

$$(1) \quad 3^x = 6561 = 3^8 \rightarrow \boxed{x=8}$$

$$2^{3x+2} = 0.25 = 2^{-2} \rightarrow 3x+2 = -2 \rightarrow \boxed{x=-4/3}$$

$$\left(\frac{1}{3}\right)^x = 81 ; \quad 3^{-x} = 3^4 \rightarrow \boxed{x=-4}$$

$$3^x = \frac{1}{\sqrt{27}} = 3^{-3/2} \rightarrow \boxed{x=-3/2}$$

$$(2) \quad 3^x = 6000 ; \quad x = \frac{\ln 6000}{\ln 3} = \boxed{7.919}$$

$$2^x = 0.3 ; \quad x = \frac{\ln 0.3}{\ln 2} = \boxed{-1.737}$$

$$2^{3x+2} = 1000 ; \quad 3x+2 = \frac{\ln 1000}{\ln 2} = 9.966 \rightarrow x = \frac{7.966}{3} = \boxed{2.655}$$

$$\left(\frac{1}{3}\right)^x = 100 ; \quad x = \frac{\ln 100}{\ln 1/3} = \boxed{-4.192}$$

$$x = \log_5 38 = \frac{\ln 38}{\ln 5} = \boxed{2.260}$$

$$(3) \quad \log_2 32 = \boxed{5} \quad \log_{10} 0.1 = \boxed{-1} \quad \log_3 \frac{27}{\sqrt{3}} = \log_3 \frac{3^3}{3^{1/2}} = \log_3 3^{5/2} = \boxed{\frac{5}{2}} \quad \ln e^2 = \boxed{2}$$

$$(4) \quad {}_{10}\log_{10} x = \boxed{x} \quad e^{\ln x} = \boxed{x}$$

$${}_4\log_2 x = (2^2)\log_2 x = 2\log_2 x = 2\log_2(x^2) = \boxed{x^2}$$

$${}_2\log_4 x = (\sqrt{4})^3 \log_4 x = 4^{\frac{3}{2}} \log_4 x = 4\log_4(x^{3/2}) = \boxed{x^{3/2}} = \boxed{\sqrt{x^3}}$$

$${}_3\log_9 x = (9^{1/2})\log_9 x = 9^{\frac{1}{2}} \log_9 x = 9\log_9(x^{1/2}) = \boxed{x^{1/2}} = \boxed{\sqrt{x}}$$

Otra forma:

$${}_3\log_9 x = 3 \frac{\log_3 x}{\log_3 9} = 3 \frac{\log_3 x}{2} = 3^{\frac{1}{2}} \log_3 x = 3\log_3(x^{1/2}) = \boxed{x^{1/2}} = \boxed{\sqrt{x}}$$

$$(5) \quad \log_x 9 = 2 \rightarrow x^2 = 9 \rightarrow x = \boxed{3}$$

No puede ser negativa la base de un logaritmo.

$$\log_x \left(\frac{1}{3}\right) = -2 \rightarrow x^{-2} = \frac{1}{3} ; \quad \frac{1}{x^2} = \frac{1}{3} ; \quad x^2 = 3 \rightarrow x = \boxed{\sqrt{3}}$$

No existen logaritmos con bases negativas.

$$\log_2 x = 3 \rightarrow 2^3 = x \rightarrow \boxed{x=8}$$

$$\log_{10} x = -1 \rightarrow 10^{-1} = x \rightarrow \boxed{x = \frac{1}{10} = 0.1}$$

$$\log_8 2 = x \rightarrow 8^x = 2 \rightarrow (2^3)^x = 2 ; \quad 2^{3x} = 2 \rightarrow 3x = 1 ; \quad \boxed{x = \frac{1}{3}}$$

⑥ a) $5^{2x} - 30 \cdot 5^x + 125 = 0$

$t = 5^x$ $t^2 - 30t + 125 = 0$; $t = \begin{cases} 25 \rightarrow 5^x = 25 & \boxed{x=2} \\ 5 \rightarrow 5^x = 5 & \boxed{x=1} \end{cases}$

b) $9^x - 2 \cdot 3^{x+2} + 81 = 0$

$3^{2x} - 2 \cdot 3^x \cdot 3^2 + 81 = 0$

$t = 3^x$ $t^2 - 18t + 81 = 0$; $t = \begin{cases} 9 \rightarrow 3^x = 9 & \boxed{x=2} \\ 9 \end{cases}$

c) $2^{x-1} + 2^x + 2^{x+1} = 7$

$\frac{2^x}{2} + 2^x + 2 \cdot 2^x = 7$

$t = 2^x$ $\frac{t}{2} + t + 2t = 7$; $t + 2t + 4t = 14$; $7t = 14$; $t = 2 \rightarrow 2^x = 2$ $\boxed{x=1}$

