

Determina centro y radio de las circunferencias:

231. $3x^2 + 3y^2 + 6x - 12y - 48 = 0$

$$x^2 + y^2 + 2x - 4y - 16 = 0$$

D E F

$$D = -2a$$

$$a = \frac{2}{-2} = -1$$

$$-16 = 1 + 4 - R^2$$

$$E = -2b$$

$$b = \frac{-4}{-2} = 2$$

$$R = \sqrt{21}$$

$$F = a^2 + b^2 - R^2$$

Centro $(-1, 2)$ Radio $= \sqrt{21}$

232. $2x^2 + 2y^2 - x + 3y + 3 = 0$

$$x^2 + y^2 - \frac{1}{2}x + \frac{3}{2}y + \frac{3}{2} = 0$$

$$D = -\frac{1}{2}$$

$$E = \frac{3}{2}$$

$$F = \frac{3}{2}$$

$$-\frac{1}{2} = -2a \Rightarrow a = \frac{1}{4}$$

$$\frac{3}{2} = \frac{1}{16} + \frac{9}{16} - R^2 \Rightarrow R^2 = \frac{10}{16} - \frac{24}{16}$$

$$\frac{3}{2} = -2b \Rightarrow b = -\frac{3}{4}$$

$$R = \sqrt{-\frac{14}{16}} \quad \text{NO EXISTE}$$

No es una circunferencia

233. $x^2 + y^2 + y = 0$

$$D = 0$$

$$0 = -2a \Rightarrow a = 0$$

$$E = 1$$

$$1 = -2b \Rightarrow b = -\frac{1}{2}$$

$$F = 0$$

$$0 = 0 + \frac{1}{4} - R^2 \Rightarrow R = \frac{1}{2}$$

Centro $(0, -\frac{1}{2})$

Radio $= \frac{1}{2}$

234. $3x^2 + 3y^2 - 2x + 3y - 6 = 0$

$$x^2 + y^2 - \frac{2}{3}x + y - 2 = 0$$

$$D = -\frac{2}{3}$$

$$E = 1$$

$$F = -2$$

$$+2a = +\frac{2}{3}$$

$$a = \frac{1}{3}$$

$$-2 = \frac{1}{9} + \frac{1}{4} - R^2$$

$$-2b = 1$$

$$b = -\frac{1}{2}$$

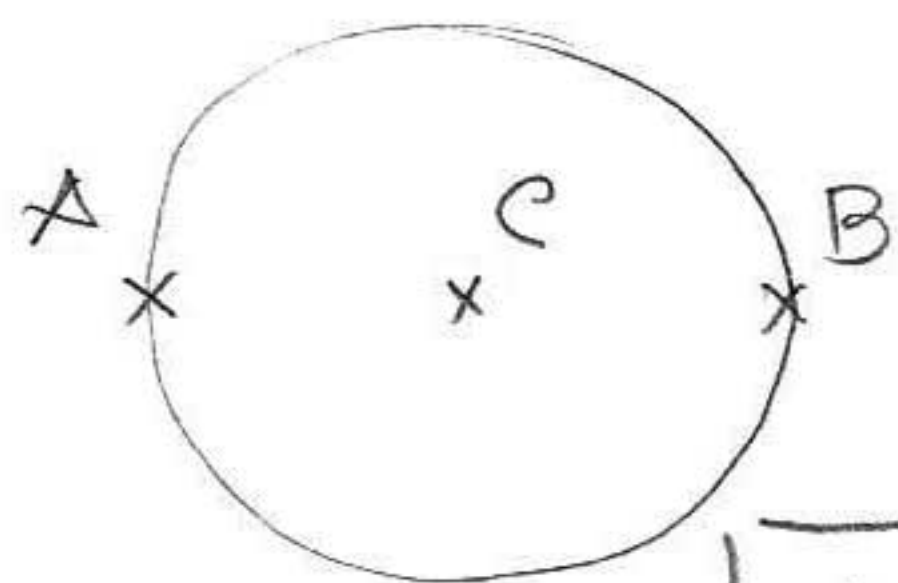
$$-\frac{2}{1} - \frac{1}{9} - \frac{1}{4} = \frac{-72 - 4 - 9}{36} = -\frac{85}{36}$$

Centro $(\frac{1}{3}, -\frac{1}{2})$

Radio $= \sqrt{\frac{85}{36}}$

Halla la ecuación de la circunferencia.

