Polinomios

Ejercicios resueltos

2.1-1 Realiza la suma de los siguientes polinomios:

$a)$ $p(x)=x^5+x^4-4x^3+6x^2+x-7$
$q(x)=x^6+2x^4+x^2+5$

$b)$ $p(x)=9x^5-2x^4+12x^3+x^2-x+10$
$q(x)=-x^5+5x^4-12x^3-2x^2+x-15$

$c)$ $p(x)=-5x^4+6x^3-2x^2+3x+8$
$q(x)=2x^4-3x^3+2x^2-4$

$d)$ $p(x)=3x^4+x^3-2x^2+x-14$
$q(x)=6x^4-8x^3+2x^2-3x$
$r(x)=2x+14$

$e)$ $p(x)=-x^6+4x^5-2x^4-7x^3+6x^2+x-2$
$q(x)=3x^6+2x^5-x^3+2x^2-2x+5$
$r(x)=-2x^6-6x^5+2x^4+8x^3-8x^2+x-3$

$f)$ $p(x)=x^4-3x^3+x^2-7x+11$
$q(x)=2x^5-3x^4+x^3-x^2-7$
$r(x)=-3x^5+2x^4-5x^3+8x^2+3x-4$

Solución

$a)$ $\left.\begin{array}{l} p(x)=x^5+x^4-4x^3+6x^2+x-7 \\ q(x)=x^6+2x^4+x^2+5 \end{array}\right\}$

$p(x)+q(x)=x^6+x^5+3x^4-4x^3+7x^2+x-2$

$b)$ $\left.\begin{array}{l} p(x)=9x^5-2x^4+12x^3+x^2-x+10 \\ q(x)=-x^5+5x^4-12x^3-2x^2+x-15 \end{array}\right\}$

$p(x)+q(x)=8x^5+3x^4-x^2-5$

www.yoquieroaprobar.es

$$c)\quad \left.\begin{array}{l} p(x)=-5x^4+6x^3-2x^2+3x+8 \\ q(x)=2x^4-3x^3+2x^2-4 \end{array}\right\}$$

$$p(x)+q(x)=-3x^4+3x^3+3x+4$$

$$d)\quad \left.\begin{array}{l} p(x)=3x^4+x^3-2x^2+x-14 \\ q(x)=6x^4-8x^3+2x^2-3x \\ r(x)=2x+14 \end{array}\right\}$$

$$\begin{aligned} p(x)+q(x)+r(x)&=\left[p(x)+q(x)\right]+r(x)= \\ &=\left[9x^4-7x^3-2x-14\right]+2x+14=9x^4-7x^3 \end{aligned}$$

$$e)\quad \left.\begin{array}{l} p(x)=-x^6+4x^5-2x^4-7x^3+6x^2+x-2 \\ q(x)=3x^6+2x^5-x^3+2x^2-2x+5 \\ r(x)=-2x^6-6x^5+2x^4+8x^3-8x^2+x-3 \end{array}\right\}$$

$$\begin{aligned} p(x)+q(x)+r(x)&=\left[p(x)+q(x)\right]+r(x)= \\ &=\left[2x^6+6x^5-2x^4-8x^3+8x^2-x+3\right]+ \\ &+\left[-2x^6-6x^5+2x^4+8x^3-8x^2+x-3\right]=0 \end{aligned}$$

$$f)\quad \left.\begin{array}{l} p(x)=x^4-3x^3+x^2-7x+11 \\ q(x)=2x^5-3x^4+x^3-x^2-7 \\ r(x)=-3x^5+2x^4-5x^3+8x^2+3x-4 \end{array}\right\}$$

$$\begin{aligned} p(x)+q(x)+r(x)&=\left[p(x)+q(x)\right]+r(x)= \\ &=\left[2x^5-2x^4-2x^3-7x+4\right]+\left[-3x^5+2x^4-5x^3+8x^2+3x-4\right]= \\ &=-x^5-7x^3+8x^2-4x \end{aligned}$$

www.yoquieroaprobar.es

2.1-2 Realiza la resta de los siguientes polinomios:

