -\ell\eta\left|\frac{1}{\sec\theta}\right| + c$$

$$= -\cancel{\ell\eta 1}^{0} + \ell\eta|\sec\theta| + c = \ell\eta|\sec\theta| + c$$

13.- $\int \frac{\eta^2}{a}\operatorname{sen}\frac{\eta}{b}d\eta$

Solución.-

Sea: $u = \frac{\eta^2}{a}$ $\quad dv = \operatorname{sen}\frac{\eta}{b}d\eta$

$du = \frac{2\eta d\eta}{a}$ $\quad v = -b\cos\frac{\eta}{b}$

$\int \frac{\eta^2}{a}\operatorname{sen}\frac{\eta}{b}d\eta = -\frac{a}{b}\eta^2\cos\frac{\eta}{b} + \frac{2b}{a}\int \eta\cos\frac{\eta}{b}d\eta(*)$, resolviendo por partes la segunda integral se tiene: $u = \eta$ $\quad dv = \cos\frac{\eta}{b}d\eta$

$du = d\eta$ $\quad v = b\operatorname{sen}\frac{\eta}{b}$

$$(*) = -\frac{a}{b}\eta^2\cos\frac{\eta}{b} + \frac{2b}{a}\left(b\eta\operatorname{sen}\frac{\eta}{b} - b\int \operatorname{sen}\frac{\eta}{b}d\eta\right)$$

$$= -\frac{a}{b}\eta^2\cos\frac{\eta}{b} + \frac{2b^2}{a}\eta\operatorname{sen}\frac{\eta}{b} + \frac{2b^3}{a}\cos\frac{\eta}{b} + c$$

14.- $\int \varphi\sec^2\varphi d\varphi$

Solución.-

Sea: $u = \varphi$ $\quad dv = \sec^2\varphi d\varphi$

$du = d\varphi$ $\quad v = \tau g\varphi$

$$\int \varphi\sec^2\varphi d\varphi = \varphi\tau g\varphi - \int \tau g\varphi d\varphi = \varphi\tau g\varphi - \ell\eta|\sec\varphi| + c$$

15.- $\int \frac{dx}{5^x}$

Solución.- Sea: $u = -x, du = -dx$

$$\int \frac{dx}{5^x} = \int 5^{-x}dx = -\int 5^u du = -\frac{5^u}{\ell\eta 5} + c = -\frac{5^{-x}}{\ell\eta 5} + c = -\frac{1}{5^x\ell\eta 5} + c$$

16.- $\int \sec^2(1-x)dx$

Solución.- Sea: $u = 1-x, du = -dx$

$$\int \sec^2(1-x)dx = -\int \sec^2 u du = -\tau gu + c = -\tau g(1-x) + c$$

17.- $\int \frac{xdx}{\sqrt{16-x^4}}$

Solución.- Sea: $u = x^2, du = 2xdx$

$$\int \frac{xdx}{\sqrt{16-x^4}} = \int \frac{xdx}{\sqrt{4^2-(x^2)^2}} = \frac{1}{2}\int \frac{2xdx}{\sqrt{4^2-(x^2)^2}} = \frac{1}{2}\int \frac{du}{\sqrt{4^2-u^2}} = \frac{1}{2}\operatorname{arcs} e\,\mathrm{n}\frac{u}{4} + c$$

$$= \frac{1}{2}\operatorname{arcs} e\,\mathrm{n}\frac{x^2}{4} + c$$

18.- $\int \frac{dy}{\sqrt{1+\sqrt{1+y}}}$

Solución.- Sea: $t = \left[1+(1+y)^{1/2}\right]^{1/2} \Rightarrow t^2 = 1+(1+y)^{1/2} \Rightarrow t^2 - 1 = (1+y)^{1/2}$

$$\Rightarrow (t^2-1)^2 = 1+y \Rightarrow y = (t^2-1)^2 - 1, dy = 4t(t^2-1)dt$$

$$\int \frac{dy}{\sqrt{1+\sqrt{1+y}}} = \int \frac{4\cancel{t}(t^2-1)dt}{\cancel{t}} = 4\int (t^2-1)dt = 4(\frac{t^3}{3} - t) + c = 4t(\frac{t^2}{3} - 1) + c$$

$$= 4\sqrt{1\sqrt{1+y}}\,(\frac{1+\sqrt{1+y}}{3} - 1) + c = \frac{4}{3}\sqrt{1\sqrt{1+y}}\,(\sqrt{1+y} - 2) + c$$

19.- $\int \frac{dx}{\sqrt{x+4}-\sqrt{x+3}}$

Solución.-

$$\int \frac{dx}{\sqrt{x+4}-\sqrt{x+3}} = \int \frac{(x+4)^{1/2}+(x+3)^{1/2}}{(x+4)-(x+3)}dx = \int \left[(x+4)^{1/2}+(x+3)^{1/2}\right]dx$$

$$\int (x+4)^{1/2} + \int (x+3)^{1/2} = \frac{(x+4)^{3/2}}{3/2} + \frac{(x+3)^{3/2}}{3/2} + c = \frac{2\sqrt{(x+4)^3}}{3} + \frac{2\sqrt{(x+3)^3}}{3} + c$$

$$= \frac{2}{3}\left(\sqrt{(x+4)^3} + \sqrt{(x+3)^3}\right) + c$$

20.- $\int \cos ec\theta d\theta$

Solución.- Sea: $u = \cos ec\theta + \mathrm{co}\,\tau g\theta, du = -(\cos ec\theta\,\mathrm{co}\,\tau g\theta + \cos ec^2\theta)d\theta$

$$\int \cos ec\theta d\theta = \int \frac{\cos ec\theta(\cos ec\theta + \mathrm{co}\,\tau g\theta)d\theta}{\cos ec\theta + \mathrm{co}\,\tau g\theta} = \int \frac{\cos ec^2\theta + \cos ec\theta\,\mathrm{co}\,\tau g\theta d\theta}{\cos ec\theta + \mathrm{co}\,\tau g\theta}$$

$$= -\int \frac{du}{u} = -\ell\eta|u| + c = -\ell\eta\left|(\cos ec\theta + \mathrm{co}\,\tau g\theta)\right| + c$$

21.- $\int t(1-t^2)^{1/2}dt$

Solución.- Sea: $u = 1-t^2, du = -2tdt$

$$\int t(1-t^2)^{1/2}dt = -\frac{1}{2}\int u^{1/2}du = -\frac{1}{\cancel{2}}\frac{u^{3/2}}{3/\cancel{2}} + c = -\frac{1}{3}u^{3/2} + c = -\frac{1}{3}(1-t^2)^{3/2} + c$$

22.- $\int t(1-t^2)^{1/2} \operatorname{arcsen} t\,dt$

Solución.-

Sea: $u = \operatorname{arcsen} t \quad dv = t(1-t^2)^{1/2}dt$

$du = \frac{dt}{\sqrt{1-t^2}} \quad v = -\frac{1}{3}(1-t^2)^{3/2}$

$$\int t(1-t^2)^{1/2} \operatorname{arcsen} t\,dt = -\frac{1}{3}(1-t^2)^{3/2}\operatorname{arcsen} t + \frac{1}{3}\int (1-t^2)\cancel{\sqrt{1-t^2}}\frac{dt}{\cancel{\sqrt{1-t^2}}}$$

$$= -\frac{(1-t^2)^{3/2}}{3}\operatorname{arcsen} t + \frac{1}{3}\int (1-t^2)dt = -\frac{(1-t^2)^{3/2}}{3}\operatorname{arcsen} t + \frac{1}{3}(t - \frac{t^3}{3}) + c$$

$$= -\frac{1}{3}\left[(1-t^2)^{3/2}\operatorname{arcsen} t - t + \frac{t^3}{3}\right] + c$$

23.- $\int \frac{1+\cos 2x}{\operatorname{sen}^2 2x}dx$

Solución.-

$$\int \frac{1+\cos 2x}{\operatorname{sen}^2 2x}dx = \int \frac{1+\cos 2x}{1-\cos^2 x}dx = \int \frac{dx}{1-\cos 2x} = \int \frac{dx}{2\left(\frac{1-\cos 2x}{2}\right)} = \frac{1}{2}\int \frac{dx}{\operatorname{sen}^2 x}$$

$$= \frac{1}{2}\int \operatorname{cosec}^2 x\,dx = -\frac{1}{2}\operatorname{co\tau g} x + c$$

24.- $\int \frac{x^2+1}{x^3-x}dx$

Solución.-

$$\int \frac{x^2+1}{x^3-x}dx = \int \frac{(x^2+1)dx}{x(x^2-1)} = \int \frac{(x^2+1)dx}{x(x+1)(x-1)} = \int \frac{Adx}{x} + \int \frac{Bdx}{(x+1)} + \int \frac{Cdx}{(x-1)} (*)$$

$$\frac{(x^2+1)}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x-1)} \Rightarrow (x^2+1) = A(x^2-1) + Bx(x-1) + Cx(x+1)$$

$x = 0 \Rightarrow 1 = -A \Rightarrow A = -1$

De donde: $x = -1 \Rightarrow 2 = B(-1)(-2) \Rightarrow B = 1$

$x = 1 \Rightarrow 2 = C(1)(2) \Rightarrow C = 1$

Entonces:

$$(*)\int \frac{(x^2+1)dx}{x(x+1)(x-1)} = -\int \frac{dx}{x} + \int \frac{dx}{(x+1)} + \int \frac{dx}{(x-1)} = -\ell\eta|x| + \ell\eta|x+1| + \ell\eta|x-1| + c$$

$$= \ell\eta\left|\frac{x^2-1}{x}\right| + c$$

25.- $\int \frac{e^x dx}{\sqrt{9-e^{2x}}}$

Solución.- Sea: $u = e^x, du = e^x dx$

$$\int \frac{e^x dx}{\sqrt{9-e^{2x}}} = \int \frac{e^x dx}{\sqrt{3^2-(e^x)^2}} = \int \frac{du}{\sqrt{3^2-u^2}} = \operatorname{arcsen}\frac{u}{3} + c = \operatorname{arcsen}\frac{e^x}{3} + c$$

26.- $\int \frac{dx}{(x-1)^3}$

Solución.-

$$\int \frac{dx}{(x-1)^3} = \int (x-1)^{-3} dx = -\frac{(x-1)^{-2}}{2} + c = -\frac{1}{(x-1)^2} + c$$

27.- $\int \frac{(3x+4)dx}{\sqrt{2x+x^2}}$

Solución.- Sea: $u = 2x+x^2, du = 2(1+x)dx$

$$\int \frac{(3x+4)dx}{\sqrt{2x+x^2}} = \int \frac{(3x+3)+1}{\sqrt{2x+x^2}}dx = 3\int \frac{(x+1)dx}{\sqrt{2x+x^2}} + \int \frac{dx}{\sqrt{2x+x^2}} = \frac{3}{2}\int \frac{du}{u^{1/2}} + \int \frac{dx}{\sqrt{2x+x^2}}$$

$$= \frac{3}{2}\int \frac{du}{u^{1/2}} + \int \frac{dx}{\sqrt{(x^2+2x+1)-1}} = \frac{3}{\cancel{2}} \frac{u^{1/2}}{1/\cancel{2}} + \int \frac{dx}{\sqrt{(x+1)^2-1}} = 3\sqrt{2x+x^2} + \int \frac{dx}{\sqrt{(x+1)^2-1}}$$

Sustituyendo por: $x+1 = \sec\theta, dx = \sec\theta\tau g\theta d\theta, \sqrt{(x+1)^2-1} = \tau g\theta$

$$= 3\sqrt{2x+x^2} + \int \frac{\sec\theta\,\cancel{\tau g\theta}}{\cancel{\tau g\theta}}d\theta = 3\sqrt{2x+x^2} + \int \sec\theta d\theta = 3\sqrt{2x+x^2} + \ell\eta\left|\sec\theta + \tau g\theta\right| + c$$

$$= 3\sqrt{2x+x^2} + \ell\eta\left|x+1+\sqrt{2x+x^2}\right| + c$$

28.- $\int \frac{ds}{\sqrt{4-s^2}}$

Solución.- Sea: $s = 2\operatorname{sen}\theta, ds = 2\cos\theta d\theta, \sqrt{4-s^2} = 2\cos\theta$

$$\int \frac{ds}{\sqrt{4-s^2}} = \int \frac{\cancel{2\cos\theta}\, d\theta}{\cancel{2\cos\theta}} = \int d\theta = \theta = \operatorname{arcsen} s/2 + c$$

