1. Desarrollar y simplificar, dando el resultado racionalizado:

$$\left(\sqrt{2}-\frac{1}{\sqrt{3}}\right)^4=$$ *(1,75 puntos)*

2. Dado α ϵ 4º cuadrante tal que $\operatorname{ctg}\alpha=-\frac{\sqrt{3}}{3}$, se pide: *(2 puntos)*

a) tg 2α

b) cos α/2

c) cos (α+240º)

d) sen (α-1920º)

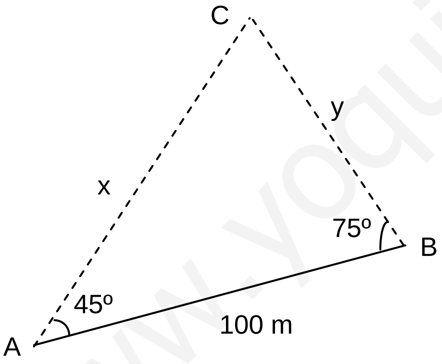
e) Razonar (sin calculadora) de qué α se trata.

3. Resolver: $\cos 2x+3\operatorname{sen}x=2$ (¡Comprobar las soluciones obtenidas!) *(2 puntos)*

4. **a)** Resolver el triángulo de datos a=4 m, b=3 m, $\hat{B}=30º$

b) Hallar su área. *(2 puntos)*

5.



Dos observadores A y B, separados 100 m, ven el punto inaccesible C bajo los ángulos que indica la figura. Hallar a qué distancia se encuentran ambos observadores de dicho punto. *(2 puntos)*

NOTA: La ortografía, sintaxis, presentación cuidada (orden en el planteamiento, limpieza, caligrafía, etc.) y corrección en el lenguaje matemático se calificarán con un máximo de 0,25 puntos.

① $\left(\sqrt{2}-\frac{1}{\sqrt{3}}\right)^4 = \overset{1}{\binom{4}{0}}\underset{4}{(\sqrt{2})^4} - \overset{4}{\binom{4}{1}}\underset{2\sqrt{2}}{(\sqrt{2})^3}\cdot\underset{\sqrt{3}/3}{\frac{1}{\sqrt{3}}} + \overset{6}{\binom{4}{2}}\underset{2}{(\sqrt{2})^2}\cdot\underset{1/3}{\left(\frac{1}{\sqrt{3}}\right)^2} - \overset{4}{\binom{4}{3}}\sqrt{2}\cdot\underset{\sqrt{3}/9}{\left(\frac{1}{\sqrt{3}}\right)^3} + \overset{1}{\binom{4}{4}}\cdot\underset{1/9}{\left(\frac{1}{\sqrt{3}}\right)^4} =$ ← 0.25

TOTAL: 1,75

0.25 →

$$= 4 - 4\cdot 2\sqrt{2}\cdot\frac{\sqrt{3}}{3} + 6\cdot 2\cdot\frac{1}{3} - 4\sqrt{2}\cdot\frac{\sqrt{3}}{9} + \frac{1}{9} = 4 - \frac{8}{3}\sqrt{6} + 4 - \frac{4}{9}\sqrt{6} + \frac{1}{9} = \boxed{\frac{73}{9} - \frac{28}{9}\sqrt{6}}$$ ← 1

0.25 (bajo $\frac{8}{3}\sqrt{6}$)

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② $ctg\,\alpha = -\frac{\sqrt{3}}{3} \Rightarrow \boxed{tg\,\alpha = \frac{1}{ctg\,\alpha} = -\frac{3}{\sqrt{3}} = -\frac{3\sqrt{3}}{3} = -\sqrt{3}}$ ← 0.1

a) $\boxed{tg\,2\alpha} = \frac{2\,tg\,\alpha}{1-tg^2\alpha} = \frac{2\cdot(-\sqrt{3})}{1-(-\sqrt{3})^2} = \frac{-2\sqrt{3}}{1-3} = \frac{-2\sqrt{3}}{-2} = \boxed{\sqrt{3}}$ ← 0.3

b) $1+tg^2\alpha = \frac{1}{\cos^2\alpha} \Rightarrow 1+(-\sqrt{3})^2 = \frac{1}{\cos^2\alpha}$; $1+3 = \frac{1}{\cos^2\alpha}$; $4 = \frac{1}{\cos^2\alpha}$; $\cos^2\alpha = \frac{1}{4}$

↗ $\cos\alpha = -\frac{1}{2}$ descartado pq $\alpha \in 4^{\circ}$ cuad.