$a)$ $p(x)=x^6+2x^5-3x^4+x^3+4x^2+4x-4$
$q(x)=-x^6+2x^5-5x^4+x^3+2x^2+3x-8$

$b)$ $p(x)=-3x^3+7x^2-3x-2$
$q(x)=5x^3+5x^2+5x+5$

$c)$ $p(x)=x^4+4x^3-2x^2+7x+10$
$q(x)=-2x^4+5x^3-8x^2+3x+11$

$d)$ $p(x)=-x^5+5x^3+4x^2-x+1$
$q(x)=x^4+9x^3-3x^2+x-1$

$e)$ $p(x)=-7x^3+x^2-12x-2$
$q(x)=-6x^3+3x^2-13x+15$

$f)$ $p(x)=x^4+3x^3-3x^2+2x+14$
$q(x)=-x^5-2x^4+3x^3-3x+14$

Solución

$a)$ $\left.\begin{array}{l} p(x)=x^6+2x^5-3x^4+x^3+4x^2+4x-4 \\ q(x)=-x^6+2x^5-5x^4+x^3+2x^2+3x-8 \end{array}\right\}$

$$\begin{aligned} p(x)-q(x)&=p(x)+\left[-q(x)\right]= \\ &=x^6+2x^5-3x^4+x^3+4x^2+4x-4- \\ &\quad-\left[-x^6+2x^5-5x^4+x^3+2x^2+3x-8\right] \end{aligned}$$

$$p(x)-q(x)=2x^6+2x^4+2x^2+x+4$$

$b)$ $\left.\begin{array}{l} p(x)=-3x^3+7x^2-3x-2 \\ q(x)=5x^3+5x^2+5x+5 \end{array}\right\}$

$$\begin{aligned} p(x)-q(x)&=p(x)+\left[-q(x)\right]= \\ &=-3x^3+7x^2-3x-2-\left[5x^3+5x^2+5x+5\right] \end{aligned}$$

$$p(x)-q(x)=-8x^3+2x^2-8x-7$$

www.yoquieroaprobar.es

$c)$ $\left.\begin{array}{l} p(x)=x^4+4x^3-2x^2+7x+10 \\ q(x)=-2x^4+5x^3-8x^2+3x+11 \end{array}\right\}$

$$p(x)-q(x)=p(x)+\left[-q(x)\right]=$$
$$=x^4+4x^3-2x^2+7x+10-\left[-2x^4+5x^3-8x^2+3x+11\right]$$

$$p(x)-q(x)=3x^4-x^3+6x^2+4x-1$$

$d)$ $\left.\begin{array}{l} p(x)=-x^5+5x^3+4x^2-x+1 \\ q(x)=x^4+9x^3-3x^2+x-1 \end{array}\right\}$

$$p(x)-q(x)=p(x)+\left[-q(x)\right]=$$
$$=-x^5+5x^3+4x^2-x+1-\left[x^4+9x^3-3x^2+x-1\right]$$

$$p(x)-q(x)=-x^5-x^4-4x^3+7x^2-2x+2$$

$e)$ $\left.\begin{array}{l} p(x)=-7x^3+x^2-12x-2 \\ q(x)=-6x^3+3x^2-13x+15 \end{array}\right\}$

$$p(x)-q(x)=p(x)+\left[-q(x)\right]=$$
$$=-7x^3+x^2-12x-2-\left[-6x^3+3x^2-13x+15\right]$$

$$p(x)-q(x)=-x^3-2x^2+x-17$$

$f)$ $p(x)=x^4+3x^3-3x^2+2x+14$
$q(x)=-x^5-2x^4+3x^3-3x+14$

$$p(x)-q(x)=p(x)+\left[-q(x)\right]=$$
$$=x^4+3x^3-3x^2+2x+14-\left[-x^5-2x^4+3x^3-3x+14\right]$$

$$p(x)-q(x)=x^5+3x^4-3x^2+5x$$

www.yoquieroaprobar.es

2.1-3 Realiza el producto de los siguientes polinomios:

$a)$ $p(x)=x^4+2x^3-x^2+3x+1$
$q(x)=2x$

$b)$ $p(x)=-x^5+x^4-x^3+x^2-x+1$
$q(x)=-5x^4$

$c)$ $p(x)=2x^6+3x^4+x^2-6$
$q(x)=x^3+x$

$d)$ $p(x)=x^2+2x+3$
$q(x)=-x^2+x+4$

$e)$ $p(x)=x^5+3x^3+6x$
$q(x)=-x^2+2x-2$
$r(x)=2x^3+5x^2-2x+3$

$f)$ $p(x)=-x^2+2x+3$
$q(x)=x^2+x+1$
$r(x)=x^4+x^3+x^2+x+4$

Solución

$a)$ $\left.\begin{array}{l} p(x)=x^4+2x^3-x^2+3x+1 \\ q(x)=2x \end{array}\right\}$

$$\begin{aligned} p(x)\cdot q(x) &= \left[x^4+2x^3-x^2+3x+1\right]\cdot 2x = \\ &= x^4\cdot 2x+2x^3\cdot 2x-x^2\cdot 2x+3x\cdot 2x+1\cdot 2x = \\ &= 2x^5+4x^4-2x^3+6x^2+2x \end{aligned}$$