235. De diámetro AB siendo A(-1,5) y B(3,-7)

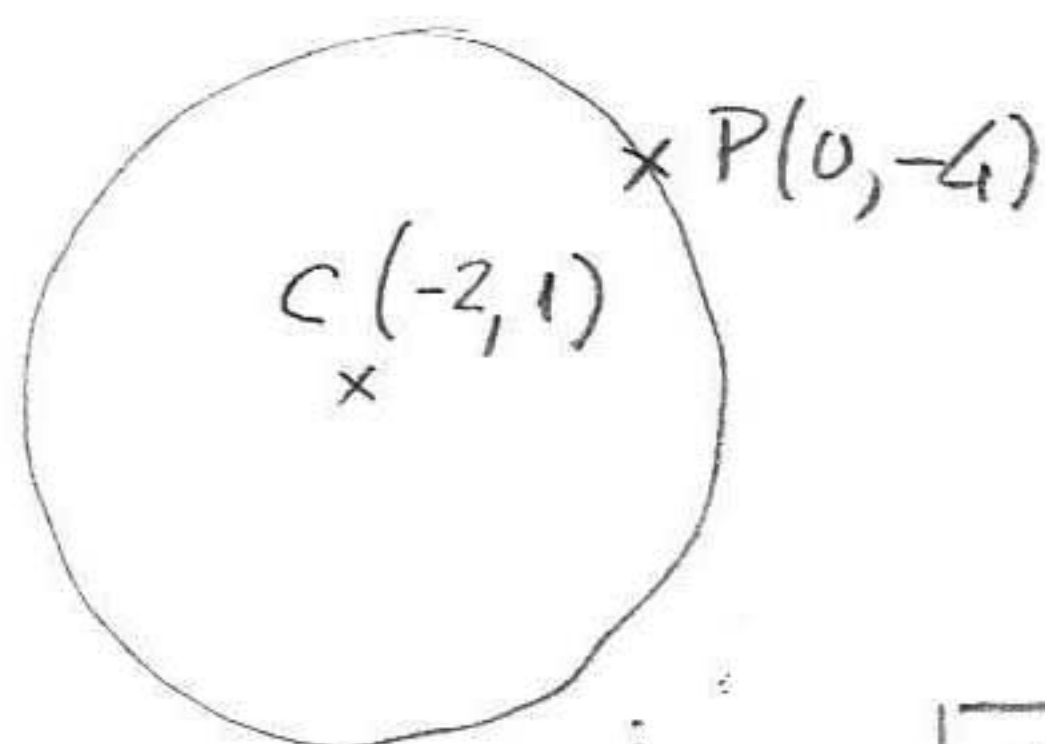


$$C\left(\frac{-1+3}{2}, \frac{5-7}{2}\right) \Rightarrow C(1, -1)$$

$$\text{Radio} = d(AC) = \sqrt{4 + 36} = \sqrt{40}$$

$$(x-1)^2 + (y+1)^2 = 40$$

236. Que pase por P(0,-4) y Centro en C(-2,1)



$$(x-a)^2 + (y-b)^2 = R^2$$

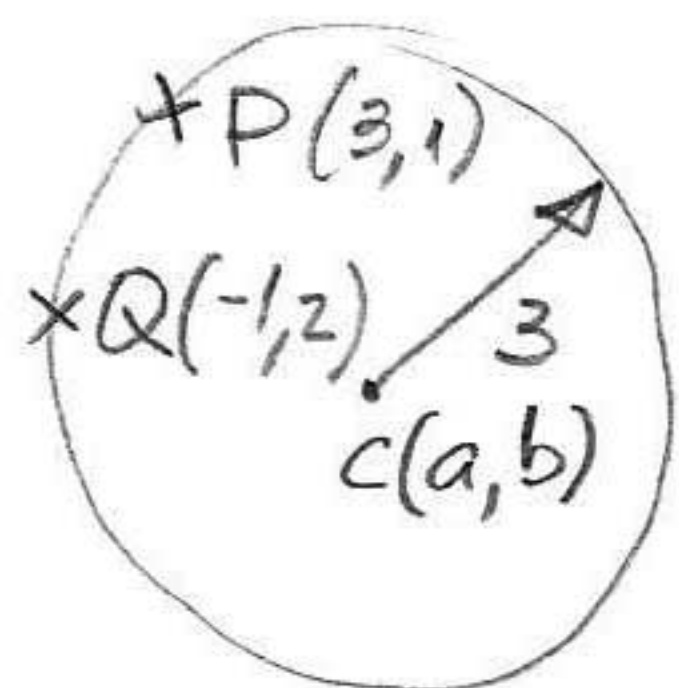
$$(x+2)^2 + (y-1)^2 = R^2$$

$$(0+2)^2 + (-4-1)^2 = R^2$$

$$4 + 25 = R^2 \rightarrow R = \sqrt{29}$$

$$(x+2)^2 + (y-1)^2 = 29$$

237. Que pase por P(3,1), Q(-1,2) y su radio es 3



Las distancias CP y CQ tienen que valer 3

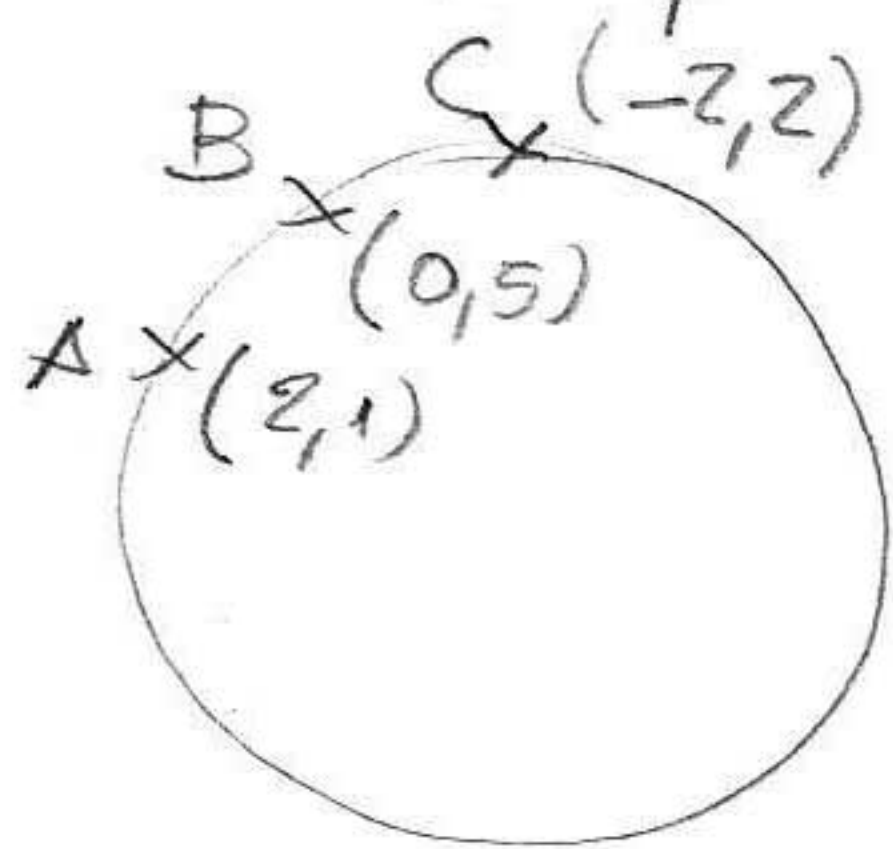
$$\left. \begin{aligned} \sqrt{(3-a)^2 + (1-b)^2} &= 3 \\ \sqrt{(-1-a)^2 + (2-b)^2} &= 3 \end{aligned} \right\} \begin{aligned} (3-a)^2 + (1-b)^2 &= 9 \\ (-1-a)^2 + (2-b)^2 &= 9 \end{aligned}$$