⑦ a) $\log_{10} x = 3 - \log_{10} 50$; $\log_{10} x + \log_{10} 50 = 3$; $\log_{10}(50x) = 3$; $50x = 10^3$ $\boxed{x=20}$

b) $\log(3-5x) - \log 4 = \frac{1}{2} \log(3-x)$

$\log \frac{3-5x}{4} = \frac{1}{2} \log(3-x)$

$2 \log \frac{3-5x}{4} = \log(3-x)$

$\log \left(\frac{3-5x}{4} \right)^2 = 3-x \rightarrow \left(\frac{3-5x}{4} \right)^2 = 3-x$; $\frac{9-30x+25x^2}{16} = 3-x$;

$9-30x+25x^2 = 48-16x$; $25x^2 - 14x - 39 = 0$ $x = \begin{cases} \cancel{15/6} \\ -1 \end{cases}$ No es solución.

c) $\log x^3 = \log 6 + 2 \log x$

$\log x^3 = \log 6 + \log x^2$

$\log x^3 = \log(6x^2) \rightarrow x^3 = 6x^2$; $x^2(x-6) = 0 \rightarrow \begin{cases} x = \cancel{0} \\ x = 6 \end{cases}$ No está definido $\log 0$

⑧ $\log_x 100 = 2 + \log_x 25$

$\log_x 100 - \log_x 25 = 2$

$\log_x \left(\frac{100}{25} \right) = 2$

$\log_x 4 = 2 \rightarrow x^2 = 4 \rightarrow x = \begin{cases} 2 \\ \cancel{-2} \end{cases}$ La base no puede ser negativa.

$$9) B = 10 \log\left(\frac{I}{10^{-16}}\right)$$

$$a) I = 10^{14} \text{ W/m}^2 \rightarrow B = 10 \log \frac{10^{14}}{10^{-16}} = 10 \log 10^2 = \boxed{20 \text{ decibelios}}$$

$$b) B = 110 \text{ decibelios} \rightarrow 110 = 10 \log\left(\frac{I}{10^{-16}}\right) ; 11 = \log\left(\frac{I}{10^{-16}}\right) \rightarrow \frac{I}{10^{-16}} = 10^{11} \rightarrow$$

$$\boxed{I = 10^{-5} \text{ W/m}^2}$$

$$10) M_m = M_0 \left(1 + \frac{35}{100}\right)^n$$

$$\boxed{M_m = M_0 \cdot 1.035^n}$$

$$M_0 = 24.000$$

$$M_m = 2 \cdot 24.000 = 48.000$$

$$\rightarrow 48.000 = 24.000 \cdot 1.035^n ; 2 = 1.035^n \rightarrow n = \frac{\ln 2}{\ln 1.035}$$

$$= \boxed{20,149 \text{ años}}$$

$$M_0 = 50.000$$

$$M_m = 2 \cdot 50.000 = 100.000$$

$$\rightarrow 100.000 = 50.000 \cdot 1.035^n ; 2 = 1.035^n$$

$$M_0$$

$$M_m = 2M_0 \rightarrow 21\% = 1\% \cdot 1.035^n ; 2 = 1.035^n$$

El tiempo de duplicación será siempre el mismo.

11)

$$P_m = P_0 \cdot 1.04^n$$

$$P_0 = 200 \cdot 10^4 = \boxed{296.05 \text{ €}}$$

12)

$$C \xrightarrow{4\%} 1.04 \cdot C \xrightarrow{5\%} 1.05 \cdot 1.04 \cdot C \xrightarrow{2.5\%} 1.025 \cdot 1.05 \cdot 1.04 \cdot C$$

$$1.025 \cdot 1.05 \cdot 1.04 \cdot C = 1.1193C, \text{ que significa una subida del } \boxed{11,93\%}$$