Calcula las derivadas de las funciones:

1 $f(x)=5$

$f'(x)=0$

2 $f(x)=-2x$

$f'(x)=-2$

3 $f(x)=-2x+2$

$f'(x)=-2$

4 $f(x)=-2x^2-5$

$f'(x)=-4x$

5 $f(x)=2x^4+x^3-x^2+4$

$f'(x)=8x^3+3x^2-2x$

6 $f(x)=\dfrac{x^3+2}{3}$

$f'(x)=x^2$

7 $f(x)=\dfrac{1}{3x^2}$

$$f'(x)=\frac{-6x}{(3x)^2}=\frac{-6x}{9x^4}=-\frac{2}{3x^3}$$

8 $f(x)=\dfrac{x+1}{x-1}$

$$f'(x)=\frac{1\cdot(x-1)-(x+1)\cdot 1}{(x-1)^2}=\frac{-2}{(x-1)^2}$$

9 $f(x)=\left(5x^2-3\right)\cdot\left(x^2+x+4\right)$

$$f'(x)=10x\left(x^2+x+4\right)+\left(5x^2-3\right)(2x+1)=20x^3+15x^2+34x-3$$

Calcula mediante la fórmula de la derivada de una potencia:

1 $f(x)=\dfrac{5}{x^5}=5x^{-5}$

$$f'(x)=-25x^{-6}=-\frac{25}{x^6}$$

2 $f(x)=\dfrac{5}{x^5}+\dfrac{3}{x^2}=5x^{-5}+3x^{-2}$

$$f'(x)=-25x^{-6}-6x^{-3}=-\frac{25}{x^6}-\frac{6}{x^3}$$

3 $f(x)=\sqrt{x}=x^{\frac{1}{2}}$

$$f'(x)=\frac{1}{2}x^{\frac{1}{2}-1}=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}}$$

4 $f(x)=\dfrac{1}{\sqrt{x}}=x^{-\frac{1}{2}}$

$$f'(x)=-\frac{1}{2}x^{-\frac{1}{2}-1}=-\frac{1}{2}x^{-\frac{3}{2}}=-\frac{1}{2\sqrt{x^3}}$$

5 $f(x)=\dfrac{1}{x\sqrt{x}}=\dfrac{1}{x\ \ x^{\frac{1}{2}}}=x^{\frac{-3}{2}}$

$$f'(x)=\frac{-3}{2}x^{\frac{-5}{2}}=-\frac{3}{2\sqrt{x^5}}$$

6 $f(x)=\sqrt[3]{x^2}+\sqrt{x}=x^{\frac{2}{3}}+x^{\frac{1}{2}}$

$$f'(x)=\frac{2}{3}x^{\frac{2}{3}-1}+\frac{1}{2}x^{\frac{1}{2}-1}=\frac{2}{3}x^{-\frac{1}{3}}+\frac{1}{2}x^{-\frac{1}{2}}=\frac{2}{3\sqrt[3]{x}}+\frac{1}{2\sqrt{x}}$$

7 $f(x)=\left(x^2+3x-2\right)^4$

$$f'(x)=x^4\left(x^2+3x-2\right)^3(2x+3)$$

Calcula mediante la fórmula de la derivada de una raíz:

1 $f(x)=\sqrt{x^2-2x+3}$

$$f'(x)=\frac{2x-2}{2\sqrt{x^2-2x+3}}=\frac{x-1}{\sqrt{x^2-2x+3}}$$

2 $f(x)=\sqrt[4]{x^5-x^3-2}$

$$f'(x)=\frac{5x^4-3x^2}{4\sqrt[4]{\left(x^5-x^3-2\right)^3}}$$

3 $f(x)=\sqrt[3]{\dfrac{x^2+1}{x^2-1}}$

$$f'(x)=\frac{\dfrac{2x\left(x^2-1\right)-\left(x^2+1\right)2x}{\left(x^2-1\right)^2}}{3\ \sqrt[3]{\left(\dfrac{x^2+1}{x^2-1}\right)^2}}= \ =\frac{\dfrac{-4x}{\left(x^2-1\right)^2}}{3\ \sqrt[3]{\left(\dfrac{x^2+1}{x^2-1}\right)^2}}=\frac{-4x}{3\left(x^2-1\right)^2\sqrt[3]{\left(\dfrac{x^2+1}{x^2-1}\right)^2}}=$$

$$\frac{-4x}{3\ \sqrt[3]{\left(\dfrac{x^2+1}{x^2-1}\right)^2\left(x^2-1\right)^4}}=\frac{-4x}{3\ \sqrt[3]{\left(x^2+1\right)^2\left(x^2-1\right)^2}}= \quad =\frac{-4x}{3\ \sqrt[3]{\left(x^4-1\right)^2}}$$