$b)$ $\left.\begin{array}{l} p(x)=-x^5+x^4-x^3+x^2-x+1 \\ q(x)=-5x^4 \end{array}\right\}$

$$\begin{aligned} p(x)\cdot q(x) &= \left[-x^5+x^4-x^3+x^2-x+1\right]\cdot\left(-5x^4\right) = \\ &= 5x^9-5x^8+5x^7-5x^6+5x^5-5x^4 \end{aligned}$$

www.yoquieroaprobar.es

c) $\left.\begin{array}{l} p(x)=2x^6+3x^4+x^2-6 \\ q(x)=x^3+x \end{array}\right\}$

$$\begin{aligned} p(x)\cdot q(x) &= \left[2x^6+3x^4+x^2-6\right]\cdot\left[x^3+x\right]= \\ &= \left[2x^6+3x^4+x^2-6\right]\cdot x^3+\left[2x^6+3x^4+x^2-6\right]\cdot x= \\ &= \left[2x^9+3x^7+x^5-6x^3\right]+\left[2x^7+3x^5+x^3-6x\right]= \\ &= 2x^9+5x^7+4x^5-5x^3-6x \end{aligned}$$

d) $\left.\begin{array}{l} p(x)=x^2+2x+3 \\ q(x)=-x^2+x+4 \end{array}\right\}$

$$\begin{aligned} p(x)\cdot q(x) &= \left[x^2+2x+3\right]\cdot\left[-x^2+x+4\right]= \\ &= \left[x^2+2x+3\right]\cdot\left(-x^2\right)+\left[x^2+2x+3\right]\cdot x+\left[x^2+2x+3\right]\cdot 4= \\ &= -x^4-2x^3-3x^2+x^3+2x^2+3x+4x^2+8x+12= \\ &= -x^4-x^3+3x^2+11x+12 \end{aligned}$$

e) $\left.\begin{array}{l} p(x)=x^5+3x^3+6x \\ q(x)=-x^2+2x-2 \\ r(x)=2x^3+5x^2-2x+3 \end{array}\right\}$

$$\begin{aligned} p(x)\cdot q(x)\cdot r(x) &= \left[p(x)\cdot q(x)\right]\cdot r(x)= \\ &= \left[\left(x^5+3x^3+6x\right)\cdot\left(-x^2+2x-2\right)\right]\cdot\left(2x^3+5x^2-2x+3\right)= \\ &= \left(-x^7-3x^5-6x^3+2x^6+6x^4+12x^2-2x^5-6x^3-12x\right)\cdot \\ &\quad\cdot\left(2x^3+5x^2-2x+3\right) \end{aligned}$$

$$\begin{aligned} p(x)\cdot q(x)\cdot r(x) &= \left(-x^7+2x^6-5x^5+6x^4-12x^3+12x^2-12x\right)\cdot \\ &\quad\cdot\left(2x^3+5x^2-2x+3\right) \end{aligned}$$

$$\begin{aligned} p(x)\cdot q(x)\cdot r(x) &= \\ &= -2x^{10}+4x^9-10x^8+12x^7-24x^6+24x^5-24x^4- \\ &\quad -5x^9+10x^8-25x^7+30x^6-60x^5+60x^4-60x^3+ \\ &\quad +2x^8-4x^7+10x^6-12x^5+24x^4-24x^3+24x^2- \\ &\quad -3x^7+6x^6-15x^5+18x^4-36x^3+36x^2-36x \end{aligned}$$

$$\begin{aligned} &p(x)\cdot q(x)\cdot r(x)= \\ &= -2x^{10}-x^9+2x^8-20x^7+22x^6-63x^5+78x^4-120x^3+60x^2-36x \end{aligned}$$

www.yoquieroaprobar.es

$$f)\quad \left.\begin{array}{l} p(x)=-x^2+2x+3 \\ q(x)=x^2+x+1 \\ r(x)=x^4+x^3+x^2+x+4 \end{array}\right\}$$

$$p(x)\cdot q(x)\cdot r(x)=\left[p(x)\cdot q(x)\right]\cdot r(x)=$$
$$=\left[\left(-x^2+2x+3\right)\cdot\left(x^2+x+1\right)\right]\cdot\left(x^4+x^3+x^2+x+4\right)$$

$$p(x)\cdot q(x)\cdot r(x)=\left[p(x)\cdot q(x)\right]\cdot r(x)=$$
$$=\left(-x^4+2x^3+3x^2-x^3+2x^2+3x-x^2+2x+3\right)\cdot$$
$$\cdot\left(x^4+x^3+x^2+x+4\right)$$

