PROBLEMAS RESUELTOS DE CÁLCULO DE DERIVADAS

1) Calcular las derivadas de:

a) $f(x\ = -\frac{2x^5}{\cos x}$

$$f'(x) = -\frac{10x^4 \cos x + 2x^5 \operatorname{sen} x}{\cos^2 x}$$

b) $g(x) = -\frac{3}{2}\ln\sqrt{7x}$

Simplificamos antes de derivar, aplicando propiedades de logaritmos neperianos: $g(x) = -\frac{3}{2}\frac{1}{2}\ln 7x = -\frac{3}{4}\ln 7x \Rightarrow g'(x) = -\frac{3}{4}\frac{7}{7x} = -\frac{3}{4x}$

c) $h(x\ = \frac{e^{3x-}}{2}$

Como $h(x) = \frac{1}{2}e^{3x-5} \Rightarrow h'(x) = \frac{1}{2}3e^{3x-5} = \frac{3}{2}e^{3x-5}$

2) Halle f ’(2), g ’(4) y h ’(0) para las funciones definidas de la siguiente forma (L designa *logaritmo neperiano*):

$$f(x) = x^2 \quad \frac{16}{x^2}; \quad g(x) = (x \quad +9)^3; \quad h(x) = \mathrm{L}(x^2+1).$$

Simplemente, aplicando las reglas de derivación, se obtiene:

$f'(x) = 2x \quad \frac{32x}{x^4} = 2x - \frac{32}{x^3} \quad \Rightarrow f'(2) = 4 - \frac{32}{8} = 4–4 = 0$

$g'(x) = 3(x^2+9) \quad 2x = 6x(x^2+9) \quad \Rightarrow g'(4) = 24(16+9)^2 = 15.000$

$h'(x) = \frac{2x}{x^2+1} \quad \Rightarrow h'(0) = \frac{0}{1} = 0$

3) Derivar y simplificar:

a) $f(x) = \frac{3x-1}{x} - (5x-x^2)^2.$

$$f'(x) = \frac{3x-(3x-1)}{x^2} - 2(5x-x^2)(5-2x) = \frac{1}{x^2} - 2(25x - 10x^2 - 5x^2 + 2x^3) =$$

$$= \frac{1}{x^2} - 2(2x^3 - 15x^2 + 25x) = \frac{1}{x^2} - 4x^3 + 30x^2 - 50x =$$

$$= \boxed{\frac{-4x^5 + 30x^4 - 50x^3 + 1}{x^2}}$$

b) $g(x) = (x^2-1)\cdot \ln x.$

$$g'(x) = 2x\ln x + \frac{x^2-1}{x} = \boxed{\frac{2x^2\ln x + x^2 - 1}{x}}$$

c) $h(x) = 2^{5x}$.

$$h'(x) = \boxed{5\cdot 2^{5x}\ln 2}$$

d) $i(x) = (x^3 - 6x)\cdot(x^2+1)^3$.

$$i'(x) = (3x^2 - 6)(x^2 + 1)^3 + (x^3 - 6x)\ 3\cdot 2x\ (x^2 + 1)^2 =$$
$$= (x^2 + 1)^2\ ((3x^2 - 6)\ (x^2 + 1) + 6x(x^3 - 6x)) =$$
$$= (x^2 + 1)^2\ (3x^4 + 3x^2 - 6x^2 - 6 + 6x^4 - 36x^2) = \boxed{(x^2 + 1)^2\ (9x^4 - 39x^2 - 6)}$$

e) $j(x) = (x+1)\cdot e^{2x+1}$.

$$j'(x) = e^{2x+1} + (x + 1)\ 2\ e^{2x+1} = e^{2x+1}(1 + 2x + 2) = \boxed{e^{2x+1}(2x + 3)}$$

f) $k(x) = 3x\cos 3x^2$.

$$k'(x) = 3\cos 3x^2 - 3x\cdot 6x\ \text{sen}\ 3x^2 = \boxed{3\cos 3x^2 - 18x^2\ \text{sen}\ 3x^2}$$

Nota: La expresión simplificada final siempre puede resultar subjetiva, y debe entenderse como una expresión cómoda para operar y para volver a derivar si es preciso. Por ejemplo, en el d y el f se podría extraer 3 factor común.