$$\begin{aligned} 9 + a^2 - 6a + 1 + b^2 - 2b &= 9 \rightarrow a^2 - 6a + (b^2 - 2b + 1) = 0 \Rightarrow \\ 1 + a^2 + 2a + 4 + b^2 - 4b &= 9 \end{aligned}$$

$$\begin{aligned} a &= \frac{6 \pm \sqrt{36 - 4(b^2 - 2b + 1)}}{2} = \frac{6 \pm \sqrt{36 - 4b^2 + 8b - 4}}{2} = \frac{6 \pm \sqrt{32 + 8b - 4b^2}}{2} \\ &= \frac{6 \pm 2\sqrt{8 + 2b - b^2}}{2} \end{aligned}$$

(...)

238. Que pasa por los puntos $(2,1)$ $(0,5)$ y $(-2,2)$



$$x^2 + y^2 + Dx + Ey + F = 0$$

$$\begin{cases} 2^2 + 1^2 + 2D - E + F = 0 \\ 0^2 + 5^2 + 0D + 5E + F = 0 \\ 4 + 4 - 2D + 2E + F = 0 \end{cases} \Rightarrow \begin{cases} 2D - E + F = -5 \\ 5E + F = -25 \\ -2D + 2E + F = -8 \end{cases}$$

$$E = \frac{-25 - F}{5}$$

$$\begin{cases} 2D - \left(\frac{-25 - F}{5}\right) + F = -5 \\ -2D + 2\left(\frac{-25 - F}{5}\right) + F = -8 \end{cases} \Rightarrow \begin{cases} 10D + 25 + F + 5F = -5 \\ -10D - 50 - 2F + 5F = -40 \end{cases}$$

$$\begin{cases} 10D + 6F = -30 \\ -10D + 3F = 10 \end{cases}$$

$$D = \frac{-30 + \frac{120}{9}}{10} = \frac{-150}{90} = -\frac{5}{3}$$

$$9F = -20 \rightarrow$$

$$F = -\frac{20}{9}$$

$$E = \frac{-25 + \frac{20}{9}}{5} = \frac{-205}{45} = -\frac{41}{9}$$

Ecuación

$$x^2 + y^2 - \frac{5}{3}x - \frac{41}{9}y - \frac{20}{9} = 0$$

239. Concéntrica con $2x^2 + 2y^2 - 3x + 4y - 5 = 0$ y radio $= \sqrt{3}$

$$x^2 + y^2 - \frac{3}{2}x + 2y - \frac{5}{2} \Rightarrow D = -\frac{3}{2} \quad E = 2$$

$$-2a = -\frac{3}{2} \Rightarrow a = \frac{3}{4} \quad -2b = 2 \Rightarrow b = -1$$