$$p(x)\cdot q(x)\cdot r(x)=\left[p(x)\cdot q(x)\right]\cdot r(x)=$$
$$=\left(-x^4+x^3+4x^2+5x+3\right)\cdot\left(x^4+x^3+x^2+x+4\right)$$

$$p(x)\cdot q(x)\cdot r(x)=\left[p(x)\cdot q(x)\right]\cdot r(x)=$$
$$=-x^8+x^7+4x^6+5x^5+3x^4-$$
$$-x^7+x^6+4x^5+5x^4+3x^3-$$
$$-x^6+x^5+4x^4+5x^3+3x^2-$$
$$-x^5+x^4+4x^3+5x^2+3x-$$
$$-4x^4+4x^3+16x^2+20x+12$$

$$p(x)\cdot q(x)\cdot r(x)=$$
$$=-x^8+4x^6+9x^5+9x^4+16x^3+24x^2+23x+12$$

www.yoquieroaprobar.es

2.1-4 Dados los siguientes polinomios, realiza la operación que se indica:

$$a)\quad \left.\begin{array}{l} p(x)=x^3+6x^2-x+2 \\ q(x)=2x^2+4x-3 \end{array}\right\} \quad \left(p(x)+q(x)\right)\cdot\left(p(x)-q(x)\right)$$

$$b)\quad \left.\begin{array}{l} p(x)=x^2-5x+2 \\ q(x)=-x^2+3x-4 \end{array}\right\} \quad \left(3\cdot p(x)+q(x)\right)\cdot\left(p(x)-2\cdot q(x)\right)$$

Solución

$$a)\quad \left.\begin{array}{l} p(x)=x^3+6x^2-x+2 \\ q(x)=2x^2+4x-3 \end{array}\right\} \quad \left(p(x)+q(x)\right)\cdot\left(p(x)-q(x)\right)$$

$$\left.\begin{array}{l} p(x)+q(x)=x^3+8x^2+3x-1 \\ p(x)-q(x)=x^3+4x^2-5x+5 \end{array}\right\}$$

$$\left(p(x)+q(x)\right)\cdot\left(p(x)-q(x)\right)=\left(x^3+8x^2+3x-1\right)\cdot\left(x^3+4x^2-5x+5\right)$$

$$\begin{aligned}\left(p(x)+q(x)\right)\cdot\left(p(x)-q(x)\right)=\ & x^6+8x^5+3x^4-x^3+ \\ & +4x^5+32x^4+12x^3-4x^2- \\ & -5x^4-40x^3-15x^2+5x+ \\ & +5x^3+40x^2+15x-5\end{aligned}$$

$$\left(p(x)+q(x)\right)\cdot\left(p(x)-q(x)\right)=x^6+12x^5+30x^4-24x^3+21x^2+20x-5$$

$$b)\quad \left.\begin{array}{l} p(x)=x^2-5x+2 \\ q(x)=-x^2+3x-4 \end{array}\right\} \quad \left(3\cdot p(x)+q(x)\right)\cdot\left(p(x)-2\cdot q(x)\right)$$

$$\left.\begin{array}{l} 3\cdot p(x)+q(x)=3x^2-15x+6-x^2+3x-4=2x^2-12x+2 \\ p(x)-2\cdot q(x)=x^2-5x+2+2x^2-6x+8=3x^2-11x+10 \end{array}\right\}$$

$$\left(3\cdot p(x)+q(x)\right)\cdot\left(p(x)-2\cdot q(x)\right)=\left(2x^2-12x+2\right)\cdot\left(3x^2-11x+10\right)$$

$$\begin{aligned}\left(3\cdot p(x)+q(x)\right)\cdot\left(p(x)-2\cdot q(x)\right)=\ & 6x^4-36x^3+6x^2- \\ & -22x^3+132x^2-22x+ \\ & +20x^2-120x+20\end{aligned}$$

$$\left(3\cdot p(x)+q(x)\right)\cdot\left(p(x)-2\cdot q(x)\right)=6x^4-58x^3+158x^2-142x+20$$

www.yoquieroaprobar.es

2.1-5 Calcula las siguientes potencias de binomios utilizando el desarrollo del binomio de Newton:

a) $(x+1)^4$

b) $(x-4)^3$

c) $(2x+3)^5$

d) $(x^2+2)^3$

e) $(x^3+1)^5$

f) $(3x^2-1)^4$

Solución

a) $$(x+1)^4=\binom{4}{0}x^4+\binom{4}{1}x^3\cdot 1^1+\binom{4}{2}x^2\cdot 1^2+\binom{4}{3}x\cdot 1^3+\binom{4}{4}1^4=$$