4) Calcule las derivadas de las siguientes funciones:

a) $f(x) = \left(\dfrac{2-5x}{3}\right)^2 + \dfrac{1-2x}{x^2}$

$$f'(x) = 2\left(\frac{2-5x}{3}\right)\frac{-5}{3} + \frac{-2x^2-(1-2x)2x}{x^4} = \frac{-10(2-5x)}{9} + \frac{-2x^2-2x+4x^2}{x^4} =$$
$$= \frac{50x-20}{9} + \frac{2x^2-2x}{x^4} = \frac{50x-20}{9} + \frac{x(2x-2)}{x^4} = \frac{50x-20}{9} + \frac{2x-2}{x^3} =$$
$$= \frac{50x^4-20x^3}{9x^3} + \frac{18x-18}{9x^3} = \boxed{\frac{50x^2-20x^3+18x-18}{9x^3}}$$

b) $g(x) = (3x + 2)^2 \ln(1 + x^2)$

$$g'(x) = 2(3x + 2)3\ln(1 + x^2) + (3x + 2)^2\frac{2x}{1+x^2} =$$
$$= \boxed{(18x + 12)\ln(1 + x^2) + \frac{2x(3x+2)^2}{1+x^2}}$$

c) $h(x) = 2^{5x} + \dfrac{1}{x^2}$

$$h'(x) = \boxed{5\cdot 2^{5x}\ln 2 - \frac{2}{x^3}}$$

5) Calcule las derivadas de las siguientes funciones:

a) $f(x) = \dfrac{3x-1}{x} - (5x - x^2)^2$.

$$f'(x) = \frac{3x-(3x-1)}{x^2} - 2(5x-x^2)(5-2x) = \frac{3x-3x+1}{x^2} - (5x-x^2)(10-4x) =$$

$$= \frac{1}{x^2} - (50x - 20x^2 - 10x^2 + 4x^3) = \frac{1 - 50x^2 + 30x^4 - 4x^5}{x^2} =$$

$$= \boxed{\frac{-4x^5 + 30x^4 - 50x^2 + 1}{x^2}}$$

b) $g(x) = (x^2 - 1) \cdot \ln x.$

$$g'(x) = \boxed{2x \ln x + \frac{x^2 - 1}{x}}$$

c) $h(x) = 2^{3x}.$

$$h'(x) = \boxed{3 \cdot 2^{3x} \ln 2}$$

d) $i(x) = (x^3 - 6x) \cdot (x^2 + 1)^3.$

$$i'(x) = (3x^2 - 6)(x^2 + 1)^3 + (x^3 - 6x)3(x^2 + 1)^2 2x =$$
$$= (3x^2 - 6)(x^2 + 1)^3 + (6x^4 - 36x^2)(x^2 + 1)^2 =$$
$$= (x^2 + 1)^2[(3x^2 - 6)(x^2 + 1) + 6x^4 - 36x^2] =$$
$$= (x^2 + 1)^2(3x^4 + 3x^2 - 6x^2 - 6 + 6x^4 - 36x^2) =$$
$$= \boxed{(x^2 + 1)^2(9x^4 - 39x^2 - 6)}$$

6) Calcular las derivadas de:

a) $y = \frac{\operatorname{sen} x}{1 + \cos x} \Rightarrow y' = \frac{\cos x(1 + \cos x) - \operatorname{sen} x(-\operatorname{sen} x)}{(1 + \cos x)^2} =$

$$= \frac{\cos x + \cos^2 x + \operatorname{sen}^2 x}{(1 + \cos x)^2} = \frac{\cos x + 1}{(1 + \cos x)^2} = \frac{(1 + \cos x)}{(1 + \cos x)^2} = \boxed{\frac{1}{1 + \cos x}}$$

b) $y = \operatorname{arctg}(e^{-2x}) \Rightarrow y' = \frac{-2e^{-2x}}{1 + (e^{-2x})^2} = \boxed{-\frac{2e^{-2x}}{1 + e^{-4x}}}$

c) $y = \operatorname{sen}^3 3x \Rightarrow y' = 3\,(\operatorname{sen}^2 3x)\,(3 \cos 3x) = \boxed{9 \operatorname{sen}^2 3x \cos 3x}$