29.- $\int \frac{dx}{x^2\sqrt{x^2+e}}$

Solución.- Sea: $x = \sqrt{e}\tau g\theta, dx = \sqrt{e}\sec^2\theta d\theta, \sqrt{x^2+e} = \sqrt{e}\sec\theta$

$$\int \frac{dx}{x^2\sqrt{x^2+e}} = \int \frac{\cancel{\sqrt{e}}\sec^{\cancel{2}}\theta d\theta}{e\tau g^2\,\cancel{\sqrt{e}\sec\theta}} = \frac{1}{e}\int \frac{\sec\theta d\theta}{\tau g^2} = \frac{1}{e}\int \frac{\frac{1}{\cancel{\cos\theta}}d\theta}{\frac{\operatorname{sen}^2\theta}{\cos^{\cancel{2}}\theta}} = \frac{1}{e}\int \frac{\cos\theta}{\operatorname{sen}^2\theta}(*)$$

Sea: $u = \operatorname{sen}\theta, du = \cos\theta d\theta$, luego:

$$(*) = \frac{1}{e}\int\frac{du}{u^2} = \frac{1}{e}\int u^{-2}du = \frac{1}{e}\frac{u^{-1}}{-1}+c = -\frac{1}{eu}+c = -\frac{1}{e\,\mathrm{sen}\,\theta}+c = -\frac{1}{e\dfrac{x}{\sqrt{x^2+e}}}+c$$

$$= -\frac{\sqrt{x^2+e}}{ex}+c$$

30.- $\int\frac{xdx}{\sqrt{1+x}}$

Solución.- Sea: $x+1=t^2 \Rightarrow x=t^2-1, dx=2tdt$

$$\int\frac{xdx}{\sqrt{1+x}} = \int\frac{(t^2-1)2\cancel{t}dt}{\cancel{t}} = 2\int(t^2-1)dt = 2(\frac{t^3}{3}-t)+c = 2t(\frac{t^2}{3}-1)+c$$

$$= 2\sqrt{x+1}(\frac{x+1}{3}-1)+c = 2\sqrt{x+1}(\frac{x-2}{3})+c$$

31.- $\int\frac{y^2dy}{\sqrt{y+1}}$

Solución.- Sea: $y+1=t^2 \Rightarrow y=t^2-1, dy=2tdt$

$$\int\frac{y^2dy}{\sqrt{y+1}} = \int\frac{(t^2-1)^2 2\cancel{t}dt}{\cancel{t}} = 2\int(t^2-1)^2dt = 2\int(t^4-2t^2+1)dt = 2\left(\frac{t^5}{5}-\frac{2t^3}{3}+t\right)+c$$

$$= 2t\left(\frac{t^4}{5}-\frac{2t^2}{3}+1\right)+c = 2\sqrt{y+1}\left(\frac{(\sqrt{y+1})^4}{5}-\frac{2(\sqrt{y+1})^2}{3}+1\right)+c$$

$$= 2\sqrt{y+1}\left(\frac{(y+1)^2}{5}-\frac{2y+2}{3}+1\right)+c = 2\sqrt{y+1}\left(\frac{y^2+2y+1}{5}-\frac{2y+2}{3}+1\right)+c$$

$$= 2\sqrt{y+1}\left(\frac{3y^2-4y+8}{15}\right)+c$$

32.- $\int\frac{y^3dy}{\sqrt{y^2-1}}$

Solución.- Sea: $u=y^2-1 \Rightarrow y^2=u+1, dy=2ydy$

$$\int\frac{y^3dy}{\sqrt{y^2-1}} = \int\frac{y^2ydy}{\sqrt{y^2-1}} = \frac{1}{2}\int\frac{(u+1)du}{u^{1/2}} = \frac{1}{2}\int(u^{1/2}+u^{-1/2})du = \frac{1}{\cancel{2}}\left(\frac{u^{3/2}}{3/\cancel{2}}+\frac{u^{1/2}}{1/\cancel{2}}\right)+c$$

$$= \frac{u^{3/2}}{3}+u^{1/2}+c = u^{1/2}(1/3\,u+1)+c = \sqrt{y^2-1}\left(\frac{y^2-1}{3}+1\right)+c = \sqrt{y^2-1}\left(\frac{y^2+2}{3}\right)+c$$

33.- $\int\frac{d\theta}{1+2\cos\theta}$

Solución.- Sea: $d\theta=\frac{2dz}{1+z^2}, \cos\theta=\frac{1-z^2}{1+z^2}, \theta=2\,\mathrm{arc}\,\tau gz$

$$\int \frac{d\theta}{1+2\cos\theta} = \int \frac{\frac{2dz}{1+z^2}}{1+\frac{2(1-z^2)}{1+z^2}} = \int \frac{2dz}{1+z^2+2(1-z^2)} = \int \frac{2dz}{1+z^2+2-2z^2} = \int \frac{2dz}{3-z^2}$$

$$= \int \frac{2dz}{3-z^2} = -2\int \frac{dz}{z^2-3} = -2\int \frac{dz}{z^2-(\sqrt{3})^2} = -\cancel{2}\,\frac{1}{\cancel{2}\sqrt{3}}\,\ell\eta\left|\frac{z-\sqrt{3}}{z+\sqrt{3}}\right| + c$$

$$= -\frac{1}{\sqrt{3}}\,\ell\eta\left|\frac{\tau g\,\theta/2-\sqrt{3}}{\tau g\,\theta/2+\sqrt{3}}\right| + c$$

34.- $\int \frac{t^4-t^3+4t^2-2t+1}{t^3+1}dt$

Solución.-

$$\int \frac{t^4-t^3+4t^2-2t+1}{t^3+1}dt = \int\left(t-1+\frac{3t^2-t+1}{t^3+t}\right)dt = \int tdt - \int dt + \int \frac{3t^2-t+1}{t^3+t}dt$$

$$= \frac{t^2}{2} - t + \int \frac{3t^2-t+1}{t^3+t}dt(*)$$

$$\frac{3t^2-t+1}{t(t^2+1)} = \frac{A}{t} + \frac{Bt+C}{(t^2+1)} \Rightarrow 3t^2-t+1 = A(t^2+1)+(Bt+C)t$$

De donde: $\left.\begin{array}{l} t=0 \Rightarrow 1=A \Rightarrow A=1 \\ t=1 \Rightarrow 3=2A+B+C \Rightarrow B+C=1 \\ t=-1 \Rightarrow 5=2A-(C-B) \Rightarrow B-C=3 \end{array}\right\} B=2, C=-1$

$$(*) = \frac{t^2}{2} - t + \int \frac{Adt}{t} + \int \frac{Bt+C}{t^2+1}dt = \frac{t^2}{2} - t + \int \frac{dt}{t} + \int \frac{2t-1}{t^2+1}dt$$

$$= \frac{t^2}{2} - t + \ell\eta|t| + \int \frac{2tdt}{t^2+1} - \int \frac{dt}{t^2+1} = \frac{t^2}{2} - t + \ell\eta|t| + \ell\eta\left|t^2+1\right| - \operatorname{arc}\tau g t + c$$

$$= \frac{t^2}{2} - t + \ell\eta\left|t(t^2+1)\right| - \operatorname{arc}\tau g t + c$$

35.- $\int \frac{d\varphi}{\ell\eta e}$

Solución.-

$$\int \frac{d\varphi}{\ell\eta e} = \int d\varphi = \varphi + c$$

36.- $\int x(10+8x^2)^9 dx$

Solución.- Sea: $u = 10+8x^2, du = 16xdx$

$$\int x(10+8x^2)^9 dx = \frac{1}{16}\int 16x(10+8x^2)^9 dx = \frac{1}{16}\int u^9 ddu = \frac{1}{16}\frac{u^{10}}{10} + c = \frac{u^{10}}{160} + c$$

$$= \frac{(10+8x^2)^{10}}{160} + c$$

37.- $\int \frac{dx}{\sqrt{(16+x^2)^3}}$

Solución.- Sea: $x = 4\tau g\theta, dx = 4\sec^2\theta d\theta$

$$\int \frac{dx}{\sqrt{(16+x^2)^3}} = \int \frac{\cancel{4\sec^2}\theta d\theta}{4^{\cancel{3}}\sec^{\cancel{3}}\theta} = \frac{1}{16}\int \frac{d\theta}{\sec\theta} = \frac{1}{16}\int \cos\theta d\theta = \frac{1}{16}\operatorname{sen}\theta + c = \frac{x}{16\sqrt{16+x^2}} + c$$

38.- $\int \frac{x^3dx}{\sqrt{x^2+4}}$

Solución.- Sea: $u = x^2+4 \Rightarrow x^2 = u-4, du = 2xdx$

$$\int \frac{x^3dx}{\sqrt{x^2+4}} = \int \frac{x^2xdx}{\sqrt{x^2+4}} = \frac{1}{2}\int \frac{(u-4)du}{u^{1/2}} = \frac{1}{2}\int (u^{1/2} - 4u^{-1/2})du = \frac{1}{2}\int u^{1/2}du - 2\int u^{-1/2}du$$

$$= \frac{1}{\cancel{2}}\frac{u^{3/2}}{3/\cancel{2}} - \frac{2u^{1/2}}{1/2} + c = \frac{u^{3/2}}{3} - 4u^{1/2} + c = u^{1/2}(\frac{u}{3} - 4) + c = \sqrt{x^2+4}(\frac{x^2+4}{3} - 4) + c$$

$$= \sqrt{x^2+4}(\frac{x^2-8}{3}) + c$$

39.- $\int \frac{x^3dx}{\sqrt{16-x^2}}$

Solución.- Sea: $u = 16-x^2 \Rightarrow x^2 = 16-u, du = -2xdx$

$$\int \frac{x^3dx}{\sqrt{16-x^2}} = \int \frac{x^2xdx}{\sqrt{16-x^2}} = -\frac{1}{2}\int \frac{(16-u)du}{u^{1/2}} = -\frac{1}{2}\int (16u^{-1/2} - u^{1/2})du$$

$$= -\frac{1}{\cancel{2}}\frac{16u^{1/2}}{1/\cancel{2}} + \frac{1}{\cancel{2}}\frac{u^{3/2}}{3/\cancel{2}} = -16u^{1/2} + \frac{u^{3/2}}{3} + c = -16u^{1/2} + \frac{\sqrt{u}u}{3} + c = \sqrt{u}(-16 + \frac{u}{3}) + c$$

$$= \sqrt{16-x^2}(-16 + \frac{16-x^2}{3}) + c = -\sqrt{16-x^2}(\frac{32+x^2}{3}) + c$$

40.- $\int a(x^2+1)^{1/2}dy$

Solución.-

$$\int a(x^2+1)^{1/2}dy = a(x^2+1)^{1/2}\int dy = a(x^2+1)^{1/2}y + c$$

41.- $\int \frac{dx}{(\sqrt{6-x^2})^3}$

Solución.- Sea: $x = \sqrt{6}\operatorname{sen}\theta, dx = \sqrt{6}\cos\theta d\theta, \sqrt{6-x^2} = \sqrt{6}\cos\theta$

$$\int \frac{dx}{(\sqrt{6-x^2})^3} = \int \frac{\cancel{\sqrt{6}}\,\cancel{\cos\theta}d\theta}{(\sqrt{6})^{\cancel{3}}\cos^{\cancel{3}}\theta} = \frac{1}{6}\int \frac{d\theta}{\cos^2\theta} = \frac{1}{6}\sec^2\theta d\theta = \frac{1}{6}\tau g\theta + c = \frac{1}{6}\frac{x}{\sqrt{6-x^2}} + c$$

42.- $\int \frac{dx}{x(3+\ell\eta x)}$

Solución.- Sea: $u = 3 + \ell\eta x, du = \frac{dx}{x}$

$$\int \frac{dx}{x(3+\ell\eta x)} = \int \frac{du}{u} = \ell\eta|u| + c = \ell\eta|3+\ell\eta x| + c$$

43.- $\int \frac{e^x}{16+e^{2x}}dx$

Solución.- Sea: $u = e^x, du = e^x dx$

$$\int \frac{e^x}{16+e^{2x}}dx = \int \frac{du}{4^2+u^2} = \frac{1}{4}\operatorname{arc}\tau g\frac{u}{4} + c = \frac{1}{4}\operatorname{arc}\tau g\frac{e^x}{4} + c$$

44.- $\int \cos\sqrt{1-x}dx$

Solución.- Sea: $1-x = t^2 \Rightarrow x = 1-t^2, dx = -2tdt$

$\int \cos\sqrt{1-x}dx = -2\int \cos t dt (*)$, integrando por partes se tiene:

Sea: $u = t \qquad dv = \cos t dt$

$du = dt \qquad v = \operatorname{sen} t$

$$(*) = -2\left(t\operatorname{sen}t - \int \operatorname{sen}tdt\right) = -2t\operatorname{sen}t + 2\int\operatorname{sen}tdt = -2t\operatorname{sen}t - 2\cos t + c$$

$$= -2\sqrt{1-x}\operatorname{sen}\sqrt{1-x} - 2\cos\sqrt{1-x} + c$$

45.- $\int \frac{x^3dx}{\sqrt{x-1}}$

Solución.- Sea: $x-1 = t^2 \Rightarrow x = t^2+1, dx = 2tdt$

$$\int \frac{x^3dx}{\sqrt{x-1}} = \int \frac{(t^2+1)^3 2\cancel{t}dt}{\cancel{t}} = 2\int (t^6+3t^4+3t^2+1)dt = \frac{2t^7}{7} + \frac{6t^5}{5} + 2t^3 + 2t + c$$

$$= t(\frac{2t^6}{7} + \frac{6t^4}{5} + 2t^2 + 2) + c = \sqrt{x-1}\left[\frac{2(x-1)^3}{7} + \frac{6(x-1)^2}{5} + 2(x-1) + 2\right] + c$$

$$= 2\sqrt{x-1}\left[\frac{(x-1)^3}{7} + \frac{3(x-1)^2}{5} + x\right] + c$$

46.- $\int \frac{2y^5-7y^4+7y^3-19y^2+7y-6}{(y-1)^2(y^2+1)^2}dy$

Solución.-

$$\int \frac{2y^5-7y^4+7y^3-19y^2+7y-6}{(y-1)^2(y^2+1)^2}dy \; (*)$$

$$\frac{2y^5-7y^4+7y^3-19y^2+7y-6}{(y-1)^2(y^2+1)^2} = \frac{A}{y-1} + \frac{B}{(y-1)^2} + \frac{Cy+D}{(y^2+1)} + \frac{Ey+F}{(y^2+1)^2}$$

$$2y^5-7y^4+7y^3-19y^2+7y-6 = A(y-1)(y^2+1)^2 + B(y^2+1)^2$$

$\Rightarrow +(Cy+D)(y-1)^2(y^2+1) + (Ey+F)(y-1)^2$, luego:

$$2y^5-7y^4+7y^3-19y^2+7y-6 = (A+C)y^5 + (-A+B-2C+D)y^4$$

$$\Rightarrow +(2A+2C-2D+E)y^3 + (-2A+2B-2C+2D-2E+F)y^2$$

$\Rightarrow +(A+C-2D+E-2F)y+(-A+B+D+F)$, Igualando coeficientes se tiene:

$$\left(\begin{array}{cccccccc} A & & +C & & & & = & 2 \\ -A & +\ B & -2C & +D & & & = & -7 \\ 2A & & +2C & -2D & +E & & = & 7 \\ -2A & +2B & -2C & +2D & -2E & +F & = & -19 \\ A & & +C & -2D & +E & -2F & = & 7 \\ -A & +\ B & & +D & & +F & = & -6 \end{array}\right) \Rightarrow \begin{array}{l} A=1, B=-4, C=1 \\ D=0, E=3, F=-1 \end{array}$$

$$(*)\int \frac{2y^5-7y^4+7y^3-19y^2+7y-6}{(y-1)^2(y^2+1)^2}dy=\int \frac{dy}{y-1}-4\int \frac{dy}{(y-1)^2}+\int \frac{ydy}{(y^2+1)}+\int \frac{(3y-1)dy}{(y^2+1)^2}$$

$$=\ell\eta\left|y-1\right|+\frac{4}{y-1}+\frac{1}{2}\ell\eta\left|y^2+1\right|+3\int \frac{ydy}{(y^2+1)}-\int \frac{dy}{(y^2+1)^2}$$

$$=\ell\eta\left|y-1\right|+\frac{4}{y-1}+\ell\eta\left|\sqrt{y^2+1}\right|-\frac{3}{2}\ell\eta\left|y^2+1\right|-\left[\frac{1}{2}\frac{y}{y^2+1}+\frac{1}{2}\text{arc}\,\tau gy\right]+c$$

$$=\ell\eta\left|(y-1)\sqrt{y^2+1}\right|+\frac{4}{y-1}-\frac{3}{2}\ell\eta\left|y^2+1\right|-\frac{y}{2(y^2+1)}-\frac{1}{2}\text{arc}\,\tau gy+c$$

$$=\ell\eta\left|\frac{(y-1)}{\sqrt{y^2+1}}\right|+\frac{4}{y-1}-\frac{y}{2(y^2+1)}-\frac{1}{2}\text{arc}\,\tau gy+c$$

47.- $\int \operatorname{sen}\sqrt{x+1}dx$

Solución.- Sea: $x+1=t^2 \Rightarrow x=t^2-1, dx=2tdt$

$\int \operatorname{sen}\sqrt{x+1}dx=2\int (\operatorname{sen}t)tdt(*)$, trabajando por partes

Sea: $\begin{array}{ll} u=t & dv=\operatorname{sen}tdt \\ du=dt & v=-\cos t \end{array}$

$$(*)2\int (\operatorname{sen}t)tdt=2\left(-t\cos t+\int \cos tdt\right)=-2t\cos t+2\operatorname{sen}t+c$$

$$=-2\sqrt{x+1}\cos\sqrt{x+1}+2\operatorname{sen}\sqrt{x+1}+c$$

48.- $\int \frac{9x^2+7x-6}{x^3-x}dx$

Solución.-

$$\int \frac{9x^2+7x-6}{x^3-x}dx=\int \frac{9x^2+7x-6}{x(x+1)(x-1)}dx=\int \frac{Adx}{x}+\int \frac{Bdx}{x+1}+\int \frac{Cdx}{x-1}(*)$$

$$\frac{9x^2+7x-6}{x^3-x}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x-1}\Rightarrow 9x^2+7x-6=A(x+1)(x-1)+Bx(x-1)+Cx(x+1)$$

De donde: $\begin{cases} x=0 \Rightarrow -6=-A \Rightarrow A=6 \\ x=1 \Rightarrow 10=2C \Rightarrow C=5 \\ x=-1 \Rightarrow -4=2B \Rightarrow B=-2 \end{cases}$

$$(*)=6\int \frac{dx}{x}-2\int \frac{dx}{x+1}+5\int \frac{dx}{x-1}=6\ell\eta\left|x\right|-2\ell\eta\left|x+1\right|+5\ell\eta\left|x-1\right|+c$$

$$= \ell\eta\left|x^6\right| - \ell\eta\left|(x+1)^2\right| + \ell\eta\left|(x-1)^5\right| + c = \ell\eta\left|\frac{x^6(x-1)^5}{(x+1)^2}\right| + c$$

49.- $\int \frac{5w^3-5w^2+2w-1}{w^4+w^2}dw$

Solución.-

$$\int \frac{5w^3-5w^2+2w-1}{w^4+w^2}dw = \int \frac{5w^3-5w^2+2w-1}{w^2(w^2+1)}dw(*)$$

$$\frac{5w^3-5w^2+2w-1}{w^2(w^2+1)} = \frac{Aw+B}{w^2} + \frac{Cw+D}{w^2+1}$$

$$5w^3-5w^2+2w-1 = (Aw+B)(w^2+1) + (Cw+D)w^2$$

$$\Rightarrow Aw^3 + Aw + Bw^2 + B + Cw^3 + Dw^2 \Rightarrow (A+C)w^3 + (B+D)w^2 + Aw + B$$

Igualando coeficientes se tiene:

$$\begin{pmatrix} A & & +C & & = & 5 \\ & B & & +D & = & -5 \\ A & & & & = & 2 \\ & B & & & = & -1 \end{pmatrix} \Rightarrow A = 2, B = -1, C = 3, D = -4$$

$$(*)\ \int \frac{Aw+B}{w^2}dw + \int \frac{Cw+D}{w^2+1}dw = \int \frac{2w-1}{w^2}dw + \int \frac{3w-4}{w^2+1}dw$$

$$= \int \frac{2wdw}{w^2} - \int w^{-2}dw + \frac{3}{2}\int \frac{2wdw}{w^2+1} - 4\int \frac{dw}{w^2+1}$$

$$= \ell\eta\left|w^2\right| + \frac{1}{w} + \ell\eta\left|\sqrt{(w^2+1)^3}\right| - 4\operatorname{arc}\tau gw + c = \ell\eta\left|w^2\sqrt{(w^2+1)^3}\right| + \frac{1}{w} - 4\operatorname{arc}\tau gw + c$$

50.- $\int \frac{3dx}{1+2x}$

Solución.- Sea: $u = 1+2x, du = 2dx$

$$\int \frac{3dx}{1+2x} = 3\int \frac{dx}{1+2x} = \frac{3}{2}\int \frac{du}{u} = \frac{3}{2}\ell\eta|u| + c = \frac{3}{2}\ell\eta|1+2x| + c = \ell\eta\left|\sqrt{(1+2x)^3}\right| + c$$

51.- $\int \frac{(1-x)^2 dx}{x}$

Solución.-

$$\int \frac{(1-x)^2 dx}{x} = \int \frac{1-2x+x^2 dx}{x} = \int \frac{dx}{x} - 2\int dx + \int xdx = \ell\eta|x| - 2x + \frac{x^2}{2} + c$$

52.- $\int \frac{xe^{-2x^2}}{2}dx$

Solución.- Sea: $u = -2x^2, du = -4xdx$

$$\int \frac{xe^{-2x^2}}{2}dx = \frac{1}{2}\int xe^{-2x^2}dx = -\frac{1}{8}\int e^u du = -\frac{1}{8}e^u + c = -\frac{1}{8}e^{-2x^2} + c$$

53.- $\int e^{2t}\cos(e^t)dt$

Solución.- Sea: $w = e^t, dw = e^t dt$

$\int e^t \cos(e^t) e^t dt = \int w \cos w dw (*)$, trabajando por partes

Sea: $u = w \quad dv = \cos w dw$

$du = dw \quad v = \operatorname{sen} w$

$$(*)\int w\cos w dw = w \operatorname{sen} w - \int \operatorname{sen} w dw = w \operatorname{sen} w + \cos w + c = e^t \operatorname{sen}(e^t) + \cos(e^t) + c$$

54.- $\int \sqrt{x}(x^{3/2} - 4)^3 dx$

Solución.- Sea: $u = x^{3/2} - 4, du = \frac{3}{2}\sqrt{x}dx$

$$\int \sqrt{x}(x^{3/2} - 4)^3 dx = \frac{2}{3}\int u^3 du = \frac{2}{3}\frac{u^4}{4} + c = \frac{1}{6}u^4 + c = \frac{(x^{3/2} - 4)^4}{6} + c$$

55.- $\int \frac{\operatorname{sen} x e^{\sec x}}{\cos^2 x} dx = \int \frac{\operatorname{sen} x}{\cos x}\frac{1}{\cos} e^{\sec x} dx = \int \tau g x \sec x e^{\sec x} dx (*)$

Solución.- Sea: $u = \sec x, du = \sec x \tau g x dx$

$$(*) = \int e^u du = e^u + c = e^{\sec x} + c$$

56.- $\int \frac{ds}{s^{1/3}(1 + s^{2/3})}$

Solución.- Sea: $t = s^{1/3} \Rightarrow s = t^3, ds = 3t^2 dt$

$$\int \frac{ds}{s^{1/3}(1 + s^{2/3})} = \int \frac{3t^{\cancel{2}} dt}{\cancel{t}(1 + t^2)} = \int \frac{3tdt}{(1 + t^2)} = 3\int \frac{tdt}{(1 + t^2)} = \frac{3}{2}\ell\eta\left|1 + t^2\right| + c$$

57.- $\int \frac{1}{z^3}\left(\frac{1 - z^2}{z^2}\right)^{10} dz$

Solución.- Sea: $u = \frac{1 - z^2}{z^2}, du = \frac{-2dz}{z^3}$

$$\int \frac{1}{z^3}\left(\frac{1 - z^2}{z^2}\right)^{10} dz = -\frac{1}{2}\int u^{10} du = -\frac{1}{2}\frac{u^{11}}{11} + c = -\frac{u^{11}}{22} + c = -\frac{1}{22}\left(\frac{1 - z^2}{z^2}\right)^{11} + c$$

58.- $\int \frac{x\ell\eta(1 + x^2)}{1 + x^2} dx$

Solución.- Sea: $u = \ell\eta(1 + x^2), du = \frac{2xdx}{1 + x^2}$

$$\int \frac{x\ell\eta(1 + x^2)}{1 + x^2} dx = \frac{1}{2}\int u du = \frac{1}{2}\frac{u^2}{2} + c = \frac{u^2}{4} + c = \frac{\left[\ell\eta(1 + x^2)\right]^2}{4} + c$$

59.- $\int \frac{\operatorname{co}\tau g x dx}{\ell\eta\left|\operatorname{sen} x\right|}$

Solución.- Sea: $u = \ell\eta\left|\operatorname{sen} x\right|, du = \operatorname{co}\tau g x dx$

$$\int \frac{\operatorname{co}\tau g x dx}{\ell\eta\left|\operatorname{sen} x\right|} = \int \frac{du}{u} = \ell\eta\left|u\right| + c = \ell\eta\left|\ell\eta\left|\operatorname{sen} x\right|\right| + c$$

