

Problema 1 Sea la matriz

$$A = \begin{pmatrix} m & 0 & -m & 2 \\ -m & 2 & 2 & -m \\ -1 & 6 & 4 & m \end{pmatrix}$$

Calcular el rango de A para los diferentes valores de m .

Solución:

$$|A_1| = \begin{vmatrix} m & 0 & -m \\ -m & 2 & 2 \\ -1 & 6 & 4 \end{vmatrix} = 6m(m-1) = 0 \implies m = 0, m = 1$$

$$|A_2| = \begin{vmatrix} m & 0 & 2 \\ -m & 2 & -m \\ -1 & 6 & m \end{vmatrix} = 4(2m^2 - 3m + 1) = 0 \implies m = 1, m = 1/2$$

$$|A_3| = \begin{vmatrix} m & -m & 2 \\ -m & 2 & -m \\ -1 & 4 & m \end{vmatrix} = -m^3 + 5m^2 - 8m + 4 = 0 \implies m = 1, m = 2$$

$$|A_4| = \begin{vmatrix} 0 & -m & 2 \\ 2 & 2 & -m \\ -6 & 4 & m \end{vmatrix} = 8(m^2 - 1) \implies m = 1, m = -1$$

Si $m \neq 1 \implies \text{Rango}(A) = 3$.

Cuando $m = 1 \implies \text{Rango}(A) = 2$, ya que en estos casos el menor $\begin{vmatrix} 2 & 2 \\ 6 & 4 \end{vmatrix} = -4 \neq 0$.

Problema 2 Dada la matriz

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Calcular A^n y en particular A^{100}

Solución:

$$A^1 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad A^2 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 2 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2^2 & 0 & 2^2 \\ 0 & 1 & 0 \\ 2^2 & 0 & 2^2 \end{pmatrix}$$

$$A^n = \begin{pmatrix} 2^{n-1} & 0 & 2^{n-1} \\ 0 & 1 & 0 \\ 2^{n-1} & 0 & 2^{n-1} \end{pmatrix}$$

$$A^{100} = \begin{pmatrix} 2^{99} & 0 & 2^{99} \\ 0 & 1 & 0 \\ 2^{99} & 0 & 2^{99} \end{pmatrix}$$

Problema 3 Resolver el siguiente sistema matricial:

$$\begin{cases} X + 2Y = \begin{pmatrix} 2 & 5 \\ 7 & -1 \end{pmatrix} \\ X - Y = \begin{pmatrix} 1 & -2 \\ 3 & 5 \end{pmatrix} \end{cases}$$

Solución:

$$\begin{cases} X + 2Y = \begin{pmatrix} 2 & 5 \\ 7 & -1 \end{pmatrix} \\ X - Y = \begin{pmatrix} 1 & -2 \\ 3 & 5 \end{pmatrix} \end{cases} \Rightarrow \begin{cases} X = \begin{pmatrix} 4/3 & 1/3 \\ 13/3 & 3 \end{pmatrix} \\ Y = \begin{pmatrix} 1/3 & 7/3 \\ 4/3 & -2 \end{pmatrix} \end{cases}$$

Problema 4 calcular todas las matrices X que cumplan $AX = XA$ donde

$$A = \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}$$

Solución:

$$\text{LLamamos } X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}:$$

$$AX = XA \Rightarrow \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix} \Rightarrow$$

$$\begin{pmatrix} -a + 2c & -b + 2d \\ a & b \end{pmatrix} = \begin{pmatrix} -a + b & 2a \\ -c + d & 2c \end{pmatrix} \Rightarrow \begin{cases} -a + 2c = b - a \\ -b + 2d = 2a \\ a = -c + d \\ b = 2c \end{cases} \Rightarrow \begin{cases} b = 2c \\ a = d - c \end{cases}$$

$$\text{LLamamos } X = \begin{pmatrix} d - c & 2c \\ c & d \end{pmatrix}$$