Problema 1 Calcular los siguientes límites:

1. $\displaystyle \lim_{x\longrightarrow\infty} \frac{3x^3+5x^2-5x-1}{4x^3-7x^2+3}$

2. $\displaystyle \lim_{x\longrightarrow\infty} \left(\frac{3x^2-6x+2}{7x^2-3x-2}\right)^{x^2-8}$

3. $\displaystyle \lim_{x\longrightarrow\infty} \left(\frac{x^2-x+2}{x^2+5}\right)^{3x}$

4. $\displaystyle \lim_{x\longrightarrow\infty} \frac{\sqrt{7x^2-5+3}}{x^2+1}$

5. $\displaystyle \lim_{x\longrightarrow 1} \frac{8x^5+2x^4-9x^3+2x^2-2x-1}{3x^5-4x^4+10x^3-10x+1}$

6. $\displaystyle \lim_{x\longrightarrow 2} \frac{4x^3-2x^2-5x-14}{2x^3-3x^2-5x+6}$

7. $\displaystyle \lim_{x\longrightarrow 3} \frac{\sqrt{7x^2-2}-\sqrt{20x+1}}{x-3}$

8. $\displaystyle \lim_{x\longrightarrow 5} \frac{\sqrt{x^2+4}-\sqrt{4x+9}}{x-5}$

Solución:

1. $\displaystyle \lim_{x\longrightarrow\infty} \frac{3x^3+5x^2-5x-1}{4x^3-7x^2+3}=\frac{3}{4}$

2. $\displaystyle \lim_{x\longrightarrow\infty} \left(\frac{3x^2-6x+2}{7x^2-3x-2}\right)^{x^2-8}=0$

3. $\displaystyle \lim_{x\longrightarrow\infty} \left(\frac{x^2-x+2}{x^2+5}\right)^{3x}=e^{-3}$

4. $\displaystyle \lim_{x\longrightarrow\infty} \frac{\sqrt{7x^2-5+3}}{x^2+1}=0$

5. $\displaystyle \lim_{x\longrightarrow 1} \frac{8x^5+2x^4-9x^3+2x^2-2x-1}{3x^5-4x^4+10x^3-10x+1}=\frac{23}{19}$

6. $\displaystyle \lim_{x\longrightarrow 2} \frac{4x^3-2x^2-5x-14}{2x^3-3x^2-5x+6}=5$

7. $\lim\limits_{x \longrightarrow 3} \dfrac{\sqrt{7x^2-2}-\sqrt{20x+1}}{x-3} = \dfrac{11\sqrt{61}}{61}$

8. $\lim\limits_{x \longrightarrow 5} \dfrac{\sqrt{x^2+4}-\sqrt{4x+9}}{x-5} = \dfrac{3\sqrt{29}}{29}$

Problema 2 Calcular las siguientes derivadas:

1. $y = e^{3x^3-x^2+5x-1}$
2. $y = \ln(5x^2+1)$
3. $y = (3x^2-2x+5)^{15}$
4. $y = (x^2+5x-1)(2x^3+3x^2-1)$
5. $y = \frac{x^2-x-3}{7x+2}$
6. $y = \ln\frac{x^2+5}{x^2-2}$

Solución:

1. $y = e^{3x^3-x^2+5x-1} \Longrightarrow y' = (9x^2-2x+5)e^{3x^3-x^2+5x-1}$
2. $y = \ln(5x^2+1) \Longrightarrow y' = \dfrac{10x}{5x^2+1}$
3. $y = (3x^2-2x+5)^{15} \Longrightarrow y' = 15(3x^2-2x+5)^{14}(6x-2)$
4. $y = (x^2+5x-1)(2x^3+3x^2-1) \Longrightarrow y' = (2x+5)(2x^3+3x^2-1) + (x^2+5x-1)(6x^2+6x)$
5. $y = \dfrac{x^2-x-3}{7x+2} \Longrightarrow y' = \dfrac{(2x-1)(7x+2)-(x^2-x-3)7}{(7x+2)^2}$
6. $y = \ln\dfrac{x^2+5}{x^2-2} = \ln(x^2+5)-\ln(x^2-2) \Longrightarrow y' = \dfrac{2x}{x^2+5} - \dfrac{2x}{x^2-2}$

Problema 3 Calcular las rectas tangente y normal de las siguientes funciones:

1. $f(x) = \dfrac{5x^2+7}{x^2-3}$ en el punto $x = 1$.
2. $f(x) = \dfrac{x^2+3}{5x-1}$ en el punto $x = 0$.

Solución:

1. $b = f(a) \Longrightarrow b = f(1) = -6$ e $y - b = m(x - a)$

$$f'(x) = -\frac{44x}{(x^2-3)^2} \Longrightarrow m = f'(1) = -11$$

Recta Tangente: $y + 6 = -11(x - 1)$

Recta Normal: $y + 6 = \frac{1}{11}(x - 1)$

2. $b = f(a) \Longrightarrow b = f(0) = -3$ e $y - b = m(x - a)$

$$f'(x) = \frac{5x^2 - 2x - 15}{(5x-1)^2} \Longrightarrow m = f'(0) = -15$$

Recta Tangente: $y + 3 = -15x$

Recta Normal: $y + 3 = \dfrac{1}{15}x$