60.- $\int \frac{ax^2-bx+c}{ax^2+bx-c}dx$

Solución.-

$$\int \frac{ax^2-bx+c}{ax^2+bx-c}dx=\frac{ax^2-bx+c}{ax^2+bx-c}\int dt=\frac{ax^2-bx+c}{ax^2+bx-c}t+c$$

61.- $\int \frac{dx}{\cos^2 5x}$

Solución.- Sea: $u=5x, du=5dx$

$$\int \frac{dx}{\cos^2 5x}=\int \sec^2 5xdx=\frac{1}{5}\int \sec^2 udu=\frac{1}{5}\tau gu+c=\frac{1}{5}\tau g5x+c$$

62.- $\int \frac{dx}{12-7x}$

Solución.- Sea: $u=12-7x, du=-7dx$

$$\int \frac{dx}{12-7x}=-\frac{1}{7}\int \frac{du}{u}=-\frac{1}{7}\ell\eta|u|+c=-\frac{1}{7}\ell\eta|12-7x|+c$$

63.- $\int \tau g16xdx$

Solución.- Sea: $u=\cos(16x), du=-16\operatorname{sen}(16x)dx$

$$\int \tau g16xdx=\int \frac{\operatorname{sen}(16x)}{\cos(16x)}dx=-\frac{1}{16}\int \frac{du}{u}=-\frac{1}{16}\ell\eta|u|+c=-\frac{1}{16}\ell\eta|\cos(16x)|+c$$

64.- $\int \tau g4\theta\sec^2 4\theta d\theta$

Solución.- Sea: $u=\tau g4\theta, du=4\sec^2 4\theta d\theta$

$$\int \tau g4\theta\sec^2 4\theta d\theta=\frac{1}{4}\int udu=\frac{1}{4}\frac{u^2}{2}+c=\frac{u^2}{8}+c=\frac{\tau g^2 4\theta}{8}+c$$

65.- $\int \frac{xdx}{\sqrt{x-5}}$

Solución.- Sea: $u=x-5\Rightarrow x=u+5, du=dx$

$$\int \frac{xdx}{\sqrt{x-5}}=\int \frac{u+5}{u^{1/2}}du=\int u^{1/2}du+5\int u^{-1/2}du=\frac{u^{3/2}}{3/2}+5\frac{u^{1/2}}{1/2}+c=\frac{2u^{3/2}}{3}+10u^{1/2}+c$$

$$=\frac{2}{3}u\sqrt{u}+10\sqrt{u}+c=\frac{2}{3}(x-5)\sqrt{x-5}+10\sqrt{x-5}+c=2\sqrt{x-5}\left(\frac{x+10}{3}\right)+c$$

66.- $\int \frac{7t-2}{\sqrt{7-2t^2}}dt$

Solución.-

$$\int \frac{7t-2}{\sqrt{7-2t^2}}dt=\int \frac{7tdt}{\sqrt{7-2t^2}}-\int \frac{2dt}{\sqrt{7-2t^2}}=-\frac{7}{4}\int \frac{-4tdt}{\sqrt{7-2t^2}}-\sqrt{2}\int \frac{dt}{\sqrt{7/2-t^2}}$$

$$=-\frac{7}{2}\sqrt{7-2t^2}-\sqrt{2}\operatorname{arcsen}\sqrt{2/7}\,t+c$$

67.- $\int (1+x)\cos\sqrt{x}dx$

Solución.- Sea: $\sqrt{x}=t \Rightarrow x=t^2, dx=2tdt$

$$\int(1+x)\cos\sqrt{x}dx=\int(1+t^2)(\cos t)2tdt=2\int(t+t^3)(\cos t)dt=2\int t\cos tdt+2\int t^3\cos tdt\ (*)$$

Trabajando por partes: $\int t^3\cos tdt$

Sea: $u=t^3 \qquad dv=\cos tdt$
$du=3t^2dt \qquad v=\operatorname{sen} t$

$$\int t^3\cos tdt=t^3\operatorname{sen} t-3\int t^2\operatorname{sen} tdt$$

Trabajando por partes: $\int t^2\operatorname{sen} tdt$

Sea: $u=t^2 \qquad dv=\operatorname{sen} tdt$
$du=2tdt \qquad v=-\cos t$

$$\int t^2\operatorname{sen} tdt=-t^2\cos t+2\int t\cos tdt$$

Trabajando por partes: $\int t\cos tdt$

Sea: $u=t \qquad dv=\cos tdt$
$du=dt \qquad v=\operatorname{sen} t$

$$\int t\cos tdt=t\operatorname{sen} t-\int\operatorname{sen} tdt=t\operatorname{sen} t+\cos t+c_1$$

$$(*)\ 2\int t\cos tdt+2\int t^3\cos tdt=2\int t\cos tdt+2\left(t^3\operatorname{sen} t-3\int t^2\operatorname{sen} tdt\right)$$

$$=2\int t\cos tdt+2t^3\operatorname{sen} t-6\int t^2\operatorname{sen} tdt=2\int t\cos tdt+2t^3\operatorname{sen} t-6\left(-t^2\cos t+2\int t\cos tdt\right)$$

$$=2\int t\cos tdt+2t^3\operatorname{sen} t+6t^2\cos t-12\int t\cos tdt=2t^3\operatorname{sen} t+6t^2\cos t-10\int t\cos tdt$$

$$=2t^3\operatorname{sen} t+6t^2\cos t-10(t\operatorname{sen} t+\cos t)+c$$

$$=2t^3\operatorname{sen} t+6t^2\cos t-10t\operatorname{sen} t-10\cos t+c$$

$$=2\sqrt{x^3}\operatorname{sen}\sqrt{x}+6x\cos\sqrt{x}-10\sqrt{x}\operatorname{sen}\sqrt{x}-10\cos\sqrt{x}+c$$

68.- $\int\frac{dx}{x(\sqrt{1+x}-1)}$

Solución.- Sea: $(1+x)^{1/2}=t \Rightarrow 1+x=t^2 \Rightarrow x=t^2-1, dx=2tdt$

$$\int\frac{dx}{x(\sqrt{1+x}-1)}=\int\frac{2tdt}{(t^2-1)(t-1)}(*)$$

$$\frac{t}{(t+1)(t^2-1)}=\frac{A}{t+1}+\frac{B}{t-1}+\frac{C}{(t-1)^2}\Rightarrow t=A(t-1)^2+B(t^2-1)+C(t+1)$$

De donde: $\begin{cases} t=1\Rightarrow 1=2C\Rightarrow C=1/2 \\ t=-1\Rightarrow -1=4A\Rightarrow A=-1/4 \\ t=0\Rightarrow 0=A-B+C\Rightarrow B=1/4 \end{cases}$

$$(*)=2\left[\int\frac{Adt}{t+1}+\int\frac{Bdt}{t-1}+\int\frac{Cdt}{(t-1)^2}\right]=2\left[-\frac{1}{4}\int\frac{dt}{t+1}+\frac{1}{4}\int\frac{dt}{t-1}+\frac{1}{2}\int\frac{dt}{(t-1)^2}\right]$$

$$= -\frac{1}{2}\int\frac{dt}{t+1}+\frac{1}{2}\int\frac{dt}{t-1}+\int\frac{dt}{(t-1)^2} = -\frac{1}{2}\ell\eta\left|t+1\right|+\frac{1}{2}\ell\eta\left|t-1\right|-\frac{1}{t-1}+c$$

$$= \frac{1}{2}\ell\eta\left|\frac{t-1}{t+1}\right|-\frac{1}{t-1}+c = \frac{1}{2}\ell\eta\left|\frac{\sqrt{1+x}-1}{\sqrt{1+x}+1}\right|-\frac{1}{\sqrt{1+x}-1}+c$$

69.- $\int\frac{dx}{\mathrm{co}\,\tau g 6x}$

Solución.- Sea: $u=\cos 6x, du=-6\,\mathrm{sen}\,6x dx$

$$\int\frac{dx}{\mathrm{co}\,\tau g6x}=\int\tau g6xdx=\int\frac{\mathrm{sen}\,6x}{\cos 6x}dx=-\frac{1}{6}\int\frac{du}{u}=-\frac{1}{6}\ell\eta\left|u\right|+c=-\frac{1}{6}\ell\eta\left|\cos 6x\right|+c$$

70.- $\int\mathrm{co}\,\tau g(2x-4)dx$

Solución.- Sea: $u=\mathrm{sen}(2x-4), du=2\cos(2x-4)dx$

$$\int\mathrm{co}\,\tau g(2x-4)dx=\int\frac{\cos(2x-4)}{\mathrm{sen}(2x-4)}dx=\frac{1}{2}\int\frac{du}{u}=\frac{1}{2}\ell\eta\left|u\right|+c=\frac{1}{2}\ell\eta\left|(2x-4)\right|+c$$

71.- $\int(e^t-e^{-2t})^2dt$

Solución.-

$$\int(e^t-e^{-2t})^2dt=\int(e^{2t}-2e^{t-2t}+e^{-4t})dt=\int e^{2t}dt-2\int e^{-t}dt+\int e^{-4t}dt$$

$$=\frac{1}{2}e^{2t}+2e^{-t}-\frac{1}{2}e^{-4t}+c$$

72.- $\int\frac{(x+1)dx}{(x+2)^2(x+3)}$

Solución.-

$$\int\frac{(x+1)dx}{(x+2)^2(x+3)}\Rightarrow\frac{(x+1)}{(x+2)^2(x+3)}=\frac{A}{x+2}+\frac{B}{(x+2)^2}+\frac{C}{x+3}\ (*)$$

$$\Rightarrow x+1=A(x+2)(x+3)+B(x+3)+C(x+2)^2$$

De donde: $\begin{cases}x=-2\Rightarrow -1=B\Rightarrow B=-1\\ x=-3\Rightarrow -2=C\Rightarrow C=-2\\ x=0\Rightarrow 1=6A+3B+4C\Rightarrow A=2\end{cases}$

$$(*)\ \int\frac{Adx}{x+2}+\int\frac{Bdx}{(x+2)^2}+\int\frac{Cdx}{x+3}=2\int\frac{dx}{x+2}-\int\frac{dx}{(x+2)^2}-2\int\frac{dx}{x+3}$$

$$=2\ell\eta\left|x+2\right|+\frac{1}{x+2}-2\ell\eta\left|x+3\right|+c=\ell\eta\left|\frac{x+2}{x+3}\right|^3+\frac{1}{x+2}+c$$

73.- $\int(\mathrm{co}\,\tau g e^x)e^xdx$

Solución.- Sea: $u=\mathrm{sen}\,e^x, du=(\cos e^x)e^xdx$

$$\int(\mathrm{co}\,\tau ge^x)e^xdx=\int\frac{(\cos e^x)e^xdx}{\mathrm{sen}\,e^x}=\int\frac{du}{u}=\ell\eta\left|u\right|+c=\ell\eta\left|\mathrm{sen}\,e^x\right|+c$$

74.- $\int\frac{\mathrm{sen}\,\theta+\theta}{\cos\theta+1}d\theta$

Solución.-

$$\int \frac{\operatorname{sen}\theta+\theta}{\cos\theta+1}d\theta=\int\frac{\operatorname{sen}\theta d\theta}{\cos\theta+1}+\int\frac{\theta d\theta}{\cos\theta+1}=-\int\frac{-\operatorname{sen}\theta d\theta}{\cos\theta+1}+\int\frac{\theta(\cos\theta-1)d\theta}{\cos^2\theta+1}$$

$$=-\ell\eta\left|\cos\theta+1\right|-\int\frac{\theta\cos\theta d\theta}{\operatorname{sen}^2\theta}+\int\frac{\theta d\theta}{\operatorname{sen}^2\theta}$$

$$=-\ell\eta\left|\cos\theta+1\right|-\int\theta\operatorname{co}\tau g\theta\cos ec\theta d\theta+\int\theta\cos ec^2\theta d\theta\ (*)$$

Trabajando por partes: $\int\theta\operatorname{co}\tau g\theta\cos ec\theta d\theta$

Sea: $u=\theta$ $\quad dv=\operatorname{co}\tau g\theta\cos ec\theta d\theta$

$du=d\theta$ $\quad v=-\cos ec\theta$

$$\int\theta\operatorname{co}\tau g\theta\cos ec\theta d\theta=-\theta\cos ec\theta+\int\cos ec\theta d\theta=-\theta\cos ec\theta-\ell\eta\left|\cos ec\theta-\operatorname{co}\tau g\theta\right|+c_1$$

