

Derivar las siguientes funciones:

1.- $y = 3 \frac{x^3}{\sqrt[4]{x}} - \frac{5}{x} + 5e \cdot \ln x - \frac{x \ln 5}{3}$

"arreglamos primero"

$$y = 3 \cdot x^{3-\frac{1}{4}} - \frac{5}{x} + 5e \ln x - x \frac{\ln 5}{3}$$

$$y = 3x^{\frac{11}{4}} - \frac{5}{x} + \underbrace{5e}_{cte} \ln x - x \underbrace{\frac{\ln 5}{3}}_{cte}$$

$$y' = \frac{33}{4} \cdot x^{\frac{7}{4}} + \frac{5}{x^2} + \underbrace{5e}_{cte} \frac{1}{x} - \frac{\ln 5}{3}$$

↑
INVERSA

2.- Simplificar al máximo: $y = \frac{x^2 - 2}{(2x^2 - 2)^3}$

$$y' = \frac{2x \cdot (2x^2 - 2)^3 - (x^2 - 2) \cdot 3(2x^2 - 2)^2 \cdot (4x)}{(2x^2 - 2)^6} = \frac{4x^3 - 4x - 12x^3 + 24x}{(2x^2 - 2)^4}$$

$$y' = \frac{-8x^3 + 24x - 4x}{(2x^2 - 2)^4} = \frac{-8x^3 + 20x}{(2x^2 - 2)^4}$$

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3.- Operar y simplificar al máximo: $y = \ln\left(\frac{3^{2x} \cdot e^{3x}}{\sqrt[3]{x-1}}\right)$

Aplicamos primero las propiedades de las raíces:

$$y = \ln(3^{2x} \cdot e^{3x}) - \ln\sqrt[3]{x-1} = \ln 3^{2x} + \ln e^{3x} - \frac{1}{3} \ln(x-1) = 2x \ln 3 + 3x - \frac{1}{3} \ln(x-1)$$

$$y' = 2 \ln 3 + 3 - \frac{1}{3(x-1)}$$

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4.- $y = (\tan x)^{\cos 2x}$

Tomamos logaritmos neperianos:

$$\ln y = \ln (\tan x)^{\cos 2x}$$

$$\ln y = \cos 2x \cdot \ln(\tan x) \Rightarrow \text{DERIVAMOS}$$

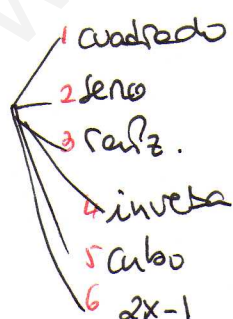
↑
PRODUCTO

$$\frac{1}{y} \cdot y' = -2 \sin 2x \cdot \ln(\tan x) + \cos 2x \cdot \frac{1 + \tan^2 x}{\tan x}$$

$$y' = \left[-2 \sin 2x \cdot \ln(\tan x) + \cos 2x \cdot \frac{1 + \tan^2 x}{\tan x} \right] \cdot (\tan x)^{\cos 2x}$$

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5.- $y = \sin^2 \sqrt{\frac{1}{(2x-1)^3}}$



Comprimos de todos ellos

$$y' = 2 \sin \sqrt{\frac{1}{(2x-1)^3}} \cdot \cos \sqrt{\frac{1}{(2x-1)^3}} \cdot \frac{1}{2 \sqrt{\frac{1}{(2x-1)^3}}} \cdot \frac{-1}{(2x-1)^2} \cdot 3(2x-1)^{-2} \cdot 2$$

$$y' = \frac{-6}{\sqrt{(2x-1)^5}} \cdot \sin \sqrt{\frac{1}{(2x-1)^3}} \cdot \cos \sqrt{\frac{1}{(2x-1)^3}} = \frac{-6}{(2x-1)^2 \sqrt{2x-1}} \cdot \sin \sqrt{\frac{1}{(2x-1)^3}} \cdot \cos \sqrt{\frac{1}{(2x-1)^3}}$$

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6.- $y = \arctg\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$ simplificando al máximo

$$y' = \frac{1}{1 + \left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)^2} \cdot \frac{1}{2 \sqrt{\frac{1-\cos x}{1+\cos x}}} \cdot \left[\frac{\sin x (1+\cos x) + (1-\cos x) \cdot \sin x}{(1+\cos x)^2} \right]$$

$$y' = \frac{1}{\cancel{1+\cos x} + \cancel{1-\cos x}} \cdot \frac{1}{2 \sqrt{\frac{1-\cos x}{1+\cos x}}} \cdot \left(\frac{\cancel{\sin x} + \cancel{\sin x} \cos x + \cancel{\sin x} - \cos x \cancel{\sin x}}{(1+\cos x)^2} \right)$$

$$y' = \frac{1}{2} \cdot \frac{1}{2} \sqrt{\frac{1+\cos x}{1-\cos x}} \cdot \frac{2 \sin x}{(1+\cos x)}$$

$$y' = \frac{\cancel{\sin x}}{2} \sqrt{\frac{1+\cancel{\cos x}}{(1-\cos x)(1+\cancel{\cos x})^2}}$$

meterlo dentro de la raíz o también se le racionaliza

$$y' = \frac{\sin x}{2} \cdot \frac{1}{\sqrt{1-\cos^2 x}}$$

al ser $1 - \cos^2 x = \sin^2 x$

$$y' = \frac{\sin x}{2} \cdot \frac{1}{\sqrt{\sin^2 x}}$$

$$y' = \frac{\cancel{\sin x}}{2} \cdot \frac{1}{\cancel{\sin x}}$$

$$y' = \frac{1}{2} \quad \text{¡¡ LUFF...!!} \quad \text{¡¡ Preciso !!}$$

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7.- Simplificar al máximo: $y = 4^{\frac{-x}{2}} \cdot e^{\sqrt{x^2+1}}$ derivamos un producto.

$$y' = 4^{-\frac{x}{2}} \cdot \ln 4 \cdot \left(\frac{-1}{2}\right) \cdot e^{\sqrt{x^2+1}} + 4^{-\frac{x}{2}} \cdot e^{\sqrt{x^2+1}} \cdot \frac{1}{2\sqrt{x^2+1}} \cdot 2x$$

$$y' = 4^{-\frac{x}{2}} \cdot e^{\sqrt{x^2+1}} \left(-\frac{1}{2} \ln 4 + \frac{x}{\sqrt{x^2+1}} \right)$$

$$y' = 4^{-\frac{x}{2}} \cdot e^{\sqrt{x^2+1}} \left(-\ln 2 + \frac{x}{\sqrt{x^2+1}} \right) //$$

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8.- Aplicando la definición de derivada, hallar la función derivada de $f(x) = \frac{3}{x} - 2$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{3}{x+h} - 2 - \frac{3}{x} + 2}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{\frac{3x - 3x - 3h}{x(x+h)}}{h} = \lim_{h \rightarrow 0} \frac{-3h}{x(x+h) \cdot h} = -\frac{3}{x^2} //$$

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