d) $y = \ln \frac{(x-2)^3}{\sqrt{2x-1}} = \ln (x-2)^3 - \ln \sqrt{2x-1} = 3 \ln(x-2) - \frac{1}{2} \ln(2x-1) \Rightarrow$

$$y' = 3\frac{1}{x-2} - \frac{1}{2}\frac{2}{2x-1} = \frac{3}{x-2} - \frac{1}{1}\frac{1}{2x-1} = \boxed{\frac{3}{x-2} - \frac{1}{2x-1}}$$

e) $y = x^3 e^{-3x} \Rightarrow y' = 3x^2 e^{-3x} + x^3 (-3) e^{-3x} = 3x^2 e^{-3x} - 3x^3 e^{-3x} =$

$$= \boxed{3x^2 e^{-3x}(1 - x)}$$

7) Derivar y simplificar: $y = \operatorname{arctg} 3x^2$; $y = \frac{x^2 e^{1-x}}{3}$; $y = \ln \sqrt[3]{\frac{(x-2)^2}{x-3}}$; $y = 2\cos^2 4x$

- $y = \operatorname{arctg} 3x^2 \Rightarrow y' = \frac{6x}{1 + (3x^2)^2} = \boxed{\frac{6x}{1 + 9x^4}}$
- $y = \frac{x^2 e^{1-x}}{3} = \frac{1}{3} x^2 e^{1-x} \Rightarrow y' = \frac{1}{3}(2xe^{1-x} + x^2(-1)e^{1-x}) = \boxed{\frac{xe^{1-x}(2-x)}{3}}$
- $y = \ln \sqrt[3]{\frac{(x-2)^2}{x-3}} = \frac{1}{3} \ln \frac{(x-2)^2}{x-3} = \frac{1}{3}\left[\ln(x-2)^2 - \ln(x-3)\right] =$

$$= \frac{1}{3}\left[2\ln(x-2) - \ln(x-3)\right] \Rightarrow y' = \frac{1}{3}\left(2\frac{1}{x-2} - \frac{1}{x-3}\right) = \boxed{\frac{2}{3(x-2)} - \frac{1}{3(x-3)}}$$

- $y = 2\cos^2 4x \Rightarrow y' = 2 \cdot 2\,(\cos 4x)\,(-\operatorname{sen} 4x)\,4 = \boxed{-16 \operatorname{sen} 4x \cos 4x} =$

$= -8 \cdot 2 \text{ sen } 4x \cos 4x = -8 \text{ sen } (2\cdot 4x) = \boxed{-8 \text{ sen } 8x}$

8) Derivar y simplificar: *(2 puntos)*

a) $y = e^{2x} \text{ tg } x \Rightarrow y' = 2e^{2x} \text{ tg } x + e^{2x}(1 + \text{tg}^2 x) = \boxed{e^{2x}(\text{tg}^2 x + 2 \text{ tg } x + 1)}$

b) $y = \ln \sqrt[3]{\dfrac{x^2}{x^2-4}} = \dfrac{1}{3}\left(2\ln x - \ln(x^2-4)\right) \Rightarrow y' = \boxed{\dfrac{1}{3}\left(\dfrac{2}{x} - \dfrac{2x}{x^2-4}\right)}$

c) $y = 2\cos^3 3x \Rightarrow y' = 2\cdot 3\cos^2 3x\,(-\text{sen } 3x)\cdot 3 = \boxed{-18 \text{ sen } 3x \cos^2 3x}$

d) $y = \text{arcsen } x^3 \Rightarrow y' = \dfrac{3x^2}{\sqrt{1-(x^3)^2}} = \boxed{\dfrac{3x^2}{\sqrt{1-x^6}}}$

9) Derivar las siguientes funciones, simplificando los resultados:

a) $y = 2e^{\cos 3x} \Rightarrow y' = 2e^{\cos 3x}(-\text{sen } 3x)3 = \boxed{-6\, e^{\cos 3x} \text{ sen } 3x}$

b) $y = \text{arctg}\sqrt{2x} \Rightarrow$

$$y' = \frac{\dfrac{2}{2\sqrt{2x}}}{1+\left(\sqrt{2x}\right)^2} = \frac{\dfrac{1}{\sqrt{2x}}}{1+2x} = \frac{\dfrac{1}{\sqrt{2x}}\dfrac{\sqrt{2x}}{\sqrt{2x}}}{1+2x} = \frac{\dfrac{\sqrt{2x}}{2x}}{1+2x} = \frac{\sqrt{2x}}{2x(1+2x)} = \boxed{\frac{\sqrt{2x}}{4x^2+2x}}$$

c) $y = \ln \sqrt[3]{\dfrac{(2x-3)^2}{x-3}} = \dfrac{1}{3}\ln\dfrac{(2x-3)^2}{x-3} = \dfrac{1}{3}\left[\ln(2x-3)^2 - \ln(x-3)\right] =$