↘ $\boxed{\cos\alpha = \frac{1}{2}}$ ← 0.1

$\boxed{\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}} = -\sqrt{\frac{1+\frac{1}{2}}{2}} = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}}$ ← 0,4

$270 < \alpha < 360$

$135 < \alpha/2 < 180 \Rightarrow \frac{\alpha}{2} \in 2^{\circ}$ cuad.

c) $\boxed{sen\,\alpha = tg\,\alpha\cdot\cos\alpha = -\sqrt{3}\cdot\frac{1}{2} = -\frac{\sqrt{3}}{2}}$ ← 0.1

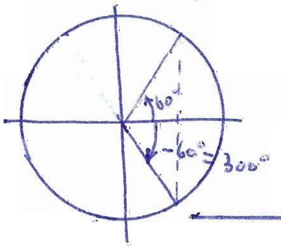
$\boxed{\cos(\alpha+240^{\circ})} = \cos\alpha\,\underset{\cos(180^{\circ}+60) = -\cos 60^{\circ} = -\frac{1}{2}}{\cos 240^{\circ}} - sen\,\alpha\,\underset{sen(180+60) = -sen 60^{\circ} = -\sqrt{3}/2}{sen\,240^{\circ}} = \frac{1}{2}\cdot\left(-\frac{1}{2}\right) - \left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) = -\frac{1}{4} - \frac{3}{4} = \boxed{-1}$ ← 0,4

d) $\boxed{sen(\alpha - 1920^{\circ})} = sen\,\alpha\cdot\underset{\cos 120^{\circ} = \cos(180-60) = -\cos 60^{\circ} = -\frac{1}{2}}{\cos 1920^{\circ}} - \cos\alpha\cdot\underset{sen 120^{\circ} = sen(180-60) = sen 60 = \frac{\sqrt{3}}{2}}{sen\,1920^{\circ}} = -\frac{\sqrt{3}}{2}\cdot\left(-\frac{1}{2}\right) - \frac{1}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = \boxed{0}$ ← 0,4

$1920^{\circ} \,|\, 360^{\circ}$

120° — 5 vueltas ⇨

e)



$tg\,\alpha = -\sqrt{3} \Rightarrow \boxed{\alpha = arctg(-\sqrt{3})} = -60^{\circ} = \boxed{300^{\circ}}$ ← 0.2

TOTAL: 2

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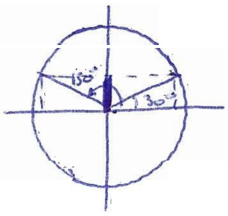
③ $\cos 2x + 3\,sen\,x = 2$; $\cos^2 x - sen^2 x + 3\,sen\,x = 2$; $1 - sen^2 x - sen^2 x + 3\,sen\,x = 2$; $-2\,sen^2 x + 3\,sen\,x - 1 = 0$

$\Rightarrow 2\,sen^2 x - 3\,sen\,x + 1 = 0 \rightarrow sen\,x = \frac{3\pm\sqrt{9-8}}{4} = \frac{3\pm 1}{4}$

↗ $sen\,x = 1 \Rightarrow x = 90^{\circ} + K\cdot 360^{\circ}$

↘ $sen\,x = \frac{1}{2} \Rightarrow \begin{cases} x = 30^{\circ} + K\cdot 360^{\circ} \\ x = 150^{\circ} + K\cdot 360^{\circ} \end{cases}$ ← 0.75

0.5



comprobación:

$x = 90^{\circ} \rightarrow \cos 180^{\circ} + 3\,sen\,90^{\circ} \overset{?}{=} 2$

$-1+3 = 2$ o.k. $\Rightarrow \boxed{x = 90^{\circ} + K\cdot 360^{\circ}}$ es solución ← 0.25

$x = 30^{\circ} \rightarrow \cos 60^{\circ} + 3\,sen\,30^{\circ} \overset{?}{=} 2$

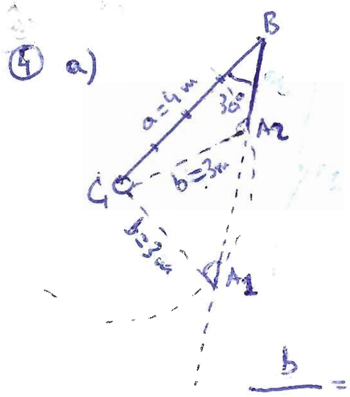
$\frac{1}{2} + 3\cdot\frac{1}{2} \overset{?}{=} 2$

$\frac{4}{2} = 2$ o.k. $\Rightarrow \boxed{x = 30^{\circ} + K\cdot 360^{\circ}}$ es soluc. ← 0.25