Centro $(\frac{3}{4}, -1)$

Radio $\sqrt{3}$

$$\text{Ecuación: } \left(x - \frac{3}{4}\right)^2 + (y + 1)^2 = 3$$

240. Concéntrica con $2x^2 + 2y^2 - 3x + 4y - 5 = 0$ y que pasa por $(4, 2)$

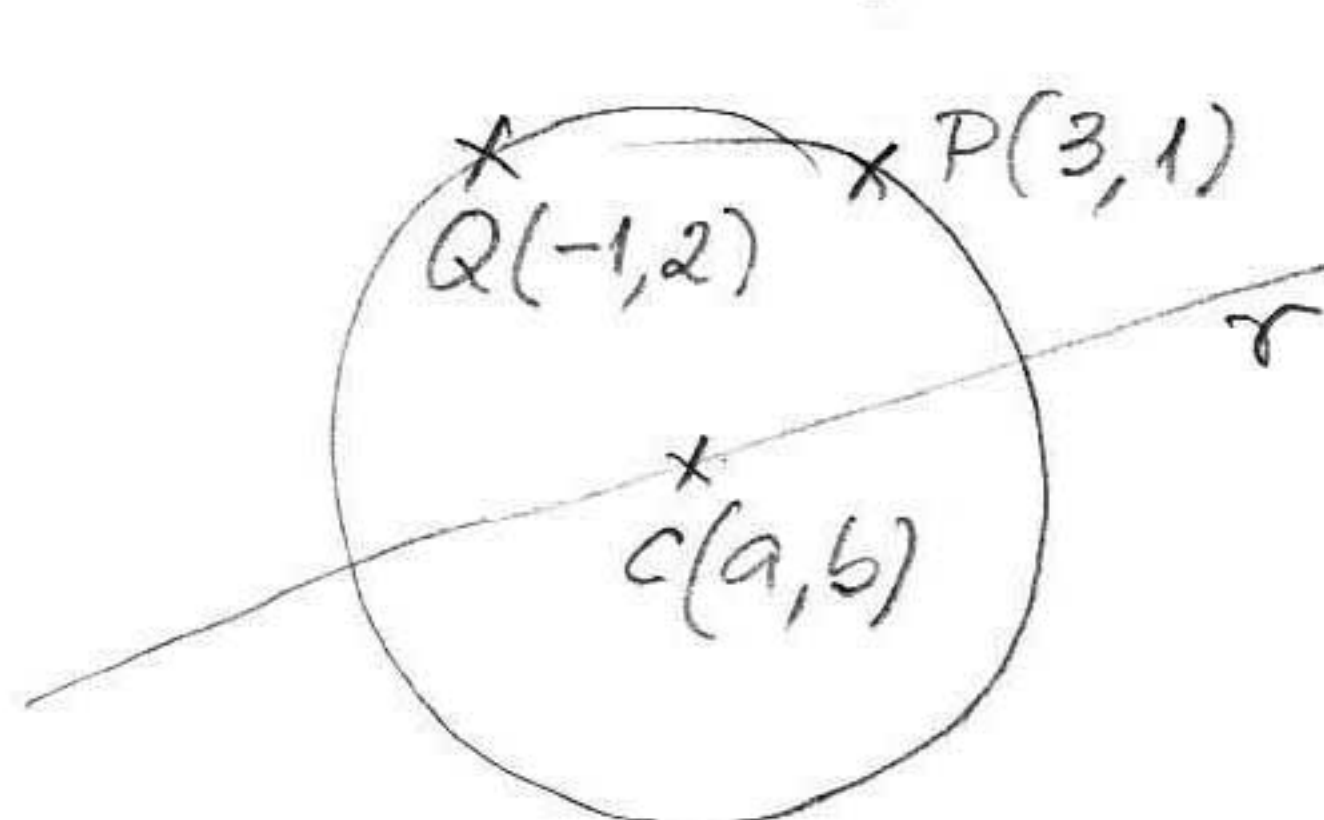
$$\text{Centro } \left(\frac{3}{4}, -1\right) \quad \left(x - \frac{3}{4}\right)^2 + (y + 1)^2 = R^2$$

$$\left(4 - \frac{3}{4}\right)^2 + (2 + 1)^2 = R^2 \Rightarrow R^2 = \left(\frac{13}{4}\right)^2 + 3^2 = \frac{169}{16} + 9 = \frac{313}{16}$$

Ecuación

$$\left(x - \frac{3}{4}\right)^2 + (y + 1)^2 = \frac{313}{16}$$

241. Que pasa por $P(3,1)$; $Q(-1,2)$ y cuyo centro esta en la recta $x-2y+1=0$



$$(x-a)^2 + (y-b)^2 = r^2$$

$$(3-a)^2 + (1-b)^2 = r^2$$

$$(-1-a)^2 + (2-b)^2 = r^2$$

$$a - 2b + 1 = 0 \rightarrow a = 2b - 1$$

$$\left. \begin{aligned} [3-(2b-1)]^2 + [1-b]^2 &= r^2 \\ [-1-(2b-1)]^2 + (2-b)^2 &= r^2 \end{aligned} \right\} \left. \begin{aligned} (4-2b)^2 + (1-b)^2 &= r^2 \\ (2b)^2 + (2-b)^2 &= r^2 \end{aligned} \right\}$$

$$\left. \begin{aligned} 16 + 4b^2 - 16b + 1 + b^2 - 2b &= r^2 \\ 4b^2 + 4 + b^2 - 4b &= r^2 \end{aligned} \right\} \left. \begin{aligned} 5b^2 - 18b + 17 &= r^2 \\ 5b^2 - 4b + 4 &= r^2 \end{aligned} \right\}$$