Trabajando por partes: $\int\theta\cos ec^2\theta d\theta$

Sea: $u=\theta$ $\quad dv=\cos ec^2\theta d\theta$

$du=d\theta$ $\quad v=-t\operatorname{co}\tau g\theta$

$$\int\theta\cos ec^2\theta d\theta=-\theta\operatorname{co}\tau g\theta+\int\operatorname{co}\tau g\theta d\theta=-\theta\operatorname{co}\tau g\theta+\ell\eta\left|\operatorname{sen}\theta\right|+c_2$$

$$(*)=-\ell\eta\left|\cos\theta+1\right|+\theta\cos ec\theta+\ell\eta\left|\cos ec\theta-\operatorname{co}\tau g\theta\right|-\theta\operatorname{co}\tau g\theta+\ell\eta\left|\operatorname{sen}\theta\right|+c$$

$$=\ell\eta\left|\frac{(\cos ec\theta-\operatorname{co}\tau g\theta)\operatorname{sen}\theta}{\cos\theta+1}\right|+\theta(\cos ec\theta-\operatorname{co}\tau g\theta)+c$$

$$=\ell\eta\left|\frac{1-\cos\theta}{1+\cos\theta}\right|+\theta\left(\frac{1-\cos\theta}{\operatorname{sen}\theta}\right)+c$$

75.- $\int\frac{\operatorname{arc}\tau gxdx}{(1+x^2)^{3/2}}$

Solución.- Sea: $x=\tau g\theta\Rightarrow\theta=\operatorname{arc}\tau gx, dx=\sec^2\theta d\theta, \sqrt{1+x^2}=\sec\theta$

$$\int\frac{\operatorname{arc}\tau gxdx}{(1+x^2)^{3/2}}=\int\frac{\theta\cancel{\sec^2\theta}d\theta}{\sec^{\cancel{3}}\theta}=\int\frac{\theta d\theta}{\sec\theta}=\int\theta\cos\theta d\theta(*)$$, trabajando por partes

Sea: $u=\theta$ $\quad dv=\cos\theta d\theta$

$du=d\theta$ $\quad v=\operatorname{sen}\theta$

$$=\theta\operatorname{sen}\theta-\int\operatorname{sen}\theta d\theta=\theta\operatorname{sen}\theta+\cos\theta+c=(\operatorname{arc}\tau gx)\frac{x}{\sqrt{1+x^2}}+\frac{1}{\sqrt{1+x^2}}+c$$

$$=\frac{1}{\sqrt{1+x^2}}\left(x\operatorname{arc}\tau gx+1\right)+c$$

76.- $\int x\operatorname{co}\tau g(x^2/5)dx$

Solución.- Sea: $u=\operatorname{sen}\frac{x^2}{5}, du=\frac{2}{5}x\cos\frac{x^2}{5}dx$

$$\int x\cot g\left(\frac{x^2}{5}\right)dx=\int\frac{x\cos\frac{x^2}{5}}{\operatorname{sen}\frac{x^2}{5}}dx=\frac{5}{2}\int\frac{du}{u}=\frac{5}{2}\ell\eta|u|+c=\frac{5}{2}\ell\eta\left|\operatorname{sen}\frac{x^2}{5}\right|+c$$

77.- $\int x\sqrt{4x^2-2}dx$

Solución.- Sea: $u=4x^2-2, dx=8xdx$

$$\int x\sqrt{4x^2-2}dx=\frac{1}{8}\int u^{1/2}du=\frac{1}{8}\frac{u^{3/2}}{3/2}+c=\frac{u^{3/2}}{12}+c=\frac{\sqrt{(4x^2-2)^3}}{12}+c$$

78.- $\int\frac{(x^2+9)^{1/2}dx}{x^4}$

Solución.- Sea: $x=3\tau g\theta, dx=3\sec^2\theta, \sqrt{x^2+9}=3\sec\theta$

$$\int\frac{(x^2+9)^{1/2}dx}{x^4}=\int\frac{3\sec\theta 3\sec^2\theta d\theta}{3^4\tau g^4\theta}=\frac{1}{9}\int\frac{\sec^3\theta d\theta}{\tau g^4\theta}=\frac{1}{9}\int\frac{\frac{1}{\cos^3\theta}d\theta}{\frac{\operatorname{sen}^4\theta}{\cos^4\theta}}=\frac{1}{9}\int\frac{\cos\theta d\theta}{\operatorname{sen}^4\theta}$$

$$=\frac{1}{9}\left(-\frac{1}{3}\frac{1}{\operatorname{sen}^3\theta}\right)+c=-\frac{1}{27\operatorname{sen}^3\theta}+c=-\frac{\cos ec^3\theta}{27}+c$$

$$=-\frac{1}{27}\left(\frac{\sqrt{x^2+9}}{x}\right)^3+c=-\frac{x^2+9}{27x^3}\sqrt{x^2+9}+c$$

79.- $\int x^2\operatorname{sen}^5x^3\cos x^3dx$

Solución.- Sea: $u=\operatorname{sen}x^3, du=3x^2\cos x^3dx$

$$\int x^2\operatorname{sen}^5x^3\cos x^3dx=\frac{1}{3}\int u^5du=\frac{1}{3}\frac{u^6}{6}+c=\frac{u^6}{18}+c=\frac{\operatorname{sen}^6x^3}{18}+c$$

80.- $\int\frac{xdx}{\sqrt{5x^2+7}}$

Solución.- Sea: $u=5x^2+7, du=10xdx$

$$\int\frac{xdx}{\sqrt{5x^2+7}}=\frac{1}{10}\int\frac{du}{u^{1/2}}=\frac{1}{10}\frac{u^{1/2}}{1/2}+c=\frac{u^{1/2}}{5}+c=\frac{(5x^2+7)^{1/2}}{5}+c=\frac{\sqrt{5x^2+7}}{5}+c$$

81.- $\int\frac{x^3dx}{x^2-x-6}$

Solución.-

$$\int\frac{x^3dx}{x^2-x-6}=\int\left(x+1+\frac{7x+6}{x^2-x-6}\right)dx=\int xdx+\int dx+\int\frac{(7x+6)dx}{(x-3)(x+2)}$$

$$=\frac{x^2}{2}+x+\int\frac{(7x+6)dx}{(x-3)(x+2)}(*)$$

$$\frac{(7x+6)}{(x-3)(x+2)}=\frac{A}{x-3}+\frac{B}{x+2}\Rightarrow 7x+6=A(x+2)+B(x-3)$$

De donde: $\begin{cases} x=-2\Rightarrow -8=-5B\Rightarrow B=\frac{8}{5} \\ x=3\Rightarrow 27=5A\Rightarrow A=\frac{27}{5} \end{cases}$

$$(*)=\frac{x^2}{2}+x+\int\frac{Adx}{x-3}+\int\frac{Bdx}{x+2}=\frac{x^2}{2}+x+\frac{27}{5}\int\frac{dx}{x-3}+\frac{8}{5}\int\frac{dx}{x+2}$$

$$=\frac{x^2}{2}+x+\frac{27}{5}\ell\eta\left|x-3\right|+\frac{8}{5}\ell\eta\left|x+2\right|+c$$

82.- $\int \operatorname{sen} 2\theta e^{\operatorname{sen}^2\theta}d\theta$

Solución.- Sea: $u=\operatorname{sen}^2\theta, du=2\operatorname{sen}\theta\cos\theta d\theta$

$$\int \operatorname{sen} 2\theta e^{\operatorname{sen}^2\theta}d\theta=\int 2\operatorname{sen}\theta\cos\theta e^{\operatorname{sen}^2\theta}d\theta=\int e^u du=e^u+c=e^{\operatorname{sen}^2\theta}+c$$

83.- $\int\frac{dx}{e^x-9e^{-x}}$

Solución.- Sea: $u=e^x, du=e^x dx$

$$\int\frac{dx}{e^x-9e^{-x}}=\int\frac{e^x dx}{e^{2x}-9}=\int\frac{e^x dx}{(e^x)^2-9}=\int\frac{du}{u^2-9}=\frac{1}{6}\ell\eta\left|\frac{u-3}{u+3}\right|+c=\frac{1}{6}\ell\eta\left|\frac{e^x-3}{e^x+3}\right|+c$$

84.- $\int\frac{dw}{1+\cos w}$

Solución.-

$$\int\frac{dw}{1+\cos w}=\int\frac{(1-\cos w)dw}{1-\cos^2 w}=\int\frac{(1-\cos w)dw}{\operatorname{sen}^2 w}=\int\cos ec^2 w dw-\int\frac{\cos w dw}{\operatorname{sen}^2 w}$$

$$=-\operatorname{co}\tau gw-\frac{(\operatorname{sen} w)^{-1}}{-1}+c=-\operatorname{co}\tau gw+\frac{1}{\operatorname{sen} w}+c=-\operatorname{co}\tau gw+\cos ecw+c$$

Nota: Este ejercicio esta desarrollado diferente en el capitulo 8.

85.- $\int e^{\left(\frac{1-\operatorname{sen}^2 x/2}{3}\right)^2}(\cos^3 x/2\operatorname{sen} x/2)dx$

Solución.- Sea: $u=\left(\frac{1-\operatorname{sen}^2 x/2}{3}\right)^2, du=-\frac{2}{9}\cos^3\frac{x}{2}\operatorname{sen}\frac{x}{2}dx$

$$\int e^{\left(\frac{1-\operatorname{sen}^2 x/2}{3}\right)^2}(\cos^3 x/2\operatorname{sen} x/2)dx=-\frac{9}{2}\int e^u du=-\frac{2}{9}e^u+c=-\frac{2}{9}e^{\left(\frac{1-\operatorname{sen}^2 x/2}{3}\right)^2}+c$$

86.- $\int\frac{x^3dx}{\sqrt{19-x^2}}$

Solución.- Sea: $x=\sqrt{19}\operatorname{sen}\theta, dx=\sqrt{19}\cos\theta d\theta, \sqrt{19-x^2}=\sqrt{19}\cos\theta$

$$\int\frac{x^3dx}{\sqrt{19-x^2}}=\int\frac{(\sqrt{19})^3\operatorname{sen}^3\theta\cancel{\sqrt{19}\cos\theta}d\theta}{\cancel{\sqrt{19}\cos\theta}}=19\sqrt{19}\int\operatorname{sen}\theta(1-\cos^2\theta)d\theta$$

$$=19\sqrt{19}\int sen\,\theta d\theta-19\sqrt{19}\int sen\,\theta\cos^2\theta d\theta=-19\sqrt{19}\cos\theta+\frac{19\sqrt{19}}{3}\cos^3\theta+c$$

$$=-19\cancel{\sqrt{19}}\,\frac{\sqrt{19-x^2}}{\cancel{\sqrt{19}}}+\frac{\cancel{19\sqrt{19}}}{3}\,\frac{\sqrt{(19-x^2)^3}}{\cancel{(\sqrt{19})^3}}+c=-19\sqrt{19-x^2}+\sqrt{(19-x^2)^3}+c$$

87.- $\int\frac{sen\,\varphi d\varphi}{\cos^{1/2}\varphi}$

Solución.- Sea: $u=\cos\varphi, du=-sen\,\varphi d\varphi$

$$\int\frac{sen\,\varphi d\varphi}{\cos^{1/2}\varphi}=-\int\frac{du}{u^{1/2}}=-\int u^{-1/2}du=-\frac{u^{1/2}}{1/2}+c=-2u^{1/2}+c=-2\sqrt{\cos\varphi}+c$$

88.- $\int(\sec\varphi+\tau g\varphi)^2d\varphi$

Solución.-

$$\int(\sec\varphi+\tau g\varphi)^2d\varphi=\int(\sec^2\varphi+2\sec\varphi\tau g\varphi+\tau g^2\varphi)d\varphi$$

$$=\int(\sec^2\varphi+2\sec\varphi\tau g\varphi+\sec^2\varphi-1)d\varphi=\int(2\sec^2\varphi+2\sec\varphi\tau g\varphi-1)d\varphi$$

$$=2\int\sec^2\varphi d\varphi+2\int\sec\varphi\tau g\varphi d\varphi-\int d\varphi=2\tau g\varphi+2\sec\varphi-\varphi+c$$

89.- $\int\frac{dt}{t(4+\ell\eta^2t)^{1/2}}$

Solución.- Sea: $u=\ell\eta t, du=\frac{dt}{t}$, además: $u=2\tau g\theta, du=2\sec^2\theta d\theta, \sqrt{4+u^2}=2\sec\theta$

$$\int\frac{dt}{t(4+\ell\eta^2t)^{1/2}}=\int\frac{du}{\sqrt{4+u^2}}=\int\frac{\cancel{2}\sec^{\cancel{2}}\theta d\theta}{\cancel{2\sec\theta}}=\int\sec\theta d\theta=\ell\eta\left|\sec\theta+\tau g\theta\right|+c$$

$$=\ell\eta\left|\frac{\sqrt{4+u^2}}{2}+\frac{u}{2}\right|+c=\ell\eta\left|\frac{\sqrt{4+u^2}+u}{2}\right|+c=\ell\eta\left|\frac{\sqrt{4+\ell\eta^2t}+\ell\eta t}{2}\right|+c$$

