Problema 1 Discutir y resolver por el método de Gauss los siguientes sistemas:

$$\left\{\begin{array}{rrrrr} 2x- & y+ & 2z= & 1 \\ x- & & z= & 2 \\ x- & 2y+ & 7z= & -4 \end{array}\right. ; \quad \left\{\begin{array}{rrrr} x+ & y & -z & =0 \\ 2x+ & y & & =2 \\ x+ & 2y & +z & =1 \end{array}\right.$$

Solución:

$$\left\{\begin{array}{rrrrr} 2x- & y+ & 2z= & 1 \\ x- & & z= & 2 \\ x- & 2y+ & 7z= & -4 \end{array}\right. \text{ Sistema Compatible Indeterminado} \Longrightarrow \left\{\begin{array}{l} x=2+\lambda \\ y=3+4\lambda \\ z=\lambda \end{array}\right.$$

$$\left\{\begin{array}{rrrr} x+ & y & -z & =0 \\ 2x+ & y & & =2 \\ x+ & 2y & +z & =1 \end{array}\right. \text{ Sistema Compatible Determinado} \Longrightarrow \left\{\begin{array}{l} x=5/4 \\ y=-1/2 \\ z=3/4 \end{array}\right.$$

Problema 2 Resolver las ecuaciones:

a) $\log(x^2-1)+1=2\log(x-2)$

b) $2^{2x-1}+2^{x+2}-1=0$

c) $\dfrac{x-1}{x^2-2x-15}-\dfrac{1}{x+3}=1-\dfrac{1}{x-5}$

d) $\dfrac{x^2+2x-15}{x^2-8x+7}\geq 0$

e) $\sqrt{x+4}-\sqrt{x-1}=1$

f) $\sqrt{2x-1}-\sqrt{x-1}=2$

Solución:

a) $\log(x^2-1)+1=2\log(x-2) \Longrightarrow \log 10(x^2-1)=\log(x-1)^2 \Longrightarrow$

$9x^2+4x-14=0 \Longrightarrow x=1,0446,\ x=-1,1231$ y no vale ninguna de ellas.

b) $2^{2x-1}+2^{x+2}-1=0 \Longrightarrow \dfrac{t^2}{2}+4t-1=0 \Longrightarrow t=0,2426\ \ t=-8,2426$(No Vale).

$2^x=0,2426 \Longrightarrow x=\dfrac{\log 0,4494}{\log 2}=-2,04310$

c) $\dfrac{x-1}{x^2-2x-15} - \dfrac{1}{x+3} = 1 - \dfrac{1}{x-5} \Longrightarrow x^2 - 3x - 22 = 0 \Longrightarrow$

$$x = 6.424428900;\ x = -3.424428900$$

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d) $\dfrac{x^2+2x-15}{x^2-8x+7} = \dfrac{(x+5)(x-3)}{(x-1)(x-7)} \geq 0 \Longrightarrow$

$$(-\infty, -5] \cup (1, 3] \cup (7, \infty)$$

$\sqrt{x+4} - \sqrt{x-1} = 1 \Longrightarrow\ x = 5$

$\sqrt{2x-1} - \sqrt{x-1} = 2 \Longrightarrow\ x = 22,58300524$