$$5b^2 - 18b + 17 = 5b^2 - 4b + 4$$

$$13 = 14b$$

$$b = \frac{13}{14}$$

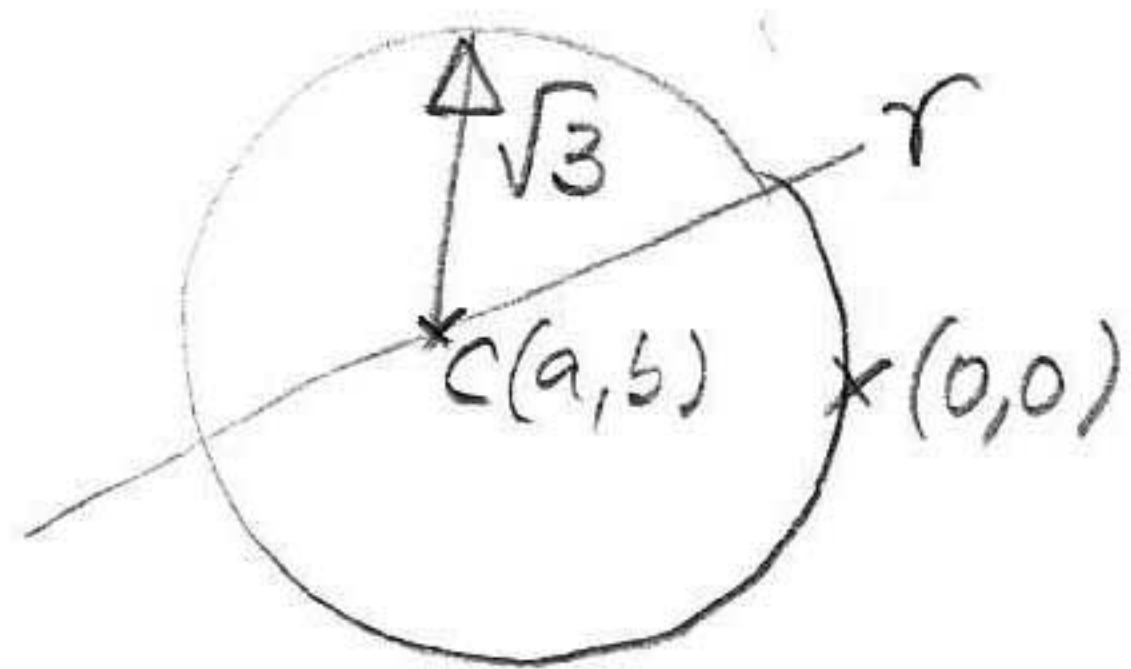
$$r^2 = 5\left(\frac{13}{14}\right)^2 - 18\left(\frac{13}{14}\right) + 17 \Rightarrow r^2 = \frac{845}{196} - \frac{234}{14} + 17 \Rightarrow$$

$$r^2 = \frac{845}{196} - \frac{3276}{196} + \frac{3332}{196} = \frac{901}{196} \Rightarrow r = \frac{\sqrt{901}}{14}$$

$$a = 2b - 1 = \frac{26}{14} - 1 = \frac{12}{14} = \frac{6}{7}$$

$$\left(x - \frac{6}{7}\right)^2 + \left(y - \frac{13}{14}\right)^2 = \frac{901}{196}$$