90.- $\int a^\theta b^{2\theta}c^{3\theta}d\theta$

Solución.- Sea: $ab^2c^3=k$,

$$\int a^\theta b^{2\theta}c^{3\theta}d\theta=\int a^\theta(b^2)^\theta(c^3)^\theta d\theta=\int(ab^2c^3)^\theta d\theta=\int k^\theta d\theta=\frac{k^\theta}{\ell\eta|k|}+c=\frac{(ab^2c^3)^\theta}{\ell\eta\left|(ab^2c^3)\right|}+c$$

91.- $\int sen^{1/2}\varphi\cos^3\varphi d\varphi$

Solución.-

$$\int sen^{1/2}\varphi\cos^3\varphi d\varphi=\int sen^{1/2}\varphi\cos^2\varphi\cos\varphi d\varphi=\int sen^{1/2}\varphi(1-sen^2\varphi)\cos\varphi d\varphi$$

$$=\int sen^{1/2}\varphi\cos\varphi d\varphi-\int sen^{5/2}\varphi\cos\varphi d\varphi=\frac{sen^{3/2}\varphi}{3/2}-\frac{sen^{7/2}\varphi}{7/2}+c$$

$$=\frac{2sen^{3/2}\varphi}{3}-\frac{2sen^{7/2}\varphi}{7}+c$$

92.- $\int \frac{\sec^2 \theta d\theta}{9+\tau g^2\theta}$

Solución.- Sea: $u = \tau g\theta, du = \sec^2 \theta d\theta$

$$\int \frac{\sec^2 \theta d\theta}{9+\tau g^2\theta} = \int \frac{du}{9+u^2} = \frac{1}{3}\text{arc}\,\tau g\,\frac{u}{3} + c = \frac{1}{3}\text{arc}\,\tau g\,\frac{(\tau g\theta)}{3} + c$$

93.- $\int \frac{dx}{\sqrt{e^{2x}-16}}$

Solución.-Sea: $u = e^x, du = e^x dx \Rightarrow dx = \frac{du}{u}$

Además: $u = 4\sec\theta, du = 4\sec\theta\tau g\theta d\theta, \sqrt{u^2-16} = 4\tau g\theta$

$$\int \frac{dx}{\sqrt{e^{2x}-16}} = \int \frac{du/u}{\sqrt{u^2-16}} = \int \frac{du}{u\sqrt{u^2-16}} = \int \frac{\cancel{4\sec\theta}\ \cancel{\tau g\theta} d\theta}{\cancel{4\sec\theta}\,4\,\cancel{\tau g\theta}} = \frac{1}{4}\int d\theta = \frac{1}{4}\theta + c$$

$$= \frac{1}{4}\text{arc}\sec\frac{u}{4} + c = \frac{1}{4}\text{arc}\sec\frac{e^x}{4} + c$$

94.- $\int (e^{2s}-1)(e^{2s}+1)ds$

Solución.-

$$\int (e^{2s}-1)(e^{2s}+1)ds = \int \left[(e^{2s})^2 - 1\right]ds = \int e^{4s}ds - \int ds = \frac{1}{4}e^{4s} + s + c$$

95.- $\int \frac{dx}{5x^2+8x+5}$

Solución.-

$$\int \frac{dx}{5x^2+8x+5} = \int \frac{dx}{5(x^2+8/5\,x+1)} = \frac{1}{5}\int \frac{dx}{x^2+8/5\,x+1}(*)$$, completando cuadrados:

$$x^2 + 8/5\,x + 1 = (x^2 + \frac{8}{5}x + \frac{16}{25}) + 1 - \frac{16}{25} = (x+4/5)^2 + 9/25 = (x+4/5)^2 + (3/5)^2$$

$$(*) = \frac{1}{5}\int \frac{dx}{(x+4/5)^2 + (3/5)^2} = \frac{1}{\cancel{5}}\frac{1}{3/\cancel{5}}\text{arc}\,\tau g\,\frac{x+4/5}{3/5} + c = \frac{1}{3}\text{arc}\,\tau g\,\frac{5x+4}{3} + c$$

96.- $\int \frac{x^3+1}{x^3-x}dx$

Solución.-

$$\int \frac{x^3+1}{x^3-x}dx = \int \left(1 + \frac{x+1}{x^3-x}\right)dx = \int dx + \int \frac{x+1}{x^3-x}dx = x + \int \frac{(x+1)dx}{x(x^2-1)}$$

$$= x + \int \frac{\cancel{(x+1)}\,dx}{x\,\cancel{(x+1)}\,(x-1)} = x + \int \frac{dx}{x(x-1)} = x + \int \frac{Adx}{x} + \int \frac{Bdx}{x-1}(*)$$

$$\frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1} \Rightarrow 1 = A(x-1) + Bx$$

De donde: $\begin{cases} x=0 \Rightarrow 1=-A \Rightarrow A=-1 \\ x=1 \Rightarrow 1=B \Rightarrow B=1 \end{cases}$

$$(*) = x - \int \frac{dx}{x} + \int \frac{dx}{x-1} = x - \ell\eta\left|x\right| + \ell\eta\left|x-1\right| + c = x + \ell\eta\left|\frac{x-1}{x}\right| + c$$

97.- $\int (\operatorname{arcsen}\sqrt{1-x^2})^0 dx$

Solución.-

$$\int (\operatorname{arcsen}\sqrt{1-x^2})^0 dx = \int dx = x + c$$

98.- $\int \frac{3dy}{1+\sqrt{y}}$

Solución.-Sea: $y^{1/2} = t \Rightarrow y = t^2, dy = 2tdt$

$$\int \frac{3dy}{1+\sqrt{y}} = 3\int \frac{dy}{1+\sqrt{y}} = 3\int \frac{2tdt}{1+t} = 6\int \frac{tdt}{1+t} = 6\int \left(1 - \frac{1}{1+t}\right)dt = 6\int dt - 6\int \frac{dt}{1+t}$$

$$= 6t - 6\ell\eta\left|1+t\right| + c = 6\sqrt{y} - 6\ell\eta\left|1+\sqrt{y}\right| + c = 6\left(\sqrt{y} - \ell\eta\left|1+\sqrt{y}\right|\right) + c$$

99.- $\int x(1+x)^{1/5} dx$

Solución.-Sea: $u = 1 + x \Rightarrow x = u - 1, du = dx$

$$\int x(1+x)^{1/5} dx = \int (u-1)u^{1/5} du = \int (u^{6/5} - u^{1/5}) du = \int u^{6/5} du - \int u^{1/5} du = \frac{u^{11/5}}{11/5} - \frac{u^{6/5}}{6/5} + c$$

$$= \left(\frac{5u^2}{11} - \frac{5u}{6}\right)u^{1/5} + c = \left(\frac{5(1+x)^2}{11} - \frac{5(1+x)}{6}\right)(1+x)^{1/5} + c$$

100.- $\int \frac{d\varphi}{a^2 \operatorname{sen}^2\varphi + b^2\cos^2\varphi}$

Solución.-Sea: $u = \tau g\varphi, du = \sec^2\varphi d\varphi$

$$\int \frac{d\varphi}{a^2 \operatorname{sen}^2\varphi + b^2\cos^2\varphi} = \int \frac{\operatorname{sen}^4\varphi d\varphi}{\frac{1}{\cos^2\varphi}(a^2\tau g^2\varphi + b^2)} = \int \frac{\operatorname{sen}^2\varphi d\varphi}{(a^2\tau g^2\varphi + b^2)} = \int \frac{du}{(a^2u^2 + b^2)}$$

$$= \frac{1}{a^2}\int \frac{du}{u^2 + (b/a)^2} = \frac{1}{a^{\cancel{2}}}\frac{1}{b/\cancel{a}} \operatorname{arc}\tau g \frac{u}{b/a} + c = \frac{1}{ab}\operatorname{arc}\tau g \frac{au}{b} + c = \frac{1}{ab}\operatorname{arc}\tau g\left(\frac{a\tau g\varphi}{b}\right) + c$$

101.- $\int \frac{tdt}{(2t+1)^{1/2}}$

Solución.-

Sea: $\begin{array}{ll} u = t & dv = \frac{dt}{\sqrt{2t+1}} \\ du = dt & v = \sqrt{2t+1} \end{array}$

$$\int \frac{tdt}{(2t+1)^{1/2}} = t\sqrt{2t+1} - \int \sqrt{2t+1}dt = t\sqrt{2t+1} - \frac{1}{\cancel{2}}\frac{(2t+1)^{3/2}}{3/\cancel{2}} + c = t\sqrt{2t+1} - \frac{(2t+1)^{3/2}}{3} + c$$

$$= \sqrt{2t+1}\left(t - \frac{2t+1}{3}\right) + c = \frac{\sqrt{2t+1}}{3}(t-1) + c$$

102.- $\int \frac{s\ell\eta|s|ds}{(1-s^2)^{1/2}}$

Solución.-

Sea: $u = \ell\eta|s|$, $du = \frac{ds}{s}$; $dv = \frac{sds}{(1-s^2)^{1/2}}$, $v = -(1-s^2)^{1/2}$, además: $s = \mathrm{sen}\,\theta, ds = \cos\theta, \sqrt{1-s^2} = \cos\theta$

$$\int \frac{s\ell\eta|s|ds}{(1-s^2)^{1/2}} = -\sqrt{1-s^2}\,\ell\eta|s| + \int \frac{\sqrt{1-s^2}}{s}ds = -\sqrt{1-s^2}\,\ell\eta|s| + \int \frac{\cos\theta\cos\theta d\theta}{\mathrm{sen}\,\theta}$$

$$= -\sqrt{1-s^2}\,\ell\eta|s| + \int \frac{(1-\mathrm{sen}^2\,\theta)d\theta}{\mathrm{sen}\,\theta} = -\sqrt{1-s^2}\,\ell\eta|s| + \int \cos ec\theta d\theta - \int \mathrm{sen}\,\theta d\theta$$

$$= -\sqrt{1-s^2}\,\ell\eta|s| + \ell\eta|\cos ec\theta - \mathrm{co}\tau g\theta| + \cos\theta + c$$

$$= -\sqrt{1-s^2}\,\ell\eta|s| + \ell\eta\left|\frac{1-\sqrt{1-s^2}}{s}\right| + \sqrt{1-s^2} + c$$

103.- $\int (2\cos\alpha\,\mathrm{sen}\,\alpha - \mathrm{sen}\,\alpha)d\alpha$

Solución.-

$$\int (2\cos\alpha\,\mathrm{sen}\,\alpha - \mathrm{sen}\,2\alpha)d\alpha = \int \cancel{(\mathrm{sen}\,2\alpha - \mathrm{sen}\,2\alpha)}^{0} d\alpha = \int 0 d\alpha = c$$

104.- $\int t^4\ell\eta^2 t dt$

Sea: $u = \ell\eta^2 t$, $du = 2\ell\eta t\frac{dt}{t}$; $dv = t^4 dt$, $v = \frac{t^5}{5}$

$\int t^4\ell\eta^2 tdt = \frac{t^5}{5}\ell\eta^2 t - \frac{2}{5}\int t^4\ell\eta t dt(*)$, trabajando por partes nuevamente:

Sea: $u = \ell\eta t$, $du = \frac{dt}{t}$; $dv = t^4 dt$, $v = \frac{t^5}{5}$

$$(*) = \frac{t^5}{5}\ell\eta^2 t - \frac{2}{5}\left(\frac{t^5}{5}\ell\eta t - \frac{1}{5}\int t^4 dt\right) = \frac{t^5}{5}\ell\eta^2 t - \frac{2t^5}{25}\ell\eta t + \frac{2}{25}\frac{t^5}{5} + c$$

$$= \frac{t^5}{5}\ell\eta^2 t - \frac{2t^5}{25}\ell\eta t + \frac{2t^5}{125} + c$$

105.- $\int u^2(1+v)^{11}dx$

Solución.-
$$\int u^2(1+v)^{11}dx = u^2(1+v)^{11}\int dx = u^2(1+v)^{11}x + c$$

106.- $\int \frac{(\varphi + \operatorname{sen} 3\varphi)d\varphi}{3\varphi^2 - 2\cos 3\varphi}$

Solución.-Sea: $u = 3\varphi^2 - 2\cos 3\varphi, du = 6(\varphi + \operatorname{sen} 3\varphi)d\varphi$
$$\int \frac{(\varphi + \operatorname{sen} 3\varphi)d\varphi}{3\varphi^2 - 2\cos 3\varphi} = \frac{1}{6}\int \frac{du}{u} = \frac{1}{6}\ell\eta|u| + c = \frac{1}{6}\ell\eta\left|3\varphi^2 - 2\cos 3\varphi\right| + c$$

