Ecuaciones trigonométricas

Resuelve las siguientes ecuaciones trigonométricas:

1) $\cos 2\alpha + \text{sen}^2\alpha = 1$

$$\cos^2\alpha - \text{sen}^2\alpha + \text{sen}^2\alpha = 1 \rightarrow \cos^2\alpha = 1 \rightarrow \cos\alpha = \pm 1 \rightarrow \begin{cases} \cos\alpha = 1 \rightarrow \alpha = 0^\circ + 360^\circ k \\ \cos\alpha = -1 \rightarrow \alpha = 180^\circ + 360^\circ k \end{cases}$$

2) $\cos 2\alpha - \text{sen}\,\alpha = 1$

$$\cos^2\alpha - \text{sen}^2\alpha - \text{sen}\,\alpha = 1 \rightarrow 1 - \text{sen}^2\alpha - \text{sen}^2\alpha - \text{sen}\,\alpha = 1 \rightarrow \text{sen}\,\alpha\left(2\,\text{sen}\,\alpha + 1\right) = 0 \rightarrow$$

$$\rightarrow \begin{cases} \text{sen}\,\alpha = 0 \rightarrow \alpha = \begin{cases} 0^\circ + 360^\circ k \\ 180^\circ + 360^\circ k \end{cases} \\ 2\,\text{sen}\,\alpha + 1 = 0 \rightarrow \text{sen}\,\alpha = -\dfrac{1}{2} \rightarrow \alpha = \begin{cases} 210^\circ + 360^\circ k \\ 330^\circ + 360^\circ k \end{cases} \end{cases}$$

3) $\cos 2\alpha = \text{sen}\,\alpha$

$$\cos^2\alpha - \text{sen}^2\alpha = \text{sen}\,\alpha \rightarrow 1 - \text{sen}^2\alpha - \text{sen}^2\alpha = \text{sen}\,\alpha \rightarrow -2\,\text{sen}^2\alpha - \text{sen}\,\alpha + 1 = 0 \xrightarrow{x = \text{sen}\,\alpha}$$

$$\rightarrow -2x^2 - x + 1 = 0 \rightarrow x = \begin{cases} \dfrac{1}{2} \\ -1 \end{cases} \rightarrow \text{sen}\,\alpha = \begin{cases} \dfrac{1}{2} \rightarrow \alpha = \begin{cases} 30^\circ + 360^\circ k \\ 150^\circ + 360^\circ k \end{cases} \\ -1 \rightarrow \alpha = 270^\circ + 360^\circ k \end{cases}$$

4) $\text{sen}\,\alpha + \cos 2\alpha = 1$

$$\text{sen}\,\alpha + \cos^2\alpha - \text{sen}^2\alpha = 1 \rightarrow \text{sen}\,\alpha + 1 - \text{sen}^2\alpha - \text{sen}^2\alpha = 1 \rightarrow \text{sen}\,\alpha - 2\,\text{sen}^2\alpha = 0 \rightarrow$$

$$\rightarrow \text{sen}\,\alpha\left(1 - 2\,\text{sen}\,\alpha\right) = 0 \rightarrow \begin{cases} \text{sen}\,\alpha = 0 \rightarrow \begin{cases} 0^\circ + 360^\circ k \\ 180^\circ + 360^\circ k \end{cases} \\ 1 - 2\,\text{sen}\,\alpha = 0 \rightarrow \text{sen}\,\alpha = \dfrac{1}{2} \rightarrow \alpha = \begin{cases} 30^\circ + 360^\circ k \\ 150^\circ + 360^\circ k \end{cases} \end{cases}$$

5) $\text{sen}^2\alpha + \cos\alpha = \dfrac{5}{4}$

$$1 - \cos^2\alpha + \cos\alpha = \frac{5}{4} \rightarrow 4\cos^2\alpha - 4\cos\alpha + 1 = 0 \rightarrow \cos\alpha = \frac{4 \pm \sqrt{16 - 16}}{8} = \frac{1}{2} \rightarrow$$

$$\rightarrow \alpha = \begin{cases} 60^\circ + 360^\circ k \\ 300^\circ + 360^\circ k \end{cases}$$

6) $\text{sen}\,2\alpha = 2\cos\alpha$

$$2\,\text{sen}\,\alpha\cos\alpha = 2\cos\alpha \rightarrow 2\,\text{sen}\,\alpha\cos\alpha - 2\cos\alpha = 0 \rightarrow \cos\alpha\left(2\,\text{sen}\,\alpha - 2\right) = 0 \rightarrow$$

$$\rightarrow \begin{cases} \cos\alpha = 0 \rightarrow \alpha = \begin{cases} 90^\circ + 360^\circ k \\ 270^\circ + 360^\circ k \end{cases} \\ 2\,\text{sen}\,\alpha - 2 = 0 \rightarrow \text{sen}\,\alpha = 1 \rightarrow \alpha = 90^\circ + 360^\circ k \end{cases}$$