242. Cuyo radio es $\sqrt{3}$, pasa por $(0,0)$ y su centro esta en la recta $x+y=0$



$$(x-a)^2 + (y-b)^2 = r^2$$

$$(-a)^2 + (-b)^2 = 3$$

$$a + b = 0$$

$$a^2 + b^2 = 3$$

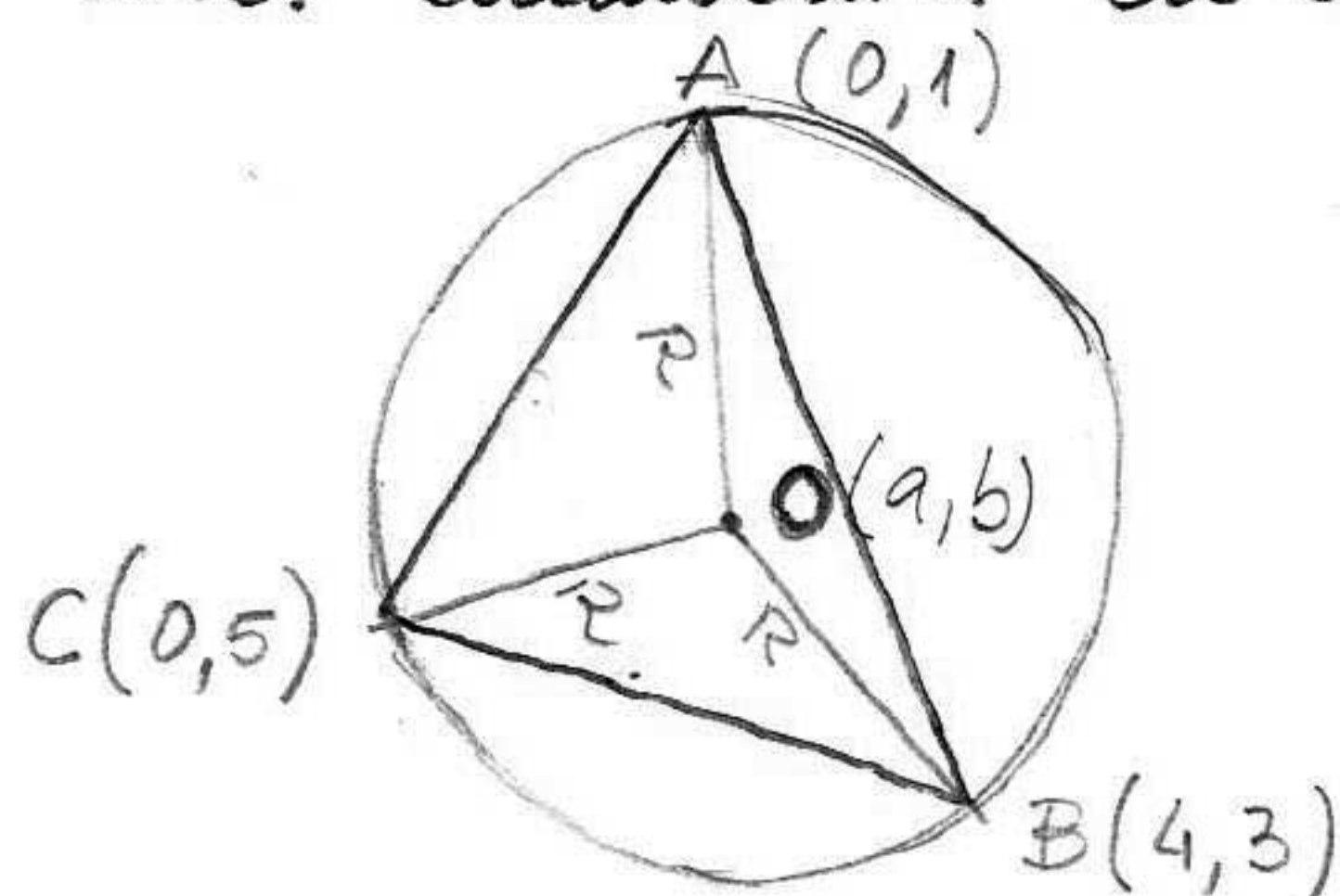
$$a + b = 0$$

$$a = -b \Rightarrow (-b)^2 + b^2 = 3 \Rightarrow 2b^2 = 3 \Rightarrow b = \sqrt{\frac{3}{2}}$$

$$a = -\sqrt{\frac{3}{2}}$$

$$\left(x + \sqrt{\frac{3}{2}}\right)^2 + \left(y - \sqrt{\frac{3}{2}}\right)^2 = 3$$

243. Circunscrita en el triángulo de vértices $(0,1); (4,3); (0,5)$



$$x^2 + y^2 + Dx + Ey + F = 0$$

$$\begin{cases} 16 + 9 + 4D + 3E + F = 0 \\ 25 + 5E + F = 0 \\ 1 + E + F = 0 \end{cases} \rightarrow E = -1 - F$$

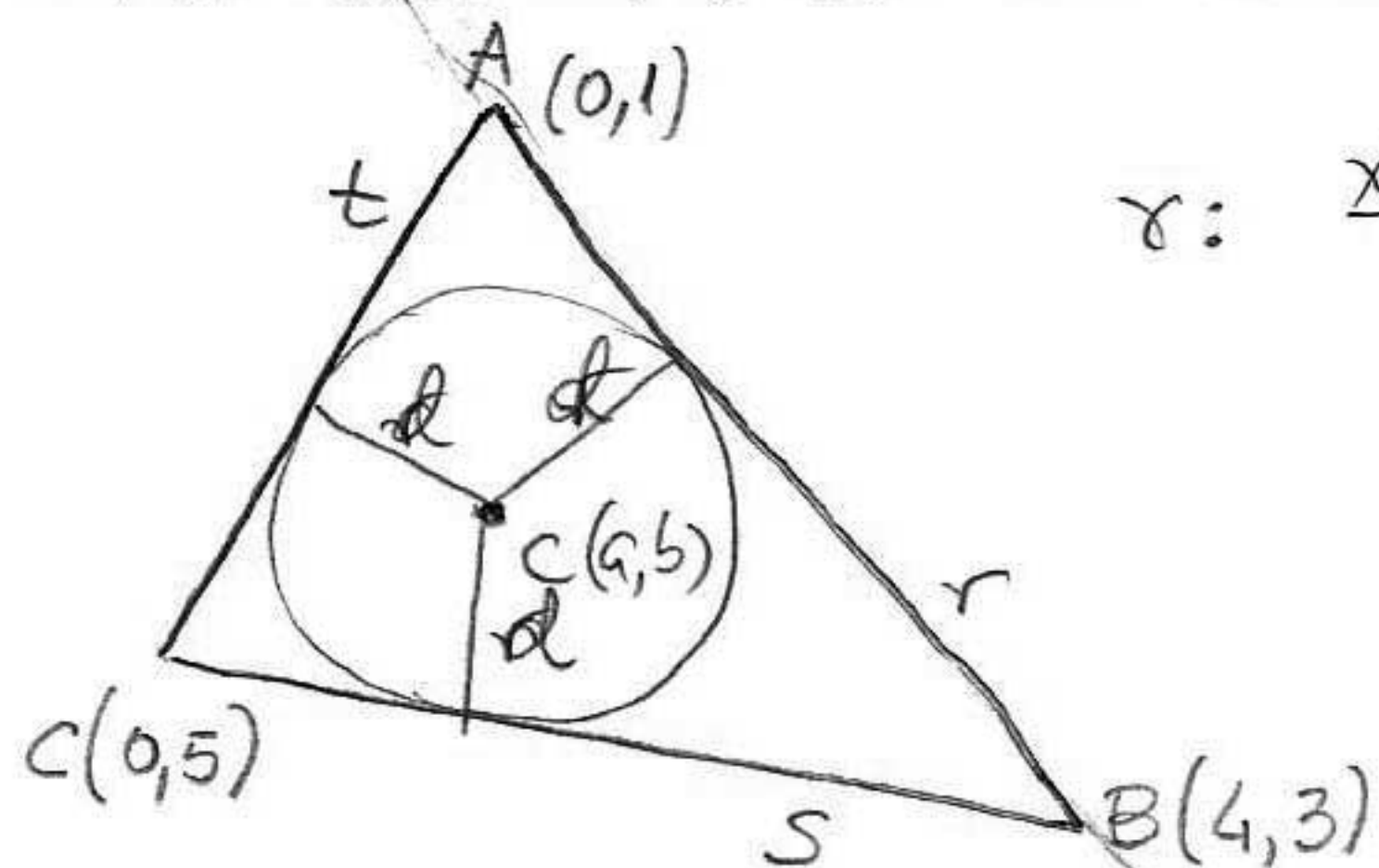
$$25 + 5(-1 - F) + F = 0$$

$$25 - 5 - 4F = 0 \Rightarrow \boxed{F = 5} \quad \boxed{E = -6}$$

$$D = \frac{-16 - 9 + 18 - 5}{4} = -3$$

$$\text{Circunferencia: } x^2 + y^2 - 3x - 6y + 5 = 0$$

244. Inscrita en el triángulo del ejercicio anterior



$$r: \frac{x-0}{4} = \frac{y-1}{2} \Rightarrow \boxed{2x - 4y + 4 = 0} \quad (r)$$