107.- $\int \frac{(y^{1/2}+1)dy}{y^{1/2}(y+1)}$

Solución.-Sea: $y^{1/2} = t \Rightarrow y = t^2, dy = 2tdt$
$$\int \frac{(y^{1/2}+1)dy}{y^{1/2}(y+1)} = \int \frac{(t+1)2\cancel{t}dt}{\cancel{t}(t^2+1)} = 2\int \frac{(t+1)dt}{(t^2+1)} = \int \frac{2tdt}{(t^2+1)} + \int \frac{dt}{(t^2+1)} = \ell\eta\left|t^2+1\right| + 2\operatorname{arc}\tau g t + c$$
$$= \ell\eta\left|y+1\right| + 2\operatorname{arc}\tau g\sqrt{y} + c$$

108.- $\int \frac{ds}{s^3(s^2-4)^{1/2}}$

Solución.-Sea: $s = 2\sec\theta, ds = 2\sec\theta\tau g\theta d\theta$
$$\int \frac{ds}{s^3(s^2-4)^{1/2}} = \int \frac{\cancel{2}\,\cancel{\sec\theta}\,\cancel{\tau g\theta}d\theta}{8\sec^{\cancel{3}}\theta\,\cancel{2}\,\cancel{\tau g\theta}} = \frac{1}{8}\int \frac{d\theta}{\sec^2\theta} = \frac{1}{8}\int \cos^2\theta d\theta = \frac{1}{16}\int (1+\cos 2\theta)d\theta$$
$$= \frac{1}{16}\theta + \frac{1}{32}\operatorname{sen} 2\theta + c = \frac{1}{16}\left(\theta + \frac{\operatorname{sen} 2\theta}{2}\right) + c = \frac{1}{16}\left(\theta + \operatorname{sen}\theta\cos\theta\right) + c$$
$$= \frac{1}{16}\left(\operatorname{arc}\sec \frac{s}{2} + \frac{2\sqrt{s^2-4}}{s^2}\right) + c$$

109.- $\int \sqrt{u}(1+u^2)^2 du$

Solución.-
$$\int \sqrt{u}(1+u^2)^2 du = \int \sqrt{u}(1+2u^2+u^4)du = \int u^{1/2}du + 2\int u^{5/2}du + \int u^{9/2}du$$
$$= \frac{u^{3/2}}{3/2} + 2\frac{u^{7/2}}{7/2} + \frac{u^{11/2}}{11/2} + c = \frac{2u^{3/2}}{3} + \frac{4u^{7/2}}{7} + \frac{2u^{11/2}}{11} + c = \frac{2u\sqrt{u}}{3} + \frac{4u^3\sqrt{u}}{7} + \frac{2u^5\sqrt{u}}{11} + c$$
$$= \sqrt{u}\left(\frac{2u}{3} + \frac{4u^3}{7} + \frac{2u^5}{11}\right) + c$$

110.- $\int \frac{(x^3+x^2)dx}{x^2+x-2}$

Solución.-
$$\int \frac{(x^3+x^2)dx}{x^2+x-2} = \int\left(x + \frac{2x}{x^2+x-2}\right)dx = \int xdx + \int \frac{2xdx}{(x+2)(x-1)} = \frac{x^2}{2} + \int \frac{2xdx}{(x+2)(x-1)}$$

$$=\frac{x^2}{2}+\int\frac{2xdx}{(x+2)(x-1)}=\frac{x^2}{2}+\int\frac{Adx}{x+2}+\int\frac{Bdx}{x-1}(*)$$

$$\frac{2x}{(x+2)(x-1)}=\frac{A}{x+2}+\frac{B}{x-1}\Rightarrow 2x=A(x-1)+B(x+2)$$

De donde: $\begin{cases} x=1\Rightarrow 2=3B\Rightarrow B=2/3 \\ x=-2\Rightarrow -4=-3A\Rightarrow A=4/3 \end{cases}$

$$(*)=\frac{x^2}{2}+\frac{4}{3}\int\frac{dx}{x+2}+\frac{2}{3}\int\frac{dx}{x-1}=\frac{x^2}{2}+\frac{4}{3}\ell\eta\left|x+2\right|+\frac{2}{3}\ell\eta\left|x-1\right|+c$$

$$=\frac{x^2}{2}+\frac{2}{3}\ell\eta\left|(x+2)^2(x-1)\right|+c$$

111- $\int adb$

Solución.-

$$\int adb=a\int db=ab+c$$

112.- $\int\frac{dx}{\sqrt{x^2-2x-8}}$

Solución.-

Completando cuadrados se tiene: $x^2-2x-8=(x^2-2x+1)-9=(x-1)^2-3^2$

Sea: $x-1=3\sec\theta, dx=3\sec\theta\tau g\theta d\theta, \sqrt{(x-1)^2-3^2}=3\tau g\theta$, luego:

$$\int\frac{dx}{\sqrt{x^2-2x-8}}=\int\frac{dx}{\sqrt{(x-1)^2-3^2}}=\int\frac{\cancel{3}\sec\theta\,\cancel{\tau g\theta}d\theta}{\cancel{3}\,\cancel{\tau g\theta}}=\int\sec\theta d\theta=\ell\eta\left|\sec\theta+\tau g\theta\right|+c$$

$$=\ell\eta\left|\frac{x-1}{3}+\frac{\sqrt{x^2-2x-8}}{3}\right|+c$$

113.- $\int\frac{(x+1)dx}{\sqrt{2x-x^2}}$

Solución.-

Completando cuadrados se tiene:

$$2x-x^2=-(x^2-2x)=-(x^2-2x+1-1)=-(x^2-2x+1)+1=1-(x^2-1)$$

Sea: $x-1=\operatorname{sen}\theta, dx=\cos\theta d\theta, \sqrt{1-(x-1)^2}=\cos\theta$, luego:

$$\int\frac{(x+1)dx}{\sqrt{2x-x^2}}=-\frac{1}{2}\int\frac{(2-2x)-4}{\sqrt{2x-x^2}}dx=-\frac{1}{2}\int\frac{(2-2x)dx}{\sqrt{2x-x^2}}+2\int\frac{dx}{\sqrt{2x-x^2}}$$

$$=-\sqrt{2x-x^2}+2\int\frac{dx}{\sqrt{2x-x^2}}=-\sqrt{2x-x^2}+2\int\frac{dx}{\sqrt{1-(x-1)^2}}$$

$$=-\sqrt{2x-x^2}+2\int\frac{\cancel{\cos\theta}d\theta}{\cancel{\cos\theta}}=-\sqrt{2x-x^2}+2\theta+c=-\sqrt{2x-x^2}+2\operatorname{arcsen}(x-1)+c$$

114.- $\int f(x)f'(x)dx$

Solución.- Sea: $u = f(x), du = f'(x)dx$

$$\int f(x)f'(x)dx = \int udu = \frac{u^2}{2} + c = \frac{\left[f(x)\right]^2}{2} + c$$

115.- $\int \frac{x^3+7x^2-5x+5}{x^2+2x-3}dx$

Solución.-

$$\int \frac{x^3+7x^2-5x+5}{x^2+2x-3}dx = \int\left(x+5+\frac{20-12x}{x^2+2x-3}\right)dx = \int xdx + 5\int dx + \int \frac{(20-12x)dx}{x^2+2x-3}$$

$$\int xdx + 5\int dx + \int \frac{(20-12x)dx}{(x+3)(x-1)} = \frac{x^2}{2} + 5x + \int \frac{Adx}{x+3} + \int \frac{B}{x-1}(*)$$

$$20-12x = A(x-1) + B(x+3)$$

De donde: $\begin{cases} x = 1 \Rightarrow 8 = 4B \Rightarrow B = 2 \\ x = -3 \Rightarrow 56 = -4A \Rightarrow A = -14 \end{cases}$

$$(*) = \frac{x^2}{2} + 5x - 14\int\frac{dx}{x+3} + 2\int\frac{dx}{x-1} = \frac{x^2}{2} + 5x + 14\ell\eta\left|x+3\right| + 2\ell\eta\left|x-1\right| + c$$

116.- $\int e^{\ell\eta\left|1+x+x^2\right|}dx$

Solución.-

$$\int e^{\ell\eta\left|1+x+x^2\right|}dx = \int(1+x+x^2)dx = x + \frac{x^2}{2} + \frac{x^3}{3} + c$$

117.- $\int \frac{(x-1)dx}{\sqrt{x^2-4x+3}}$

Solución.-

Completando cuadrados se tiene: $x^2-4x+3 = x^2-4x+4-1 = (x-2)^2-1$

Sea: $x-2 = \sec\theta, dx = \sec\theta\tau g\theta d\theta, \sqrt{(x-2)^2-1} = \tau g\theta$, luego:

$$\int \frac{(x-1)dx}{\sqrt{x^2-4x+3}} = \frac{1}{2}\int\frac{(2x-4)+2}{\sqrt{x^2-4x+3}}dx = \frac{1}{2}\int\frac{(2x-4)dx}{\sqrt{x^2-4x+3}} + \int\frac{dx}{\sqrt{x^2-4x+3}}$$

$$= \sqrt{x^2-4x+3} + \int\frac{dx}{\sqrt{x^2-4x+3}} = \sqrt{x^2-4x+3} + \int\frac{dx}{\sqrt{(x-2)^2-1}}$$

$$= \sqrt{x^2-4x+3} + \int\frac{\sec\theta\,\cancel{\tau g\theta}\,d\theta}{\cancel{\tau g\theta}} = \sqrt{x^2-4x+3} + \int\sec\theta d\theta$$

$$= \sqrt{x^2-4x+3} + \ell\eta\left|\sec\theta + \tau g\theta\right| + c$$

$$= \sqrt{x^2-4x+3} + \ell\eta\left|x-2+\sqrt{x^2-4x+3}\right| + c$$

118.- $\int \frac{xdx}{\sqrt{x^2+4x+5}}$

Solución.-

Completando cuadrados se tiene: $x^2+4x+5=x^2+4x+4+1=(x+2)^2+1$

Sea: $x+2=\tau g\theta, dx=\sec^2\theta d\theta, \sqrt{(x+2)^2+1}=\sec\theta$, luego:

$$\int\frac{xdx}{\sqrt{x^2+4x+5}}=\int\frac{xdx}{\sqrt{(x+2)^2+1}}=\int\frac{(\tau g\theta-2)\sec^{\cancel{2}}\theta d\theta}{\cancel{\sec\theta}}=\int\tau g\theta\sec\theta d\theta-2\int\sec\theta d\theta$$

$$=\sec\theta-2\ell\eta\left|\sec\theta+\tau g\theta\right|+c=\sqrt{x^2+4x+5}-2\ell\eta\left|\sqrt{x^2+4x+5}+x+2\right|+c$$

119.- $\int\frac{4dx}{x^3+4x}$

Solución.-

$$\int\frac{4dx}{x^3+4x}=\int\frac{(3x^2+4)-3x^2}{x^3+4x}dx=\int\frac{(3x^2+4)dx}{x^3+4x}-3\int\frac{x^2dx}{x^3+4x}$$

$$=\ell\eta\left|x^3+4x\right|-\frac{3}{2}\int\frac{2xdx}{x^2+4}=\ell\eta\left|x^3+4x\right|-\frac{3}{2}\ell\eta\left|x^2+4\right|+c$$

$$=\ell\eta\left|\frac{x(x^2+4)}{(x^2+4)^{3/2}}\right|+c=\ell\eta\left|\frac{x}{\sqrt{x^2+4}}\right|+c$$

120.- $\int\frac{\operatorname{co}\tau gxdx}{\ell\eta\left|\operatorname{sen}x\right|}$

Solución.- Sea: $u=\ell\eta\left|\operatorname{sen}x\right|, du=\operatorname{co}\tau gxdx$

$$\int\frac{\operatorname{co}\tau gxdx}{\ell\eta\left|\operatorname{sen}x\right|}=\int\frac{du}{u}=\ell\eta\left|u\right|+c=\ell\eta\left|\ell\eta\left|\operatorname{sen}x\right|\right|+c$$

121.- $\int\ell\eta\exp\sqrt{x-1}dx$

Solución.-

$$\int\ell\eta\exp\sqrt{x-1}dx=\int\sqrt{x-1}dx=\frac{(x-1)^{3/2}}{3/2}+c=\frac{2(x-1)\sqrt{(x-1)}}{3}+c$$