$$=\frac{4!}{4!\cdot 0!}x^4+\frac{4!}{3!\cdot 1!}x^3+\frac{4!}{2!\cdot 2!}x^2+\frac{4!}{1!\cdot 3!}x+\frac{4!}{0!\cdot 4!}=$$

$$=x^4+4x^3+6x^2+4x+1$$

b) $$(x-4)^3=\binom{3}{0}x^3+\binom{3}{1}x^2\cdot(-4)^1+\binom{3}{2}x\cdot(-4)^2+\binom{3}{3}(-4)^3=$$

$$=\frac{3!}{3!\cdot 0!}x^3-4\frac{3!}{2!\cdot 1!}x^2+16\frac{3!}{1!\cdot 2!}x-64\frac{3!}{0!\cdot 3!}=$$

$$=x^3-12x^2+48x-64$$

c) $$(2x+3)^5=$$

$$=\binom{5}{0}(2x)^5+\binom{5}{1}(2x)^4\cdot(3)^1+\binom{5}{2}(2x)^3\cdot(3)^2+$$

$$+\binom{5}{3}(2x)^2\cdot(3)^3+\binom{5}{4}(2x)^1\cdot(3)^4+\binom{5}{5}(3)^5=$$

$$=32\frac{5!}{5!\cdot 0!}x^5+48\frac{5!}{4!\cdot 1!}x^4+72\frac{5!}{3!\cdot 2!}x^3+$$

$$+108\frac{5!}{2!\cdot 3!}x^2+162\frac{5!}{1!\cdot 4!}x+243\frac{5!}{0!\cdot 5!}=$$

$$=32x^5+240x^4+720x^3+1080x^2+810x+243$$

www.yoquieroaprobar.es

$$d)\quad \left(x^2+2\right)^3=\binom{3}{0}\left(x^2\right)^3+\binom{3}{1}\left(x^2\right)^2\cdot(2)^1+\binom{3}{2}\left(x^2\right)^1\cdot(2)^2+\binom{3}{3}(2)^3=$$

$$=\frac{3!}{3!\cdot 0!}x^6+2\frac{3!}{2!\cdot 1!}x^4+4\frac{3!}{1!\cdot 2!}x^2+8\frac{3!}{0!\cdot 3!}=$$

$$=x^6+6x^4+12x^2+8$$

$$e)\quad \left(x^3+1\right)^5=$$

$$=\binom{5}{0}\left(x^3\right)^5+\binom{5}{1}\left(x^3\right)^4\cdot(1)^1+\binom{5}{2}\left(x^3\right)^3\cdot(1)^2+$$

$$+\binom{5}{3}\left(x^3\right)^2\cdot(1)^3+\binom{5}{4}\left(x^3\right)^1\cdot(1)^4+\binom{5}{5}(1)^5=$$

$$=\frac{5!}{5!\cdot 0!}x^{15}+\frac{5!}{4!\cdot 1!}x^{12}+\frac{5!}{3!\cdot 2!}x^9+\frac{5!}{2!\cdot 3!}x^6+\frac{5!}{1!\cdot 4!}x^3+\frac{5!}{0!\cdot 5!}=$$

$$=x^{15}+5x^{12}+10x^9+10x^6+5x^3+1$$

$$f)\quad \left(3x^2-1\right)^4=$$

$$=\binom{4}{0}\left(3x^2\right)^4+\binom{4}{1}\left(3x^2\right)^3\cdot(-1)^1+$$

$$+\binom{4}{2}\left(3x^2\right)^2\cdot(-1)^2+\binom{4}{3}\left(3x^2\right)^1\cdot(-1)^3+\binom{4}{4}(-1)^4=$$

$$=\frac{4!}{4!\cdot 0!}81x^8-\frac{4!}{3!\cdot 1!}27x^6+\frac{4!}{2!\cdot 2!}9x^4-\frac{4!}{1!\cdot 3!}3x^2+\frac{4!}{0!\cdot 4!}=$$

$$=81x^8-108x^6+54x^4-12x^2+1$$

www.yoquieroaprobar.es

2.1-6 Realiza la división entera de los siguientes polinomios:

$a)$ $p(x)=x^2-6x+4$
$q(x)=x^2-2$

$b)$ $p(x)=x^5+3x^3+6x-2$
$q(x)=x^3+x$

$c)$ $p(x)=x^6+3$
$q(x)=x^2+2x-4$

$d)$ $p(x)=x^6+2x^5-x^4+2x^3+6x^2-x+3$
$q(x)=x^3+10x^2-2x+3$

Solución

$a)$ $\left.\begin{array}{l} p(x)=x^2-6x+4 \\ q(x)=x^2-2 \end{array}\right\} \Rightarrow p(x)=c(x)\cdot q(x)+r(x)$

$$\begin{array}{r|l} x^2-6x+4 & x^2-2 \\ \cline{2-2} -x^2+2 & 1 \\ \hline -6x+6 & \end{array}$$

