

Problema 1 Sea la matriz

$$A = \begin{pmatrix} m & -1 & 3 \\ m+1 & m & 4 \\ m & 2 & 1 \end{pmatrix}$$

1. Calcular los valores de m para los que la matriz A es inversible.
2. Calcular A^{-1} para $m = 0$.

Solución:

1.

$$\begin{vmatrix} m & -1 & 3 \\ m+1 & m & 4 \\ m & 2 & 1 \end{vmatrix} = -2m^2 - 5m + 7 = 0 \implies m = 1, \quad m = -7/2$$

Si $m = 1$ o $m = -7/2 \implies |A| = 0 \implies$ no existe A^{-1} .

Si $m \neq 1$ y $m \neq -7/2 \implies |A| \neq 0 \implies$ existe A^{-1} .

2.

$$A = \begin{pmatrix} 0 & -1 & 3 \\ 1 & 0 & 4 \\ 0 & 2 & 5 \end{pmatrix} \implies A^{-1} = \begin{pmatrix} -8/7 & 1 & -4/7 \\ -1/7 & 0 & 3/7 \\ 2/7 & 0 & 1/7 \end{pmatrix}$$

Problema 2 Resolver la ecuación matricial $AX + BX = C + I$. Donde

$$A = \begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix}; \quad B = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}; \quad C = \begin{pmatrix} 2 & 2 \\ 1 & -3 \end{pmatrix}$$

Solución:

$$AX + BX = C + I \implies (A + B)X = C + I \implies X = (A + B)^{-1}(C + I)$$

$$(A + B)^{-1} = \begin{pmatrix} 1/3 & -2/3 \\ 0 & 1/2 \end{pmatrix}, \quad C + I = \begin{pmatrix} 3 & 2 \\ 1 & -2 \end{pmatrix}$$

$$X = (A + B)^{-1}(C + I) = \begin{pmatrix} 1/3 & 2 \\ 1/2 & -1 \end{pmatrix}$$

Problema 3 Resolver, utilizando las propiedades de los determinantes, la ecuación:

$$\begin{vmatrix} x & 1 & 0 & 1 \\ 1 & x & 1 & 0 \\ 1 & 0 & x & 1 \\ 0 & 1 & 1 & x \end{vmatrix} = 0$$

Solución:

$$\begin{vmatrix} x & 1 & 0 & 1 \\ 1 & x & 1 & 0 \\ 1 & 0 & x & 1 \\ 0 & 1 & 1 & x \end{vmatrix} = \begin{bmatrix} F_1 + F_2 + F_3 + F_4 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{vmatrix} x+2 & x+2 & x+2 & x+2 \\ 1 & x & 1 & 0 \\ 1 & 0 & x & 1 \\ 0 & 1 & 1 & x \end{vmatrix} =$$

$$(x+2) \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & x & 1 & 0 \\ 1 & 0 & x & 1 \\ 0 & 1 & 1 & x \end{vmatrix} = \begin{bmatrix} F_1 \\ F_2 - F_1 \\ F_3 - F_1 \\ F_4 \end{bmatrix} = (x+2) \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & x-1 & 0 & -1 \\ 0 & -1 & x-1 & 0 \\ 0 & 1 & 1 & x \end{vmatrix} =$$

$$(x+2) \begin{vmatrix} x-1 & 0 & -1 \\ -1 & x-1 & 0 \\ 1 & 1 & x \end{vmatrix} = x(x+2)(x^2 - 2x + 2) \implies x = 0, \quad x = -2$$