Deriva las funciones exponenciales:

1 $f(x)=10^{\sqrt{x}}$

$f'(x)=\frac{1}{2\sqrt{x}}\cdot 10^{\sqrt{x}}\cdot \ln 10$

2 $f(x)=e^{3-x^2}$

$f'(x)=-2x\cdot e^{3-x^2}$

3 $f(x)=\frac{e^x+e^{-x}}{2}$

$f'(x)=\frac{e^x-e^{-x}}{2}$

4 $f(x)=3^{2x^2}\cdot\sqrt{x}$

$f^{-1}(x)=4x\cdot 3^{2x^2}\cdot\ln 3\cdot\sqrt{x}+\frac{3^{2x^2}}{2\sqrt{x}}=$

$=3^{2x^2}\left(4x\cdot\sqrt{x}\cdot\ln 3+\frac{1}{2\sqrt{x}}\right)$

5 $f(x)=\frac{e^{2x}}{x^2}$

$f'(x)=\frac{2\cdot e^{2x}\cdot x^2-e^{2x}\cdot 2x}{x^4}=\frac{2x\cdot e^{2x}(x-1)}{x^4}=$

$=\frac{2\cdot e^{2x}(x-1)}{x^3}$

Calcula la derivada de la funciones logarítmicas:

1 $f(x)=\ln\left(2x^4-x^3+3x^2-3x\right)$

$f'(x)=\frac{8x^3-3x^2+6x-3}{2x^4-x^3+3x^2-3x}$

2 $f(x)=\ln\left(\frac{e^x+1}{e^x-1}\right)$

Aplicando las **propiedades de los logarítmos** obtenemos:

$f(x)=\ln\left(e^x+1\right)-\ln\left(e^x-1\right)$

$f'(x)=\frac{e^x}{e^x+1}-\frac{e^x}{e^x-1}=\frac{e^{2x}-e^x-e^{2x}-e^x}{\left(e^x+1\right)\left(e^x-1\right)}=$

$=\frac{-2e^x}{e^{2x}-1}$

3 $f(x) = \log\sqrt{\frac{1+x}{1-x}}$

Aplicando las **propiedades de los logarítmos** obtenemos:

$$f(x) = \frac{1}{2}\left[\log(1+x) - \log(1-x)\right]$$

$$f'(x) = \frac{1}{2}\left(\frac{1}{1+x} - \frac{-1}{1-x}\right)\cdot \log e = \frac{1}{2}\cdot\frac{1-x+1+x}{1-x^2}\cdot \log e =$$

$$= \frac{2}{1-x^2}\cdot \log e$$

4 $f(x) = \ln\sqrt{x(1-x)}$

Aplicando las **propiedades de los logarítmos** obtenemos:

$$f(x) = \frac{1}{2}\left[\ln x + \ln(1-x)\right]$$

$$f'(x) = \frac{1}{2}\left(\frac{1}{x} + \frac{-1}{1-x}\right) = \frac{1}{2}\cdot\frac{1-x-x}{x(1-x)} =$$

$$= \frac{1-2x}{2x(1-x)}$$

5 $f(x) = \ln\sqrt[3]{\frac{3x}{x+2}}$

Aplicando las **propiedades de los logarítmos** obtenemos:

$$f(x) = \frac{1}{3}\left[\ln 3x - \ln(x+2)\right]$$

$$f'(x) = \frac{1}{3}\left(\frac{3}{3x} - \frac{1}{x+2}\right) = \frac{1}{3}\cdot\frac{x+2-x}{x(x+2)} = \qquad = \frac{2}{3x(x+2)}$$

Calcula la derivada de la funciones trigonométricas:

1 $f(x) = sen\,\frac{1}{2}x$

$$f'(x) = \frac{1}{2}\cos\frac{1}{2}x$$

2 $f(x)=\cos(7-2x)$

$f'(x)=-(-2)\cdot sen(7-2x)=2\cdot sen(7-2x)$

3 $f(x)=3tg2x$

$f'(x)=6\left(1+tg^2 2x\right)$

4 $f(x)=\sec(5x+2)$

$f'(x)=5\ tg(5x+2)\cdot\sec(5x+2)$

5 $f(x)=\sqrt[3]{senx}$

$$f'(x)=\frac{\cos x}{3\sqrt[3]{sen^2x}}$$

6 $f(x)=sen^3\ 3x$

$f'(x)=3\cdot sen^2\ 3x\cdot 3\cdot\cos 3x=9\cdot sen^2\ 3x\cdot\cos 3x$

7 $f(x)=cotg(3-2x)$

$$f'(x)=\frac{2}{sen^2(3-2x)}$$

8 $f(x)=\cos\dfrac{x+1}{x-1}$

$$f'(x)=-\frac{x-1-(x+1)}{(x-1)^2}sen\frac{x+1}{x-1}=\frac{2}{(x-1)^2}\cdot sen\frac{x+1}{x-1}$$

