

**Problema 1** Discutir y resolver por el método de Gauss los siguientes sistemas:

$$\begin{cases} x- & y+ & z = 1 \\ 2x+ & y- & 2z = 2 \\ x+ & 2y- & 3z = 1 \end{cases} ; \begin{cases} x+ & y+ & z = 1 \\ 2x+ & y- & z = 2 \\ 2x+ & & z = 3 \end{cases}$$

**Solución:**

$$\begin{cases} x- & y+ & z = 1 \\ 2x+ & y- & 2z = 2 \\ x+ & 2y- & 3z = 1 \end{cases} \text{ Sistema Compatible Indeterminado} \implies \begin{cases} x = 1 + \frac{1}{3}z \\ y = \frac{4}{3}z \\ z = z \end{cases}$$

$$\begin{cases} x+ & y+ & z = 1 \\ 2x+ & y- & z = 2 \\ x+ & & z = 3 \end{cases} \text{ Sistema Compatible Determinado} \implies \begin{cases} x = 7/5 \\ y = -3/5 \\ z = 1/5 \end{cases}$$

**Problema 2** Resolver las siguientes ecuaciones e inecuaciones:

- $\ln(2-x) - \ln(x+2) = 1$
- $2^{2x-1} - 2^x - 1 = 0$
- $\frac{2x-5}{x^2-4x-21} - 1 = \frac{x}{x+3} - \frac{2}{7-x}$
- $\frac{x^2-4x-21}{x+1} \leq 0$
- $\sqrt{x+3} + \sqrt{x} = 2$

**Solución:**

- $\ln(2-x) - \ln(x+2) = 1 \implies x = \frac{2(1-e)}{1+e}$
- $2^{2x-1} - 2^x - 1 = 0 \implies x = 0,4499$
- $\frac{2x-5}{x^2-4x-21} - 1 = \frac{x}{x+3} - \frac{2}{7-x} \implies x = 6,2943, x = -0,79436$
- $\frac{x^2-4x-21}{x+1} \leq 0 \implies (-\infty, -3] \cup (-1, 7]$
- $\sqrt{x+3} + \sqrt{x} = 2 \implies x = \frac{1}{16}$

**Problema 3** Calcular los siguientes límites:

1.  $\lim_{x \rightarrow 1} \frac{x^5 + x - 2}{2x^3 - x - 1}$
2.  $\lim_{x \rightarrow -\infty} \frac{x^3 + 3x - 1}{-x^2 + 2}$
3.  $\lim_{x \rightarrow \infty} \left( \frac{2x + 1}{2x} \right)^{x-3}$
4.  $\lim_{x \rightarrow 1} \frac{\sqrt{2x^2 - 1} - \sqrt{2x - 1}}{x - 1}$

**Solución:**

1.  $\lim_{x \rightarrow 1} \frac{x^5 + x - 2}{2x^3 - x - 1} = \frac{6}{5}$
2.  $\lim_{x \rightarrow -\infty} \frac{x^3 + 3x - 1}{-x^2 + 2} = +\infty$
3.  $\lim_{x \rightarrow \infty} \left( \frac{2x + 1}{2x} \right)^{x-3} = e^{1/2}$
4.  $\lim_{x \rightarrow 1} \frac{\sqrt{2x^2 - 1} - \sqrt{2x - 1}}{x - 1} = 1$

**Problema 4** Calcular las derivadas de las siguientes funciones:

1.  $y = (x^2 - x)^{10}(x + 1)^5$
2.  $y = \frac{3x^2 - x - 1}{x + 2}$
3.  $y = (3x - 1)^{2x+1}$
4.  $y = \ln \sqrt{\frac{2x + 1}{x + 3}}$
5.  $y = e^{\sqrt{x-2}}$
6.  $y = x^{\sqrt{x}}$

**Solución:**

1.  $y = (x^2 - x)^{10}(x + 1)^5 \implies$   
 $y' = 10(x^2 - x)^9(2x - 1)(x + 1)^5 + 5(x^2 - x)^{10}(x + 1)^4$
2.  $y = \frac{3x^2 - x - 1}{x + 2} \implies y' = \frac{3x^2 + 12x - 1}{(x + 2)^2}$
3.  $y = (3x - 1)^{2x+1} \implies y' = (3x - 1)^{2x+1} \left[ 2 \ln(3x - 1) + \frac{3(2x+1)}{3x+1} \right]$

$$4. y = \ln \sqrt{\frac{2x+1}{x+3}} \implies y' = \frac{1}{2} \ln \sqrt{\frac{2x+1}{x+3}} \left[ \frac{2}{2x+1} - \frac{1}{x+3} \right]$$

$$5. y = e^{\sqrt{x-2}} \implies y' = \frac{1}{2\sqrt{x-2}} e^{\sqrt{x-2}}$$

$$6. y = x^{\sqrt{x}} \implies y' = x^{\sqrt{x}} \left[ \frac{\ln x}{2\sqrt{x}} + \frac{\sqrt{x}}{x} \right]$$

**Problema 5** Calcular las rectas tangente y normal a la función  $f(x) = \frac{3x-1}{x+2}$  en el punto de abscisa  $x = 1$ .

**Solución:**

$$a = 1, \quad f(a) = f(1) = \frac{2}{3}$$

$$f'(x) = \frac{7}{(x+2)^2} \implies m = f'(1) = \frac{7}{9}$$

$$\text{Recta Tangente: } y - \frac{2}{3} = \frac{7}{9}(x - 1)$$

$$\text{Recta Normal: } y - \frac{2}{3} = -\frac{9}{7}(x - 1)$$