122.- $\int\frac{\sqrt{1+x^3}}{x}dx$

Solución.- Sea: $\sqrt{1+x^3}=t\Rightarrow t^2=1+x^3\Rightarrow x=\sqrt[3]{t^2-1}, dx=\frac{2tdt}{3(t^2-1)^{2/3}}$

$$\int\frac{\sqrt{1+x^3}}{x}dx=\int\frac{t\frac{2tdt}{3(t^2-1)^{2/3}}}{(t^2-1)^{1/3}}=\frac{2}{3}\int\frac{t^2dt}{t^2-1}=\frac{2}{3}\int\left(1+\frac{1}{t^2-1}\right)dt=\frac{2}{3}\int dt+\frac{2}{3}\int\frac{dt}{t^2-1}$$

$$=\frac{2}{3}t+\frac{1}{3}\ell\eta\left|\frac{t-1}{t+1}\right|+c=\frac{2}{3}\sqrt{1+x^3}+\frac{1}{3}\ell\eta\left|\frac{\sqrt{1+x^3}-1}{\sqrt{1+x^3}+1}\right|+c$$

123.- $\int\sqrt{\frac{x-1}{x+1}}\frac{1}{x}dx$

Solución.- Sea: $\sqrt{\frac{x-1}{x+1}}=t\Rightarrow t^2=\frac{x-1}{x+1}\Rightarrow x(1-t^2)=t^2\Rightarrow x=\frac{1+t^2}{1-t^2}, dx=\frac{4tdt}{(1-t^2)^2}$

$$\int\sqrt{\frac{x-1}{x+1}}\frac{1}{x}dx=\int t\frac{(1-t^2)}{(1+t^2)}\frac{4tdt}{(1-t^2)^2}=4\int\frac{t^2\cancel{(1-t^2)}dt}{(1+t^2)(1-t^2)^{\cancel{2}}}=4\int\frac{t^2dt}{(1+t^2)(1-t^2)}$$

$$=4\int\frac{t^2dt}{(1+t)(1-t)(1+t^2)}=4\left[\int\frac{Adt}{1+t}+\int\frac{Bdt}{1-t}+\int\frac{Ct+D}{1+t^2}dt\right](*)$$

$$\frac{t^2}{(1+t)(1-t)(1+t^2)}=\frac{A}{1+t}+\frac{B}{1-t}+\frac{Ct+D}{1+t^2}$$

$$\Rightarrow t^2=A(1-t)(1+t^2)+B(1+t)(1+t^2)+(Ct+D)(1-t^2)$$

De donde: $\begin{cases} t=1\Rightarrow 1=4B\Rightarrow B=\frac{1}{4} \\ t=-1\Rightarrow 1=4A\Rightarrow A=\frac{1}{4} \\ t=0\Rightarrow 0=A+B+D\Rightarrow D=-\frac{1}{2} \end{cases}$

$t=2\Rightarrow 4=-5A+15B+(2C+D)(-3)\Rightarrow C=0$

$$(*)=4\left(\frac{1}{4}\int\frac{dt}{1+t}+\frac{1}{4}\int\frac{dt}{1-t}-\frac{1}{2}\int\frac{dt}{1+t^2}\right)=\int\frac{dt}{1+t}-\int\frac{dt}{t-1}-2\int\frac{dt}{1+t^2}$$

$$=\ell\eta\left|t+1\right|-\ell\eta\left|t-1\right|-2\operatorname{arc}\tau gt+c=\ell\eta\left|\frac{t+1}{t-1}\right|-2\operatorname{arc}\tau gt+c$$

$$=\ell\eta\left|\frac{\sqrt{\frac{x+1}{x-1}}+1}{\sqrt{\frac{x+1}{x-1}}-1}\right|-2\operatorname{arc}\tau g\sqrt{\frac{x+1}{x-1}}+c=\ell\eta\left|\frac{\sqrt{x-1}+\sqrt{x+1}}{\sqrt{x-1}-\sqrt{x+1}}\right|-2\operatorname{arc}\tau g\sqrt{\frac{x+1}{x-1}}+c$$

124.- $\int\frac{\operatorname{sen}xdx}{1+\operatorname{sen}x+\cos x}$

Solución.- Sea: $\operatorname{sen}x=\frac{2z}{1+z^2},\cos x=\frac{1-z^2}{1+z^2}, z=\tau g\frac{x}{2}, dx=\frac{2dz}{1+z^2}$

$$\int\frac{\operatorname{sen}xdx}{1+\operatorname{sen}x+\cos x}=\int\frac{\left(\frac{2z}{1+z^2}\right)\left(\frac{2}{1+z^2}\right)}{1+\left(\frac{2z}{1+z^2}\right)\left(\frac{1-z^2}{1+z^2}\right)}dz=\int\frac{\frac{4z}{1+z^2}dz}{1+z^2+2z+1-z^2}$$

$$\int\frac{4zdz}{(1+z^2)(2+2z)}=\int\frac{2zdz}{(1+z)(1+z^2)}=\int\frac{Adz}{1+z}+\int\frac{Bz+C}{1+z^2}dz(*)$$

$$\frac{2z}{(1+z)(1+z^2)}=\frac{A}{1+z}+\frac{Bz+C}{1+z^2}$$

De donde: $\begin{cases} z=-1\Rightarrow -2=2A\Rightarrow A=-1 \\ z=0\Rightarrow 0=A+C\Rightarrow C=1 \\ z=1\Rightarrow 2=2A+2B+2C\Rightarrow B=1 \end{cases}$

$$(*) = -\int \frac{dz}{1+z} + \int \frac{z+1}{1+z^2}dz = -\ell\eta\left|1+z\right| + \frac{1}{2}\int \frac{2zdz}{z^2+1} + \int \frac{dz}{z^2+1}$$

$$= -\ell\eta\left|1+z\right| + \frac{1}{2}\ell\eta\left|z^2+1\right| + \operatorname{arc}\tau gz + c = \ell\eta\left|\frac{\sqrt{z^2+1}}{z+1}\right| + \operatorname{arc}\tau gz + c$$

$$= \ell\eta\left|\frac{\sqrt{\tau g^2 \,{}^{x}\!/_{2}+1}}{\tau g \,{}^{x}\!/_{2}+1}\right| + \operatorname{arc}\tau gz + c$$

125.- $\int \frac{dx}{3+2\cos x}$

Solución.- Sea: $\operatorname{s}e\operatorname{n} x = \frac{2z}{1+z^2}, \cos x = \frac{1-z^2}{1+z^2}, z = \tau g\frac{x}{2}, dx = \frac{2dz}{1+z^2}$

$$\int \frac{dx}{3+2\cos x} = \int \frac{\frac{2z}{1+z^2}}{3+2\left(\frac{1-z^2}{1+z^2}\right)}dz = \int \frac{2dz}{3+3z^2+2-2z^2} = 2\int \frac{dz}{5+z^2} = \frac{2}{\sqrt{5}}\operatorname{arc}\tau g\frac{z}{\sqrt{5}} + c$$

$$= \frac{2\sqrt{5}}{5}\operatorname{arc}\tau g\left(\frac{\sqrt{5}}{5}\tau g\frac{x}{2}\right) + c$$

126.- $\int \frac{xdx}{\sqrt{x^2-2x+5}}$

Solución.-

Completando cuadrados se tiene: $x^2-2x+5 = x^2-2x+1+4 = (x-1)^2+2^2$,

Sea: $x-1 = 2\tau g\theta, dx = 2\sec^2\theta d\theta, \sqrt{(x-1)^2+2^2} = 2\sec\theta$,luego:

$$\int \frac{xdx}{\sqrt{x^2-2x+5}} = \frac{1}{2}\int \frac{(2x-2+2)dx}{\sqrt{x^2-2x+5}} = \frac{1}{2}\int \frac{(2x-2)dx}{\sqrt{x^2-2x+5}} + \int \frac{dx}{\sqrt{x^2-2x+5}}$$

$$= \sqrt{x^2-2x+5} + \int \frac{dx}{\sqrt{x^2-2x+5}} = \sqrt{x^2-2x+5} + \int \frac{dx}{\sqrt{(x-1)^2+2^2}}$$

$$= \sqrt{x^2-2x+5} + \int \frac{\cancel{2}\sec^{\cancel{2}}\theta d\theta}{\cancel{2\sec\theta}} = \sqrt{x^2-2x+5} + \int \sec\theta d\theta$$

$$= \sqrt{x^2-2x+5} + \ell\eta\left|\sec\theta + \tau g\theta\right| + c$$

127.- $\int \frac{(1+\operatorname{s}e\operatorname{n} x)dx}{\operatorname{s}e\operatorname{n} x(2+\cos x)}$

Solución.- Sea: $\operatorname{s}e\operatorname{n} x = \frac{2z}{1+z^2}, \cos x = \frac{1-z^2}{1+z^2}, z = \tau g\frac{x}{2}, dx = \frac{2dz}{1+z^2}$

$$\int \frac{(1+\operatorname{sen} x)dx}{\operatorname{sen} x(2+\cos x)} = \int \frac{\left(1+\frac{2z}{1+z^2}\right)\frac{\cancel{2}}{\cancel{1+z^2}}}{\frac{\cancel{2}z}{\cancel{1+z^2}}\left(2+\frac{1-z^2}{1+z^2}\right)}dz = \int \frac{(1+z^2+2z)dz}{2z(1+z^2)+z(1-z^2)}$$

$$= \int \frac{(z^2+2z+1)dz}{z^3+3z} = \int \frac{(z^2+2z+1)dz}{z(z^2+3)} = \int \frac{Adz}{z} + \int \frac{Bz+C}{(z^2+3)}dz(*)\}$$

$$\frac{(z^2+2z+1)}{z(z^2+3)} = \frac{A}{z} + \frac{Bz+C}{(z^2+3)} \Rightarrow z^2+2z+1 = A(z^2+3)+(Bz+C)z$$

$\Rightarrow Az^2+3A+Bz^2+Cz \Rightarrow (A+B)z^2+Cz+3A$, igualando coeficientes se tiene:

$$\begin{pmatrix} A & +B & =1 \\ & C & =2 \\ 3A & & =1 \end{pmatrix} \Rightarrow A = 1/3, B = 2/3, C = 2$$

$$(*) = \frac{1}{3}\int \frac{dz}{z} + \int \frac{2/3\, z+2}{(z^2+3)}dz = \frac{1}{3}\int \frac{dz}{z} + \frac{1}{3}\int \frac{2zdz}{(z^2+3)} + 2\int \frac{dz}{(z^2+3)}$$

$$= \frac{1}{3}\ell\eta\left|\tau g\, x/2\right| + \frac{1}{3}\ell\eta\left|\tau g^2\, x/2 + 3\right| + \frac{2}{\sqrt{3}}\operatorname{arc}\tau g\left(\frac{\tau g^2\, x/2}{\sqrt{3}}\right) + c$$

128.- $\int \frac{dx}{x^4+4}$

Solución.- Sea: $x^4+4 = x^4+4x^2+4-4x^2 = (x^2+2)^2-(2x)^2 = (x^2+2x+2)(x^2-2x+2)$

$$\int \frac{dx}{x^4+4} = \int \frac{dx}{(x^2+2x+2)(x^2-2x+2)} = \int \frac{(Ax+B)dx}{(x^2+2x+2)} + \int \frac{(Cx+D)dx}{(x^2-2x+2)}(*)$$

$$\frac{1}{(x^4+4)} = \frac{(Ax+B)}{(x^2+2x+2)} + \frac{(Cx+D)}{(x^2-2x+2)}$$

$$1 = (Ax+B)(x^2-2x+2)+(Cx+D)(x^2+2x+2)$$

$$1 = (A+C)x^3+(-2A+B+2C+D)x^2+(2A-2B+2C+2D)x+(2B+2D)$$

Igualando coeficientes se tiene:

$$\begin{pmatrix} A & + C & =0 \\ -2A & +B+2C+ D & =0 \\ 2A-2B+2C+2D & & =0 \\ 2B & +2D & =1 \end{pmatrix} \Rightarrow A = 1/8, B = 1/4, C = -1/8, D = 1/4$$

$$(*) = \frac{1}{8}\int \frac{(x+2)dx}{(x^2+2x+2)} - \frac{1}{8}\int \frac{(x-2)dx}{(x^2-2x+2)}$$

$$= \frac{1}{8}\int \frac{(x+1)dx}{(x+1)^2+1} + \frac{1}{8}\int \frac{dx}{(x+1)^2+1} - \frac{1}{8}\int \frac{(x-1)dx}{(x-1)^2+1} + \frac{1}{8}\int \frac{dx}{(x-1)^2+1}$$

$$= \frac{1}{16}\ell\eta\left|x^2+2x+2\right| + \frac{1}{8}\operatorname{arc}\tau g(x+1) - \frac{1}{16}\ell\eta\left|x^2-2x+2\right| + \frac{1}{8}\operatorname{arc}\tau g(x-1) + c$$

$$= \frac{1}{16}\ell\eta\left|\frac{x^2+2x+2}{x^2-2x+2}\right| + \frac{1}{8}\left[\operatorname{arc}\tau g(x+1) + \operatorname{arc}\tau g(x-1)\right] + c$$

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