7) $\text{sen}\,4\alpha = \text{sen}\,2\alpha$

$$2\,\text{sen}\,2\alpha\cos 2\alpha = \text{sen}\,2\alpha \rightarrow 2\,\text{sen}\,2\alpha\cos 2\alpha - \text{sen}\,2\alpha = 0 \rightarrow \text{sen}\,2\alpha\left(2\cos 2\alpha - 1\right) = 0 \rightarrow$$

$$\rightarrow \begin{cases} \operatorname{sen} 2\alpha = 0 \rightarrow \alpha = \begin{cases} 0^\circ + 360^\circ k \\ 90^\circ + 360^\circ k \end{cases} \\ \cos 2\alpha = \dfrac{1}{2} \rightarrow 2\alpha = \begin{cases} 60^\circ + 360^\circ k \\ 300^\circ + 360^\circ k \end{cases} \rightarrow \alpha = \begin{cases} 30^\circ + 180^\circ k \\ 150^\circ + 180^\circ k \end{cases} \end{cases}$$

8) $2\operatorname{sen}\alpha = \operatorname{tg}\alpha$

$$2\operatorname{sen}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} \rightarrow 2\operatorname{sen}\alpha\cos\alpha = \operatorname{sen}\alpha \rightarrow 2\operatorname{sen}\alpha\cos\alpha - \operatorname{sen}\alpha = 0 \rightarrow \operatorname{sen}\alpha(2\cos\alpha - 1) = 0$$

$$\rightarrow \begin{cases} \operatorname{sen}\alpha = 0 \rightarrow \alpha = \begin{cases} 0^\circ + 360^\circ k \\ 180^\circ + 360^\circ k \end{cases} \\ 2\cos\alpha - 1 = 0 \rightarrow \cos\alpha = \dfrac{1}{2} \rightarrow \alpha = \begin{cases} 60^\circ + 360^\circ k \\ 300^\circ + 360^\circ k \end{cases} \end{cases}$$

9) $\operatorname{sen}\alpha = 1 + 2\cos^2\alpha$

$$\operatorname{sen}\alpha = 1 + 2(1 - \operatorname{sen}^2\alpha) \rightarrow \operatorname{sen}\alpha = 1 + 2 - 2\operatorname{sen}^2\alpha \rightarrow 2\operatorname{sen}^2\alpha + \operatorname{sen}\alpha - 3 = 0 \rightarrow \operatorname{sen}\alpha = 1 \rightarrow$$
$$\rightarrow \alpha = 90^\circ + 360^\circ k$$

10) $\sec\alpha + \operatorname{tg}\alpha = 0$

$$\frac{1}{\cos\alpha} + \frac{\operatorname{sen}\alpha}{\cos\alpha} = 0 \rightarrow 1 + \operatorname{sen}\alpha = 0 \rightarrow \operatorname{sen}\alpha = -1 \rightarrow \alpha = 270^\circ + 360^\circ k$$

11) $6\cos^2\dfrac{\alpha}{2} + \cos\alpha = 1$

$$6\left(\sqrt{\frac{1+\cos\alpha}{2}}\right)^2 + \cos\alpha = 1 \rightarrow 6\left(\frac{1+\cos\alpha}{2}\right) + \cos\alpha = 1 \rightarrow 3 + 3\cos\alpha + \cos\alpha = 1 \rightarrow \cos\alpha = -\frac{1}{2}$$

$$\rightarrow \alpha = \begin{cases} 120^\circ + 360^\circ k \\ 240^\circ + 360^\circ k \end{cases}$$

12) $6\cos^2\alpha + 6\operatorname{sen}^2\alpha = 5 + \operatorname{sen}\alpha$

$$6(\cos^2\alpha + \operatorname{sen}^2\alpha) = 5 + \operatorname{sen}\alpha \rightarrow 6 = 5 + \operatorname{sen}\alpha \rightarrow \operatorname{sen}\alpha = 1 \rightarrow \alpha = 90^\circ + 360^\circ k$$