$$s: \frac{x-0}{4} = \frac{y-5}{-2} \Rightarrow -2x - 4y + 20 = 0$$

$$2x + 4y - 20 = 0 \Rightarrow \boxed{x + 2y - 10 = 0} \quad (s)$$

$$t: \frac{x-0}{0} = \frac{y-1}{4} \Rightarrow 4x = 0 \Rightarrow \boxed{x = 0} \quad (t)$$

distancia del centro a cada recta

$$\frac{|2a - 4b + 4|}{\sqrt{4 + 16}} = \frac{|a + 2b - 10|}{\sqrt{1 + 4}} = \frac{|a|}{\sqrt{1}}$$

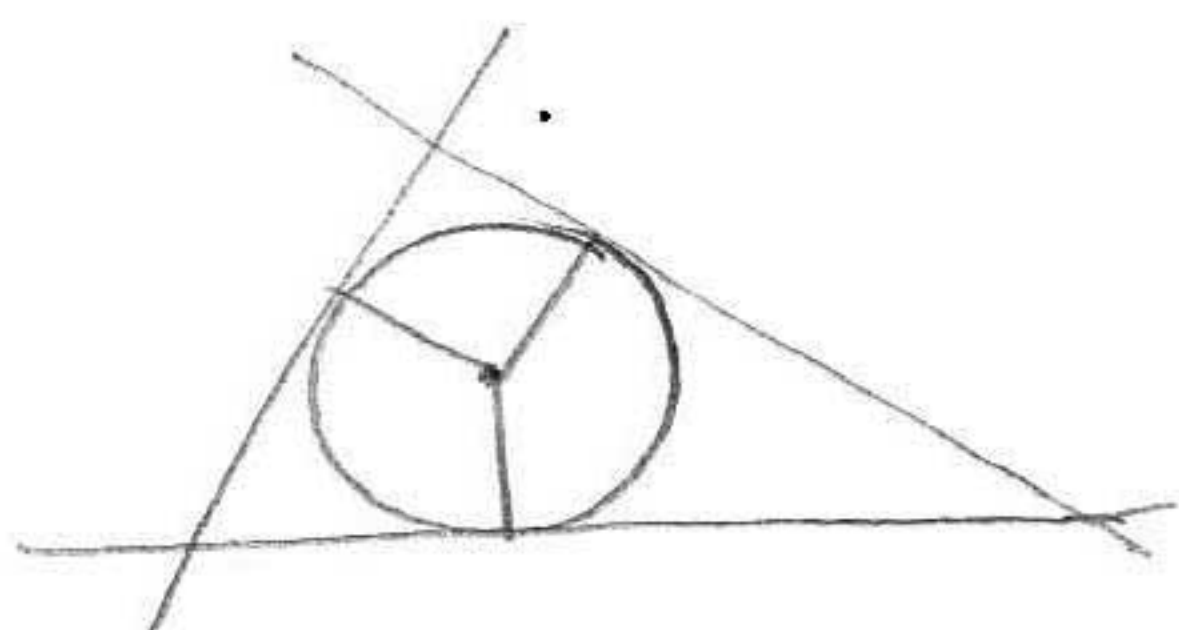
$$\begin{cases} 2a - 4b + 4 = \sqrt{20}a \\ a + 2b - 10 = \sqrt{5}a \end{cases} \quad \begin{cases} 2a - 4b + 4 = \sqrt{20}a \\ 2a + 4b - 20 = 2\sqrt{5}a \end{cases} \quad \begin{cases} 4a - 4\sqrt{5}a = 16 \\ a = \frac{16}{4 - 4\sqrt{5}} = -3'236 \end{cases}$$

$$b = \frac{\sqrt{5} \cdot (-3'236) - (-3'236) + 10}{2} = 3 \quad \text{Centro } (-3'236, 3)$$

$$\text{Radio} = 3'236$$

$$\text{Circunferencia } (x + 3'236)^2 + (y - 3)^2 = 10'472$$

245. Inscrita en el triángulo de lados $x-y=0$ $x-2y+4=0$
 $x+y+2=0$



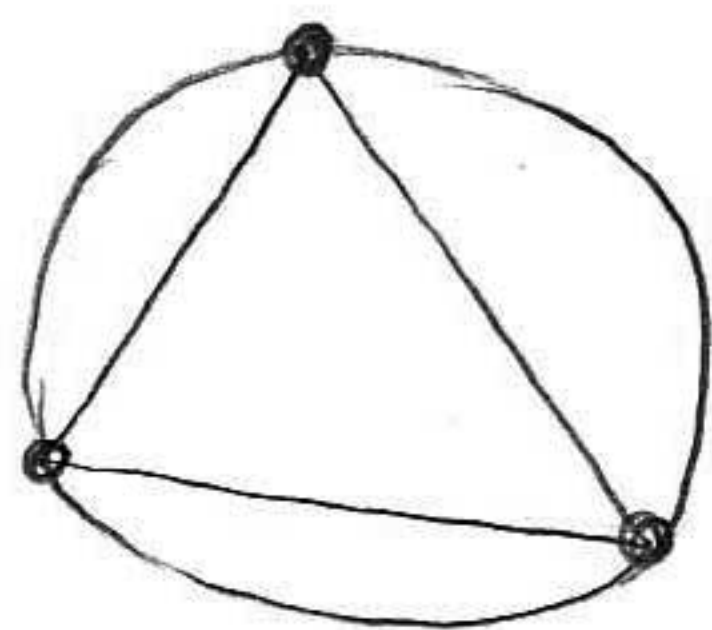
$$\frac{a-b}{\sqrt{2}} = \frac{a-2b+4}{\sqrt{5}} = \frac{a+b+2}{\sqrt{2}}$$

