Problema 1 Sea la matriz

$$A = \begin{pmatrix} m & -m & 2 \\ 2 & m & 3 \\ m & -4 & 3 \end{pmatrix}$$

1. Calcular los valores de m para los que la matriz A es inversible.

2. Calcular A^{-1} para $m = 0$.

Solución:

1.

$$\begin{vmatrix} m & -m & 2 \\ 2 & m & 3 \\ m & -4 & 3 \end{vmatrix} = -2(m^2 - 9m + 8) = 0 \Longrightarrow m = 1, \ \ m = 8$$

Si $m = 1$ o $m = 8 \Longrightarrow |A| = 0 \Longrightarrow$ no existe A^{-1}.

Si $m \neq 1$ y $m \neq 8 \Longrightarrow |A| \neq 0 \Longrightarrow$ existe A^{-1}.

2.

$$A = \begin{pmatrix} 0 & 0 & 2 \\ 2 & 0 & 3 \\ 0 & -4 & 3 \end{pmatrix} \Longrightarrow A^{-1} = \begin{pmatrix} -3/4 & 1/2 & 0 \\ 3/8 & 0 & -1/4 \\ 1/2 & 0 & 0 \end{pmatrix}$$

Problema 2 Resolver la ecuación matricial $AX + B = C + X$. Donde

$$A = \begin{pmatrix} 6 & 1 \\ 0 & 2 \end{pmatrix}; \ \ B = \begin{pmatrix} 1 & 7 \\ -2 & 3 \end{pmatrix}; \ \ C = \begin{pmatrix} 6 & 1 \\ 1 & 0 \end{pmatrix}$$

Solución:

$$AX + B = C + X \Longrightarrow X = (A - I)^{-1}(C - B)$$

$$A - I = \begin{pmatrix} 6 & 1 \\ 0 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 1 \\ 0 & 1 \end{pmatrix}, \ \ (A + I)^{-1} = \begin{pmatrix} 1/5 & -1/5 \\ 0 & 1 \end{pmatrix}$$

$$C - B = \begin{pmatrix} 6 & 1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 7 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 5 & -6 \\ 3 & -3 \end{pmatrix}$$

$$X = (A - I)^{-1}(C - B) = \begin{pmatrix} 1/5 & -1/5 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 5 & -6 \\ 3 & -3 \end{pmatrix} = \begin{pmatrix} 2/5 & -3/5 \\ 3 & -3 \end{pmatrix}$$

Problema 3 Resolver, utilizando las propiedades de los determinantes, calcular:

$$\begin{vmatrix} x & -1 & 1 & 0 \\ -1 & x & 0 & 1 \\ 1 & 0 & x & -1 \\ 0 & 1 & -1 & x \end{vmatrix}$$

Solución:

$$\begin{vmatrix} x & -1 & 1 & 0 \\ -1 & x & 0 & 1 \\ 1 & 0 & x & -1 \\ 0 & 1 & -1 & x \end{vmatrix} = \begin{bmatrix} \mathrm{F}_1 + \mathrm{F}_2 + \mathrm{F}_3 + \mathrm{F}_4 \\ \mathrm{F}_2 \\ \mathrm{F}_3 \\ \mathrm{F}_4 \end{bmatrix} = \begin{vmatrix} x & x & x & x \\ -1 & x & 0 & 1 \\ 1 & 0 & x & -1 \\ 0 & 1 & -1 & x \end{vmatrix} =$$

$$x \begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & x & 0 & 1 \\ 1 & 0 & x & -1 \\ 0 & 1 & -1 & x \end{vmatrix} = \begin{bmatrix} \mathrm{F}_1 \\ \mathrm{F}_2 + \mathrm{F}_1 \\ \mathrm{F}_3 - \mathrm{F}_1 \\ \mathrm{F}_4 \end{bmatrix} = x \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & x+1 & 1 & 2 \\ 0 & -1 & x-1 & -2 \\ 0 & 1 & -1 & x \end{vmatrix} =$$

$$x \begin{vmatrix} x+1 & 1 & 2 \\ -1 & x-1 & -2 \\ 1 & -1 & x \end{vmatrix} = x^2(x^2-4)$$