Problema 1 Sea la matriz

$$A = \begin{pmatrix} -2 & m & 0 \\ m & 2 & -m \\ 1 & -3 & m \end{pmatrix}$$

1. Calcular los valores de m para los que la matriz A es inversible.
2. Calcular A^{-1} para $m = -1$.

Solución:

1.

$$\begin{vmatrix} -2 & m & 0 \\ m & 2 & -m \\ 1 & -3 & m \end{vmatrix} = -m^3 - m^2 + 2m = 0 \Longrightarrow m = 0, \ \ m = 1, \ \ m = -2$$

Si $m = 0$ o $m = 1$ o $m = -2 \Longrightarrow |A| = 0 \Longrightarrow$ no existe A^{-1}.

Si $m = 0$ y $m = 1$ y $m = -2 \Longrightarrow |A| \neq 0 \Longrightarrow$ existe A^{-1}.

2.

$$A = \begin{pmatrix} -2 & -1 & 0 \\ -1 & 2 & 1 \\ 1 & -3 & -1 \end{pmatrix} \Longrightarrow A^{-1} = \begin{pmatrix} -1/2 & 1/2 & 1/2 \\ 0 & -1 & -1 \\ -1/2 & 7/2 & 5/2 \end{pmatrix}$$

Problema 2 Resolver la ecuación matricial $AX - X = B - CX$. Donde

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}; \quad B = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}; \quad C = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$$

Solución:

$$AX - X = B - CX \Longrightarrow (A + C - I)X = B \Longrightarrow X = (A + C - I)^{-1}B$$

$$(A + C - I)^{-1} = \begin{pmatrix} 4/3 & -1 \\ -1 & 1 \end{pmatrix}$$

$$X = (A + C - I)^{-1}B = \begin{pmatrix} -7/3 & 11/3 \\ 2 & -3 \end{pmatrix}$$

Problema 3 Resolver, utilizando las propiedades de los determinantes, calcular:

$$\begin{vmatrix} x & 1 & 1 & x \\ 1 & x & x & 1 \\ x & 1 & x & 1 \\ 1 & x & 1 & x \end{vmatrix}$$

Solución:

$$\begin{vmatrix} x & 1 & 1 & x \\ 1 & x & x & 1 \\ x & 1 & x & 1 \\ 1 & x & 1 & x \end{vmatrix} = \begin{bmatrix} \mathrm{F}_1 + \mathrm{F}_2 + \mathrm{F}_3 + \mathrm{F}_4 \\ \mathrm{F}_2 \\ \mathrm{F}_3 \\ \mathrm{F}_4 \end{bmatrix} = \begin{vmatrix} 2x+2 & 2x+2 & 2x+2 & 2x+2 \\ 1 & x & x & 1 \\ x & 1 & x & 1 \\ 1 & x & 1 & x \end{vmatrix} =$$

$$2(x{+}1) \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & x & x & 1 \\ x & 1 & x & 1 \\ 1 & x & 1 & x \end{vmatrix} = \begin{bmatrix} \mathrm{C}_1 \\ \mathrm{C}_2 - \mathrm{C}_1 \\ \mathrm{C}_3 - \mathrm{C}_1 \\ \mathrm{C}_4 - \mathrm{C}_1 \end{bmatrix} = 2(x{+}1) \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & x-1 & x-1 & 0 \\ x & 1-x & 0 & 1-x \\ 1 & x-1 & 0 & x-1 \end{vmatrix} =$$

$$2(x+1) \begin{vmatrix} x-1 & x-1 & 0 \\ 1-x & 0 & x-1 \\ x-1 & 0 & x-1 \end{vmatrix} = 2(x-1)(x+1) \begin{vmatrix} 1-x & 1-x \\ x-1 & x-1 \end{vmatrix} = 0$$