$\left.\begin{array}{l} c(x)=1 \\ r(x)=-6x+6 \end{array}\right\} \Rightarrow x^2-6x+4=1\cdot(x^2-2)+(-6x+6)$

$b)$ $\left.\begin{array}{l} p(x)=x^5+3x^3+6x-2 \\ q(x)=x^3+x \end{array}\right\} \Rightarrow p(x)=c(x)\cdot q(x)+r(x)$

$$\begin{array}{r|l} x^5+3x^3+6x-2 & x^3+x \\ \cline{2-2} -x^5-x^3 & x^2+2 \\ \hline 2x^3+6x-2 & \\ -2x^3-2x & \\ \hline 4x-2 & \end{array}$$

$\left.\begin{array}{l} c(x)=x^2+2 \\ r(x)=4x-2 \end{array}\right\} \Rightarrow x^5+3x^3+6x-2=(x^2+2)\cdot(x^3+x)+(4x-2)$

www.yoquieroaprobar.es

c) $\left.\begin{array}{l} p(x)=x^6+3 \\ q(x)=x^2+2x-4 \end{array}\right\} \Rightarrow p(x)=c(x)\cdot q(x)+r(x)$

$$
\begin{array}{r|l}
x^6+3 & x^2+2x-4 \\ \cline{2-2}
-x^6-2x^5+4x^4 & x^4-2x^3+8x^2-24x+80 \\ \cline{1-1}
-2x^5+4x^4+3 & \\
2x^5+4x^4-8x^3 & \\ \cline{1-1}
8x^4-8x^3+3 & \\
-8x^4-16x^3+32x^2 & \\ \cline{1-1}
-24x^3+32x^2+3 & \\
24x^3+48x^2-96x & \\ \cline{1-1}
80x^2-96x+3 & \\
-80x^2-160x+320 & \\ \cline{1-1}
-256x+323 &
\end{array}
$$

$$\left.\begin{array}{l} c(x)=x^4-2x^3+8x^2-24x+80 \\ r(x)=-256x+323 \end{array}\right\}$$

$$\Rightarrow x^6+3=\left(x^4-2x^3+8x^2-24x+80\right)\cdot\left(x^2+2x-4\right)+\left(-256x+323\right)$$

d) $\left.\begin{array}{l} p(x)=x^6+2x^5-x^4+2x^3+6x^2-x+3 \\ q(x)=x^3+10x^2-2x+3 \end{array}\right\} \Rightarrow p(x)=c(x)\cdot q(x)+r(x)$

$$
\begin{array}{r|l}
x^6+2x^5-x^4+2x^3+6x^2-x+3 & x^3+10x^2-2x+3 \\ \cline{2-2}
-x^6-10x^5+2x^4-3x^3 & x^3-8x^2+81x-827 \\ \cline{1-1}
-8x^5+x^4-x^3+6x^2-x+3 & \\
8x^5+80x^4-16x^3+24x^2 & \\ \cline{1-1}
81x^4-17x^3+30x^2-x+3 & \\
-81x^4-810x^3+162x^2-243x & \\ \cline{1-1}
-827x^3+192x^2-244x+3 & \\
827x^3+8270x^2-1654x+2481 & \\ \cline{1-1}
8462x^2-1898x+2484 &
\end{array}
$$

$$\left.\begin{array}{l} c(x)=x^3-8x^2+81x-827 \\ r(x)=8462x^2-1898x+2484 \end{array}\right\}$$

$$\Rightarrow x^6+2x^5-x^4+2x^3+6x^2-x+3=$$

$$=\left(x^3-8x^2+81x-827\right)\cdot\left(x^3+10x^2-2x+3\right)+\left(8462x^2-1898x+2484\right)$$

www.yoquieroaprobar.es

2.1-7 Factoriza según sus raíces reales los siguientes polinomios:

$a)\ \ p(x)=x^2-4x-5$

$b)\ \ p(x)=x^3-5x^2+6x$

$c)\ \ p(x)=x^4-13x^2+36$

$d)\ \ p(x)=x^4-16$

Solución

$a)\ \ p(x)=x^2-4x-5$

$$x^2-4x-5=0\Rightarrow x=\frac{4\pm\sqrt{16+20}}{2}=\frac{4\pm 6}{2}=5,-1$$

$$p(x)=x^2-4x-5=(x-5)\cdot(x+1)$$

$b)\ \ p(x)=x^3-5x^2+6x$

$$x^3-5x^2+6x=x\cdot\left(x^2-5x+6\right)=0\Rightarrow\begin{cases}x=0\\x^2-5x+6=0\end{cases}$$