13) $\operatorname{tg}2\alpha\operatorname{tg}\alpha = 1$

$$\frac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}\operatorname{tg}\alpha = 1 \rightarrow \frac{2\operatorname{tg}^2\alpha}{1-\operatorname{tg}^2\alpha} = 1 \rightarrow 2\operatorname{tg}^2\alpha = 1 - \operatorname{tg}^2\alpha \rightarrow \operatorname{tg}\alpha = \pm\frac{1}{\sqrt{3}} \rightarrow \alpha = \begin{cases} 30^\circ + 180^\circ k \\ 150^\circ + 180^\circ k \end{cases}$$

14) $\operatorname{sen}2\alpha\cos\alpha = 6\operatorname{sen}^3\alpha$

$$2\operatorname{sen}\alpha\cos\alpha\cos\alpha - 6\operatorname{sen}^3\alpha = 0 \rightarrow 2\operatorname{sen}\alpha(\cos^2\alpha - 3\operatorname{sen}^2\alpha) = 0 \rightarrow$$

$$\rightarrow \begin{cases} \operatorname{sen}\alpha = 0 \rightarrow \alpha = \begin{cases} 0^\circ + 360^\circ k \\ 180^\circ + 360^\circ k \end{cases} \\ \cos^2\alpha - 3\operatorname{sen}^2\alpha = 0 \rightarrow \cos^2\alpha - 3(1-\cos^2\alpha) = 0 \rightarrow \cos\alpha = \pm\dfrac{\sqrt{3}}{2} \rightarrow \alpha = \begin{cases} 30^\circ + 180^\circ k \\ 150^\circ + 360^\circ k \end{cases} \end{cases}$$

15) $\operatorname{sen}\alpha + \cos\alpha = \sqrt{2}$

$$\frac{1}{2}\operatorname{sen}\alpha + \frac{\sqrt{3}}{2}\cos\alpha = 1 \rightarrow \operatorname{sen}\left(\alpha + 60^\circ\right) = 1 \rightarrow \alpha + 60^\circ = 90^\circ + 360^\circ k \rightarrow \alpha = 30^\circ + 360^\circ k$$

16) $\operatorname{sen}\alpha + \cos\alpha = \frac{5}{2}$

Esta ecuación no tiene solución ya que $\operatorname{sen}\alpha + \cos\alpha > 2$, y esto es imposible ya que $\operatorname{sen}\alpha, \cos\alpha \in \left[-1,1\right]$, y por tanto, $-2 \le \operatorname{sen}\alpha + \cos\alpha \le 2$.

17) $2\cos^2\alpha + \cos\alpha - 1 = 0$

$$\cos\alpha = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4} = \begin{cases} \frac{1}{2} \rightarrow \alpha = \begin{cases} 60^\circ + 360^\circ k \\ 300^\circ + 360^\circ k \end{cases} \\ -1 \rightarrow \alpha = 180^\circ + 360^\circ k \end{cases}$$

18) $2\operatorname{sen}^2\alpha - 1 = 0$

$$\operatorname{sen}^2\alpha = \frac{1}{2} \rightarrow \operatorname{sen}\alpha = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$

$$\text{Si } \operatorname{sen}\alpha = \frac{\sqrt{2}}{2} \rightarrow \alpha = \begin{cases} 45^\circ + 360^\circ k \\ 135^\circ + 360^\circ k \end{cases}$$

$$\text{Si } \operatorname{sen}\alpha = -\frac{\sqrt{2}}{2} \rightarrow \alpha = \begin{cases} -45^\circ + 360^\circ k \\ 225^\circ + 360^\circ k \end{cases} = \begin{cases} 315^\circ + 360^\circ k \\ 225^\circ + 360^\circ k \end{cases}$$

19) $\operatorname{tg}^2\alpha - \operatorname{tg}\alpha = 0$

$$\operatorname{tg}\alpha\left(\operatorname{tg}\alpha - 1\right) = 0 \rightarrow \begin{cases} \operatorname{tg}\alpha = 0 \rightarrow \alpha = 0^\circ + 180^\circ k \\ \operatorname{tg}\alpha - 1 = 0 \rightarrow \operatorname{tg}\alpha = 1 \rightarrow \alpha = 45^\circ + 180^\circ k \end{cases}$$

20) $2\operatorname{sen}^2\alpha + 3\cos\alpha = 3$

$$2\left(1 - \cos^2\alpha\right) + 3\cos\alpha = 3 \rightarrow 2 - 2\cos^2\alpha + 3\cos\alpha = 3 \rightarrow 2\cos^2\alpha - 3\cos\alpha + 1 = 0 \rightarrow$$

$$\rightarrow \cos\alpha = \begin{cases} 1 \rightarrow \alpha = 0^\circ + 360^\circ k \\ \frac{1}{2} \rightarrow \alpha = \begin{cases} 60^\circ + 360^\circ k \\ 300^\circ + 360^\circ k \end{cases} \end{cases}$$

