

Problema 1 Sea la matriz

$$A = \begin{pmatrix} -1 & 2 & m \\ m+1 & 1 & 3 \\ m-1 & 5 & 5 \end{pmatrix}$$

1. Calcular los valores de m para los que la matriz A es inversible.
2. Calcular A^{-1} para $m = 0$.

Solución:

1.

$$\begin{vmatrix} -1 & 2 & m \\ m+1 & 1 & 3 \\ m-1 & 5 & 5 \end{vmatrix} = 2(2m^2 + m - 3) = 0 \implies m = 1, \quad m = -3/2$$

Si $m = 1$ o $m = -3/2 \implies |A| = 0 \implies$ no existe A^{-1} .

Si $m \neq 1$ y $m \neq -3/2 \implies |A| \neq 0 \implies$ existe A^{-1} .

2.

$$A = \begin{pmatrix} -1 & 2 & 0 \\ 1 & 1 & 3 \\ -1 & 5 & 5 \end{pmatrix} \implies A^{-1} = \begin{pmatrix} 5/3 & 5/3 & -1 \\ 4/3 & 5/6 & -1/2 \\ -1 & -1/2 & 1/2 \end{pmatrix}$$

Problema 2 Resolver la ecuación matricial $AX - I = C - BX$. Donde

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}; \quad B = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}; \quad C = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$$

Solución:

$$AX + BX = C + I \implies (A + B)X = C + I \implies X = (A + B)^{-1}(C + I)$$

$$(A + B)^{-1} = \begin{pmatrix} -3/16 & 4/16 \\ 4/16 & 0 \end{pmatrix}, \quad C + I = \begin{pmatrix} 4 & 1 \\ 0 & 2 \end{pmatrix}$$

$$X = (A + B)^{-1}(C + I) = \begin{pmatrix} -3/4 & 5/16 \\ 1 & 1/4 \end{pmatrix}$$

Problema 3 Resolver, utilizando las propiedades de los determinantes, la ecuación:

$$\begin{vmatrix} 1 & x & 1 & 0 \\ 1 & 1 & 0 & x \\ x & 1 & 1 & 0 \\ 1 & 0 & x & 1 \end{vmatrix} = 0$$

Solución:

$$\begin{aligned} \begin{vmatrix} 1 & x & 1 & 0 \\ 1 & 1 & 0 & x \\ x & 1 & 1 & 0 \\ 1 & 0 & x & 1 \end{vmatrix} &= \begin{bmatrix} C_1 + C_2 + C_3 + C_4 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{vmatrix} x+2 & x & 1 & 0 \\ x+2 & 1 & 0 & x \\ x+2 & 1 & 1 & 0 \\ x+2 & 0 & x & 1 \end{vmatrix} = \\ (x+2) \begin{vmatrix} 1 & x & 1 & 0 \\ 1 & 1 & 0 & x \\ 1 & 1 & 1 & 0 \\ 1 & 0 & x & 1 \end{vmatrix} &= \begin{bmatrix} F_1 \\ F_2 - F_1 \\ F_3 - F_1 \\ F_4 - F_1 \end{bmatrix} = (x+2) \begin{vmatrix} 1 & x & 1 & 0 \\ 0 & 1-x & -1 & x \\ 0 & 1-x & 0 & 0 \\ 0 & -x & x-1 & 1 \end{vmatrix} = \\ (x+2) \begin{vmatrix} 1-x & -1 & x \\ -1-x & 0 & 0 \\ -x & x-1 & 1 \end{vmatrix} &= (x-1)(x+2) \begin{vmatrix} -1 & x \\ x-1 & 1 \end{vmatrix} = \\ (x-1)(x+2)(-x^2+x-1) &= 0 \implies x=1, \quad x=-2 \end{aligned}$$