$$x^2-5x+6=0\Rightarrow x=\frac{5\pm\sqrt{25-24}}{2}=\frac{5\pm 1}{2}=3,2$$

$$p(x)=x^3-5x^2+6x=x\cdot(x-3)\cdot(x-2)$$

$c)\ \ p(x)=x^4-13x^2+36$

$$x^2=t\Rightarrow x^4-13x^2+36=0\Rightarrow t^2-13t+36=0$$

$$t^2-13t+36=0\Rightarrow x^2=t=\frac{13\pm\sqrt{169-144}}{2}=\frac{13\pm 5}{2}=9,4$$

$$x=\pm\sqrt{9},\pm\sqrt{4}=3,-3,2,-2$$

$$p(x)=x^4-13x^2+36=(x-3)\cdot(x+3)\cdot(x-2)\cdot(x+2)$$

$d)\ \ p(x)=x^4-16=\left(x^2-4\right)\cdot\left(x^2+4\right)$

$$\left(x^2-4\right)\cdot\left(x^2+4\right)=0\Rightarrow\begin{cases}x^2-4=0\\x^2+4=0\end{cases}\Rightarrow\begin{cases}x=2,-2\\x^2=\pm\sqrt{-4}\notin\mathbb{R}\end{cases}$$

$$p(x)=x^4-16=\left(x^2-4\right)\cdot\left(x^2+4\right)=(x-2)\cdot(x+2)\cdot\left(x^2+4\right)$$

www.yoquieroaprobar.es

2.1-8 Factoriza según sus raíces reales los siguientes polinomios utilizando la regla de Ruffini:

$a)\ \ p(x)=x^3-3x^2-6x+8$

$b)\ \ p(x)=x^3+3x^2-13x-15$

$c)\ \ p(x)=x^3-7x^2+x-7$

$d)\ \ p(x)=x^4-2x^3-x+2$

$e)\ \ p(x)=x^5-x^4-10x^3+9x^2+9x$

$f)\ \ p(x)=x^4-\frac{5}{6}x^3-\frac{7}{3}x^2-\frac{1}{6}x+\frac{1}{3}$

Solución

$a)\ \ p(x)=x^3-3x^2-6x+8$

Divisores enteros del término independiente: 1, –1, 2, –2, 4, –4, 8, –8

	1	–3	–6	8
1		1	–2	–8
	1	–2	–8	0

Resto cero

$$p(x)=x^3-3x^2-6x+8=\left(x^2-2x-8\right)\cdot(x-1)$$

Ahora, el polinomio de grado 2 se puede factorizar obteniendo sus raíces:

$$x^2-2x-8=0\Rightarrow x=\frac{2\pm\sqrt{4+32}}{2}=\frac{2\pm 6}{2}=4,-2$$

Por tanto:

$$p(x)=x^3-3x^2-6x+8=(x-4)\cdot(x+2)\cdot(x-1)$$

$b)\ \ p(x)=x^3+3x^2-13x-15$

Divisores enteros del término independiente: 1, –1, 3, –3, 5, –5, 15, –15

	1	3	–13	–15
–1		–1	–2	15
	1	2	–15	0

Resto cero

www.yoquieroaprobar.es

$$p(x)=x^3+3x^2-13x-15=\left(x^2+2x-15\right)\cdot(x+1)$$

Ahora, el polinomio de grado 2 se puede factorizar obteniendo sus raíces:

$$x^2+2x-15=0\Rightarrow x=\frac{-2\pm\sqrt{4+60}}{2}=\frac{-2\pm 8}{2}=3,-5$$

Por tanto:

$$p(x)=x^3+3x^2-13x-15=(x-3)\cdot(x+5)\cdot(x+1)$$

c) $p(x)=x^3-7x^2+x-7$

Divisores enteros del término independiente: 1, –1, 7, –7

	1	–7	1	–7
7		7	0	7
	1	0	1	0

Resto cero

$$p(x)=x^3-7x^2+x-7=\left(x^2+1\right)\cdot(x-7)$$

Ahora, el polinomio de grado 2 se puede factorizar obteniendo sus raíces:

$x^2+1=0\Rightarrow x^2=-1\Rightarrow x=\pm\sqrt{-1}$, que no son raíces reales.