$= \dfrac{1}{3}\left[2\ln(2x-3) - \ln(x-3)\right]$. Derivando:

$$y' = \frac{1}{3}\left[2\frac{2}{2x-3} - \frac{1}{x-3}\right] = \boxed{\frac{1}{3}\left[\frac{4}{2x-3} - \frac{1}{x-3}\right]}$$

d) $y = 3x \text{ tg } 4x \Rightarrow y' = 3 \text{ tg } 4x + 3x\dfrac{4}{\cos^2 4x} = \boxed{3 \text{ tg } 4x + \dfrac{12x}{\cos^2 4x}}$

e) $y = 2 \text{ sen}^2 3x \Rightarrow$

$y' = 4(\text{sen } 3x \cos 3x)\,3 = 6 \cdot 2 \text{ sen } 3x \cos 3x = 6 \text{ sen } 2\cdot 3x = \boxed{6 \text{ sen } 6x}$

10) Derivar las siguientes funciones, simplificando los resultados:

a) $y = 2xe^{\text{sen } 3x}$

$$y' = 2e^{\text{sen } 3x} + 2xe^{\text{sen } 3x}(\cos 3x)3 = \boxed{2e^{\text{sen } 3x}(1 + 3x\cos 3x)}$$

b) $y = \ln \sqrt[3]{\dfrac{(4x-3)^2}{x-1}}$

Simplificando la expresión antes de derivar, aplicando propiedades de logaritmos:

$$y = \frac{1}{3}[\ln(4x-3)^2 - \ln(x-1)] = \frac{1}{3}[2\ln(4x-3) - \ln(x-1)] \Rightarrow$$

$$\Rightarrow y' = \frac{1}{3}\left(2\frac{4}{4x-3} - \frac{1}{x-1}\right) = \boxed{\frac{1}{3}\left(\frac{8}{4x-3} - \frac{1}{x-1}\right)}$$

c) $y = 3x^{\cos 2x}$

Tomamos ln antes de derivar: $\ln y = \ln 3x^{\cos 2x} = \ln 3 + \ln x^{\cos 2x} = \ln 3 + \cos 2x \ln x$

Derivando miembro a miembro:

$$\frac{y'}{y} = 0 + -2\text{sen }2x\ln x + (\cos 2x)\frac{1}{x} = -2\text{sen }2x\ln x + \frac{\cos 2x}{x} = \frac{-2x\text{sen }2x\ln x + \cos 2x}{x}$$

$$\Rightarrow \quad y' = 3x^{\cos 2x}\,\frac{-2x\text{sen }2x\ln x + \cos 2x}{x} = \boxed{3x^{\cos 2x-1}(\cos 2x - 2x\text{sen }2x\ln x)}$$

d) $y = 2\,\text{sen}^2 3x$

$$y' = 2\cdot 2\text{ sen }3x\,(\cos 3x)\,3 = 6\cdot 2\text{ sen }3x\cos 3x = 6\text{ sen }2\cdot 3x = \boxed{6\text{ sen }6x}$$

11) Derivar las siguientes funciones, simplificando los resultados:

a) $y = 3xe^{\cos x^2}$

$$y' = 3e^{\cos x^2} + 3xe^{\cos x^2}2x(-\text{sen }x^2) = \boxed{3e^{\cos x^2}(1 - 2x^2\text{ sen }x^2)}$$

b) $y = \ln\sqrt[5]{\dfrac{4x^2 -}{(x-1)^2}}$

Simplificando la expresión antes de derivar, aplicando propiedades de logaritmos:

$$y = \frac{1}{5}[\ln(4 \quad -3) - \ln(x-1)^2] = \frac{1}{5}[\ln(4 \;{}^2 - 3) - 2\ln(x-1)] \Rightarrow$$