9 $f(x)=\sqrt{\dfrac{1-senx}{1+senx}}$

$$f'(x)=\frac{1}{2\sqrt{\dfrac{1-senx}{1+senx}}}\cdot\frac{-\cos x(1+senx)-(1+senx)\cos x}{(1+senx)^2}=$$

$$=\frac{1}{2\sqrt{\dfrac{1-senx}{1+senx}}}\cdot\frac{-\cos x-senx\cdot\cos x-\cos x+senx\cdot\cos x}{(1+senx)^2}=$$

$$= \frac{1}{2\sqrt{\frac{1-senx}{1+senx}}} \cdot \frac{-2\cos x}{(1+senx)^2} = \frac{-2\cos x}{2\sqrt{\frac{(1-senx)(1+senx)^4}{1+senx}}} =$$

$$= -\frac{\cos x}{\sqrt{(1-senx)(1+senx)^3}} = -\frac{\cos x}{\sqrt{(1-senx)(1+senx)(1+senx)^2}} =$$

$$= -\frac{\cos x}{\sqrt{1-senx} \cdot (1+senx)} = -\frac{\cos x}{\cos x \cdot (1+senx)} =$$

$$= -\frac{1}{1+senx}$$

Calcula la derivada de la funciones trigonométricas inversas:

1 $f(x) = arc\ sen\left(1-2x^2\right)$

$$f'(x) = \frac{-4x}{\sqrt{1-\left(1-2x^2\right)^2}}$$

2 $f(x) = arc\ sen\sqrt{x^2-4}$

$$f'(x) = \frac{1}{\sqrt{1-\left(x^2-4\right)}} \cdot \frac{2x}{2\sqrt{x^2-4}} = \frac{x}{\sqrt{5-x^2} \cdot \sqrt{x^2-4}}$$

3 $f(x) = arc\ \cos e^x$

$$f'(x) = -\frac{e^x}{\sqrt{1-e^{2x}}}$$

4 $f(x) = arc\ tg\sqrt{x}$

$$f'(x) = \frac{1}{1+x}\ \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}\,(1+x)}$$

5 $f(x) = arctg\,\frac{1+x}{1-x}$

$$f'(x) = \frac{1}{1+\left(\frac{1+x}{1-x}\right)^2} \cdot \frac{1-x+1+x}{(1-x)^2} =$$

$$= \frac{1}{1+\frac{(1+x)^2}{(1-x)^2}} \cdot \frac{2}{(1-x)^2} = \frac{2}{(1-x)^2+(1+x)^2} =$$

$$= \frac{2}{1-2x+x^2+1+2x+x^2} = \frac{2}{2+2x^2} =$$

$$= \frac{1}{1+x^2}$$

Derivar por la regla de la cadena las funciones:

1 $f(x) = \ln sen\, x$

$$f'(x) = \frac{\cos x}{sen\, x} = cotg\, x$$

2 $f(x) = \ln \cos 2x$

$$f'(x) = \frac{-2sen\, 2x}{\cos 2x} = -2\, tg\, 2x$$

3 $f(x) = \ln tg\,(1-x)$

$$f'(x) = -\frac{1+tg^2\,(1-x)}{tg\,(1-x)}$$

4 $f(x) = \ln \sqrt{\frac{1+sen\, x}{1-sen\, x}}$

$$f(x) = \frac{1}{2}[\ln(1+sen\, x) - \ln(1-sen\, x)]$$

$$f'(x) = \frac{1}{2}\left(\frac{\cos x}{1+sen\, x} - \frac{-\cos x}{1-sen\, x}\right) =$$

$$= \frac{1}{2} \cdot \frac{\cos x - sen\, x \cos x + \cos x + sen\, x \cos x}{1-sen^2\, x} =$$

$$= \frac{1}{2} \cdot \frac{2\cos x}{\cos^2 x} = \frac{1}{\cos x} = \sec x$$

5 $f(x) = sen\sqrt{\ln(1-3x)}$

$$f'(x) = \cos\sqrt{\ln(1-3x)} \cdot \frac{1}{2\sqrt{\ln(1-3x)}} \cdot \frac{1}{1-3x} \cdot (-3)$$

6 $f(x) = tg\left(sen\sqrt{5x}\right)$

$$f'(x) = \left[1+tg^2\left(sen\sqrt{5x}\right)\right] \cdot \cos\sqrt{5x} \cdot \frac{1}{2\sqrt{5x}} \cdot 5$$

7 $f(x) = sen^2(\cos 2x)$

$$f'(x) = 2\, sen(\cos 2x) \cdot \cos(\cos 2x) \cdot (-sen\, 2x) \cdot 2$$