Por tanto:

$$p(x)=x^3-7x^2+x-7=\left(x^2+1\right)\cdot(x-7)$$

d) $p(x)=x^4-2x^3-x+2$

Divisores enteros del término independiente: 1, –1, 2, –2

	1	–2	0	–1	2
1		1	–1	–1	–2
	1	–1	–1	–2	0

Resto cero

$$p(x)=x^4-2x^3-x+2=\left(x^3-x^2-x-2\right)\cdot(x-1)$$

Para el polinomio de grado 3 que resulta utilizamos otra vez Ruffini con los divisores enteros del término independiente: 1, –1, 2, –2

	1	–1	–1	–2
2		2	2	2
	1	1	1	0

Resto cero

www.yoquieroaprobar.es

Por tanto:

$$p(x)=x^4-2x^3-x+2=\left(x^2+x+1\right)\cdot(x-2)\cdot(x-1)$$

Ahora, el polinomio de grado 2 se puede factorizar obteniendo sus raíces:

$x^2+x+1=0 \Rightarrow x=\dfrac{-1\pm\sqrt{1-4}}{2}$, que no son raíces reales.

Por tanto:

$$p(x)=x^4-2x^3-x+2=\left(x^2+x+1\right)\cdot(x-2)\cdot(x-1)$$

e) $p(x)=x^5-x^4-10x^3+9x^2+9x$

Podemos factorizar fácilmente el polinomio sacando *x* factor común:

$$p(x)=x^5-x^4-10x^3+9x^2+9x=\left(x^4-x^3-10x^2+9x+9\right)\cdot x$$

Para el polinomio de grado 4 que resulta utilizamos Ruffini con los divisores enteros del término independiente: 1, –1, 3, –3, 9, –9

	1	–1	–10	9	9
3		3	6	–12	–9
	1	2	–4	–3	0

Resto cero

$$p(x)=x^5-x^4-10x^3+9x^2+9x=\left(x^3+2x^2-4x-3\right)\cdot(x-3)\cdot x$$

Para el polinomio de grado 3 que resulta utilizamos otra vez Ruffini con los divisores enteros del término independiente: 1, –1, 3, –3

	1	2	–4	–3
–3		–3	3	3
	1	–1	–1	0

Resto cero

Por tanto:

$$p(x)=x^5-x^4-10x^3+9x^2+9x=\left(x^2-x-1\right)\cdot(x+3)\cdot(x-3)\cdot x$$

Ahora, el polinomio de grado 2 se puede factorizar obteniendo sus raíces:

$x^2-x-1=0 \Rightarrow x=\dfrac{1\pm\sqrt{1+4}}{2}=\dfrac{1+\sqrt{5}}{2},\dfrac{1-\sqrt{5}}{2}$, que son raíces reales.

Por tanto:

www.yoquieroaprobar.es

$$p(x)=\left(x-\frac{1+\sqrt{5}}{2}\right)\cdot\left(x-\frac{1-\sqrt{5}}{2}\right)\cdot(x+3)\cdot(x-3)\cdot x$$

f) $p(x)=x^4-\frac{5}{6}x^3-\frac{7}{3}x^2-\frac{1}{6}x+\frac{1}{3}$

Como el término independiente no es entero y aparecen términos racionales, multiplicamos todo el polinomio por 6:

$$6p(x)=6x^4-5x^3-14x^2-x+2$$

Para este polinomio de grado 4 que resulta utilizamos Ruffini con los divisores enteros del término independiente: 1, –1, 2, –2

	6	–5	–14	–1	2
2		12	14	0	–2
	6	7	0	–1	0

Resto cero

$$6p(x)=6x^4-5x^3-14x^2-x+2=\left(6x^3+7x^2-1\right)\cdot(x-2)$$

Para el polinomio de grado 3 que resulta utilizamos otra vez Ruffini con los divisores enteros del término independiente: 1, –1

	6	7	0	–1
–1		–6	–1	1
	6	1	–1	0

Resto cero

Por tanto:

$$6p(x)=6x^4-5x^3-14x^2-x+2=\left(6x^2+x-1\right)\cdot(x+1)\cdot(x-2)$$

Ahora, el polinomio de grado 2 se puede factorizar obteniendo sus raíces:

$6x^2+x-1=0\Rightarrow x=\frac{-1\pm\sqrt{1+24}}{12}=\frac{-1\pm5}{12}=\frac{1}{3},-\frac{1}{2}$, son raíces reales.

Por tanto:

$$6p(x)=6x^4-5x^3-14x^2-x+2=6\cdot\left(x-\frac{1}{3}\right)\cdot\left(x+\frac{1}{2}\right)\cdot(x+1)\cdot(x-2)$$

Finalmente:

$$p(x)=x^4-\frac{5}{6}x^3-\frac{7}{3}x^2-\frac{1}{6}x+\frac{1}{3}=\left(x-\frac{1}{3}\right)\cdot\left(x+\frac{1}{2}\right)\cdot(x+1)\cdot(x-2)$$

www.yoquieroaprobar.es