TOTAL: 2

$x = 150^{\circ} \rightarrow \underset{\cos 300^{\circ} = \cos(-60^{\circ}) = \cos 60^{\circ} = \frac{1}{2}}{\cos 300^{\circ}} + 3\,\underset{sen 150^{\circ} = sen(180-30^{\circ}) = sen 30^{\circ} = 1/2}{sen\,150^{\circ}} \overset{?}{=} 2$; $\frac{1}{2} + 3\cdot\frac{1}{2} = 2 \Rightarrow \boxed{x = 150^{\circ} + K\cdot 360^{\circ}}$ es soluc. ← 0.25

④ a)



$$\frac{a}{\operatorname{sen} A}=\frac{b}{\operatorname{sen} B}\Rightarrow\frac{4}{\operatorname{sen} A}=\frac{3}{\operatorname{sen} 30^\circ}\Rightarrow \operatorname{sen} A=\frac{4\operatorname{sen} 30^\circ}{3}=\frac{2}{3}\Rightarrow$$

$$\Rightarrow \hat{A}=\operatorname{arc\,sen}\frac{2}{3}\begin{cases}\boxed{\hat{A}_1\simeq 41^\circ 48' 37''}\Rightarrow\boxed{\hat{C}_1=180^\circ-(\hat{A}_1+\hat{B})\simeq 108^\circ 11' 23''}\\ \boxed{\hat{A}_2\simeq 138^\circ 11' 23''}\Rightarrow\boxed{\hat{C}_2=180^\circ-(\hat{A}_2+\hat{B})\simeq 11^\circ 48' 37''}\end{cases}$$

← 0.4 } 0.8 .4

$$\frac{b}{\operatorname{sen} B}=\frac{c_1}{\operatorname{sen} \hat{C}_1}\Rightarrow\frac{3}{\operatorname{sen} 30^\circ}=\frac{c_1}{\operatorname{sen} 108^\circ 11' 23''}\Rightarrow\boxed{c_1=\frac{3\operatorname{sen} 108^\circ 11' 23''}{\operatorname{sen} 30^\circ}\simeq 5{,}7\,m}$$ ← 0.3

$$\frac{b}{\operatorname{sen} B}=\frac{c_2}{\operatorname{sen} \hat{C}_2}\Rightarrow\frac{3}{\operatorname{sen} 30^\circ}=\frac{c_2}{\operatorname{sen} 11^\circ 48' 37''}\Rightarrow\boxed{c_2=\frac{3\operatorname{sen} 11^\circ 48' 37''}{\operatorname{sen} 30^\circ}\simeq 1{,}23\,m}$$ ← 0.3

b)

$$\boxed{S_{A_1BC_1}=\frac{1}{2}ab\operatorname{sen}\hat{C}_1=\frac{1}{2}4\cdot 3\cdot\operatorname{sen} 108^\circ 11' 23''\simeq 5{,}70\,m^2}$$ ← 0.3

$$\boxed{S_{A_2BC_2}=\frac{1}{2}ab\operatorname{sen}\hat{C}_2=\frac{1}{2}4\cdot 3\cdot\operatorname{sen} 11^\circ 48' 37''\simeq 1{,}23\,m^2}$$ ← 0.3

TOTAL: 2

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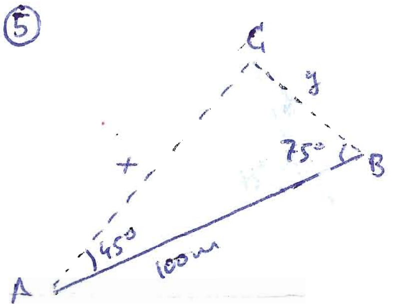
⑤



$$\hat{C}=180-(\hat{A}+\hat{B})=60^\circ$$

$$\frac{x}{\operatorname{sen} 75^\circ}=\frac{100}{\operatorname{sen} 60^\circ}\Rightarrow\boxed{x=\frac{100\operatorname{sen} 75^\circ}{\operatorname{sen} 60^\circ}\simeq 111{,}54\,m}$$ 1

$$\frac{y}{\operatorname{sen} 45^\circ}=\frac{100}{\operatorname{sen} 60^\circ}\Rightarrow\boxed{y=\frac{100\operatorname{sen} 45^\circ}{\operatorname{sen} 60^\circ}\simeq 81{,}65\,m}$$ 1

TOTAL: 2

LIMPIEZA - - - - - - - - - - - - - - 0.10
CORRECCIÓN LENGUAJE MATEMÁTICO - - - - - 0.05
ORDEN EN LA EXPOSICIÓN - - - - - 0.05
CALIGRAFÍA, ORTOGRAFÍA, SINTAXIS - - - - - 0.05