$$\Rightarrow \quad y' = \frac{1}{5}\left(\frac{8x}{4x^2-3} - 2\frac{1}{\quad 1}\right. = \boxed{\frac{1}{5}\left(\frac{8x}{4x^2-3} - \frac{2}{\quad 1}\right.}$$

c) $y = \text{arctg }\sqrt{3x}$

$$y' = \frac{\dfrac{3}{2\sqrt{\;x}}}{+\left(\sqrt{3x}\right)^2} = \frac{\dfrac{3}{2\sqrt{\;x}}}{1+\;x} = \boxed{\frac{3}{2\sqrt{3x}(1+3x)}}$$

d) $y = 3\cos^2 5x$

$$y' = 3\cdot 2\cos 5x\,(-5\text{ sen }5x) = -15\cdot 2\text{ sen }5x\cos 5x = -15\text{ sen }2\cdot 5x = \boxed{-15\text{ sen }10x}$$

12) Calcule las derivadas de las siguientes funciones:

$$f(x) = \frac{2^x + x^2}{x}\;; \quad g(x) = (x^2+1)^2\cdot\ln(e^{3x}+4)$$

$$f'(x) = \frac{(2^x\ln 2 + 2x)x - (2^x + x^2)\cdot 1}{x^2} = \frac{2^x x\ln 2 + 2x^2 - 2^x - x^2}{x^2} = \boxed{\frac{2^x x\ln 2 + x^2 - 2^x}{x^2}}$$

$$g'(x) = 2(x^2+1)2x\ln(e^{3x}+4) + (x^2+1)^2\frac{3e^{3x}}{e^{3x}+4} =$$

$$= \boxed{(4x^3+4x)\ln(e^{3x}+4) + \frac{3e^{3x}(x^2+1)^2}{e^{3x}+4}}$$

13) Derivar y simplificar:

a) $y = 2(7x^3 - 3x)^6$

$$y' = 2\cdot 6(7x^3-3x)^5(21x^2-3) = 12(21x^2-3)(7x^3-3x)^5 =$$

$$= \boxed{(252x^2-36)(7x^3-3x)^5}$$

b) $y = \dfrac{3x^2 - 12}{x - 1}$

$$y' = \frac{6x(x-1) - (3x^2 - 12)\cdot 1}{(x-1)^2} = \frac{6x^2 - 6x - 3x^2 + 12}{(x-1)^2} = \boxed{\frac{3x^2 - 6x + 12}{(x-1)^2}}$$

c) $y = \sqrt{2x^2 + 1}$

$$y' = \frac{4x}{2\sqrt{2x^2+1}} = \boxed{\frac{2x}{\sqrt{2x^2+1}}}$$

d) $y = (x + 1)e^{2x + 1}$

$$y' = 1\cdot e^{2x+1} + (x+1)2\,e^{2x+1} = e^{2x+1}[1 + 2(x+1)] = e^{2x+1}(1 + 2x + 2) =$$
$$= \boxed{e^{2x+1}(2x+3)}$$

14) Derivar y simplificar:

a) $y = 2(7x^2 - 3x)^5$

$$y' = 2\cdot 5(7x^2 - 3x)^4(14x - 3) = \boxed{10(14x - 3)\,(7x^2 - 3x)^4}$$

b) $y = \dfrac{x - 1}{3x^4 - 2}$

$$y' = \frac{3x^4 - 2 - 12x^3(x-1)}{(3x^4 - 2)^2} = \frac{3x^4 - 2 - 12x^4 + 12x^3}{(3x^4-2)^2} = \boxed{\frac{-9x^4 + 12x^3 - 2}{(3x^4 - 2)^2}}$$

c) $y = \operatorname{sen} \sqrt{2x}$

$$y' = \frac{2}{2\sqrt{2x}}\cos\sqrt{2x} = \boxed{\frac{\cos\sqrt{2x}}{\sqrt{2x}}}$$

d) $y = e^{2x + 1} \ln 3x$

$$y' = 2e^{2x+1}\ln 3x + e^{2x+1}\frac{3}{3x} = e^{2x+1}\left(2\ln 3x + \frac{1}{x}\right) = \boxed{e^{2x+1}\frac{1 + 2x\ln 3x}{x}}$$