21) $\operatorname{tg}2\alpha + 2\cos\alpha = 0$

$$\frac{2\operatorname{tg}\alpha}{1 - \operatorname{tg}^2\alpha} + 2\cos\alpha = 0 \rightarrow \frac{2\frac{\operatorname{sen}\alpha}{\cos\alpha}}{1 - \frac{\operatorname{sen}^2\alpha}{\cos^2\alpha}} + 2\cos\alpha = 0 \rightarrow \frac{\operatorname{sen}\alpha\cos\alpha}{\cos^2\alpha - \operatorname{sen}^2\alpha} + \cos\alpha = 0 \rightarrow$$

$$\rightarrow \operatorname{sen}\alpha\cos\alpha + \cos\alpha\left(\cos^2\alpha - \operatorname{sen}^2\alpha\right) = 0 \rightarrow \cos\alpha\left(\operatorname{sen}\alpha + \cos^2\alpha - \operatorname{sen}^2\alpha\right) = 0 \rightarrow$$

$$\rightarrow \begin{cases} \cos\alpha = 0 \rightarrow \alpha = \begin{cases} 90^{\circ} + 360^{\circ}k \\ 270^{\circ} + 360^{\circ}k \end{cases} \\ \operatorname{sen}\alpha + \cos^2\alpha - \operatorname{sen}^2\alpha = 0 \rightarrow \operatorname{sen}\alpha + 1 - \operatorname{sen}^2\alpha - \operatorname{sen}^2\alpha = 0 \rightarrow -2\operatorname{sen}^2\alpha + \operatorname{sen}\alpha + 1 = 0 \rightarrow \end{cases}$$

$$\rightarrow \operatorname{sen}\alpha = \begin{cases} -\dfrac{1}{2} \rightarrow \alpha = \begin{cases} 330^{\circ} + 360^{\circ}k \\ 210^{\circ} + 360^{\circ}k \end{cases} \\ 1 \rightarrow \alpha = 90^{\circ} + 360^{\circ}k \end{cases}$$

22) $\sqrt{2}\cos\dfrac{\alpha}{2} - \cos\alpha = 1$

$$\sqrt{2}\sqrt{\frac{1+\cos\alpha}{2}} - \cos\alpha = 1 \rightarrow \sqrt{1+\cos\alpha} - \cos\alpha = 1 \rightarrow \sqrt{1+\cos\alpha} = 1 + \cos\alpha \rightarrow$$

$$\rightarrow 1 + \cos\alpha = 1 + \cos^2\alpha + 2\cos\alpha \rightarrow \cos^2\alpha + \cos\alpha = 0 \rightarrow \cos\alpha\left(\cos\alpha + 1\right) = 0 \rightarrow$$

$$\rightarrow \begin{cases} \cos\alpha = 0 \rightarrow \alpha = \begin{cases} 90^{\circ} + 360^{\circ}k \\ \cancel{270^{\circ} + 360^{\circ}k} \end{cases} \\ \cos\alpha = -1 \rightarrow \alpha = 180^{\circ} + 360^{\circ}k \end{cases}$$

23) $2\operatorname{sen}\alpha\cos^2\alpha - 6\operatorname{sen}^3\alpha = 0$

$$2\operatorname{sen}\alpha\left(\cos^2\alpha - 3\operatorname{sen}^2\alpha\right) = 0 \rightarrow \begin{cases} \operatorname{sen}\alpha = 0 \\ \cos^2\alpha - 3\operatorname{sen}^2\alpha = 0 \rightarrow \cos^2\alpha + \operatorname{sen}^2\alpha - 4\operatorname{sen}^2\alpha \rightarrow \end{cases}$$

$$\rightarrow 1 - 4\operatorname{sen}^2\alpha = 0 \rightarrow \operatorname{sen}^2\alpha = \frac{1}{4} \rightarrow \operatorname{sen}\alpha = \pm\frac{1}{2} \rightarrow \begin{cases} \operatorname{sen}\alpha = \dfrac{1}{2} \rightarrow \alpha = \begin{cases} 30^{\circ} + 360^{\circ}k \\ 150^{\circ} + 360^{\circ}k \end{cases} \\ \operatorname{sen}\alpha = -\dfrac{1}{2} \rightarrow \alpha = \begin{cases} 210^{\circ} + 360^{\circ}k \\ 330^{\circ} + 360^{\circ}k \end{cases} \end{cases}$$