$$\left. \begin{aligned} \sqrt{5}a - \sqrt{5}b &= \sqrt{2}a - 2\sqrt{2}b + 4\sqrt{2} \\ \sqrt{2}a - \sqrt{2}b &= \sqrt{2}a + \sqrt{2}b + 2\sqrt{2} \end{aligned} \right\}$$

$$b = \frac{2\sqrt{2}}{-2\sqrt{2}} = \boxed{-1}$$

$$\sqrt{5}a + \sqrt{5} = \sqrt{2}a + 2\sqrt{2} + 4\sqrt{2} \Rightarrow a = \frac{6\sqrt{2} - \sqrt{5}}{\sqrt{5} - \sqrt{2}}$$

246. Circunscrita al triángulo del ejercicio anterior



$$x^2 + y^2 + Dx + Ey + F = 0$$

$$\left. \begin{aligned} x-y &= 0 \\ x-2y+4 &= 0 \end{aligned} \right\} \quad x=4 \quad \boxed{A(4,4)}$$

$$y-4=0 \rightarrow y=4$$

$$\left. \begin{aligned} x-y &= 0 \\ x+y+2 &= 0 \end{aligned} \right\} \quad y=-1$$

$$2x+2=0 \quad x=-1$$

$$\boxed{B(-1,-1)}$$

$$\left. \begin{aligned} x-2y+4 &= 0 \\ x+y+2 &= 0 \end{aligned} \right\}$$

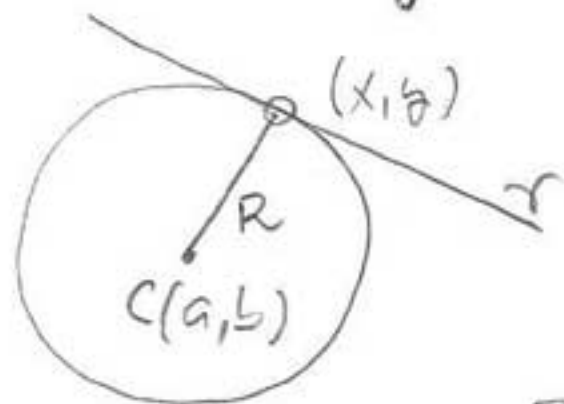
$$-3y+2=0$$

$$x = -\frac{8}{3}$$

$$y = \frac{2}{3}$$

$$\boxed{C(-\frac{8}{3}, \frac{2}{3})}$$

247. Cuyo centro es el punto $C(1, -5)$ y es tangente a la recta $3x - 4y + 1 = 0$



$$(x-1)^2 + (y+5)^2 = R^2$$

$$R = d(C, r) = \frac{|3 + 20 + 1|}{\sqrt{9 + 16}} = \frac{24}{5}$$

$$(x-1)^2 + (y+5)^2 = \frac{576}{25}$$

¿Cuáles de las siguientes ecuaciones corresponde a una circunferencia? Hallar el centro y el radio

$$x^2 + y^2 - 3x + 5 = 0$$

$$-x^2 - y^2 + 5x - 2y + 1 = 0$$

$$x^2 + y^2 + 2y - 3 = 0$$

$$-2x^2 - 2y^2 + 6x - 4y + 1 = 0$$

$$3x^2 + 3y^2 - 5x + 1 = 0$$

$$x^2 + y^2 - 2y + 4 = 0$$

$$x^2 + y^2 - 3x + 5 = 0 \Rightarrow D = -3 \quad E = 0 \quad F = 5$$

$$D = -2a \rightarrow -3 = -2a \Rightarrow a = 3/2$$

$$E = -2b \rightarrow 0 = -2b \Rightarrow b = 0$$

$$F = a^2 + b^2 - R^2 \rightarrow 5 = \frac{9}{4} - R^2 \Rightarrow R = \sqrt{\frac{9}{4} - 5} \quad \text{No existe}$$

$$x^2 + y^2 + 2y - 3 = 0 \Rightarrow D = 0 \quad E = 2 \quad F = -3 \quad \text{NO}$$

$$D = -2a \rightarrow 0 = -2a \Rightarrow a = 0$$

$$E = -2b \rightarrow 2 = -2b \Rightarrow b = -1$$

$$F = a^2 + b^2 - R^2 \rightarrow -3 = 1 - R^2 \Rightarrow R^2 = 4 \rightarrow R = 2 \quad \text{SI}$$

$$3x^2 + 3y^2 - 5x + 1 = 0 \Rightarrow D = -\frac{5}{3} \quad E = 0 \quad F = \frac{1}{3}$$

$$D = -2a \Rightarrow -\frac{5}{3} = -2a \rightarrow a = \frac{5}{6}$$

$$E = -2b \Rightarrow 0 = -2b \rightarrow b = 0$$

$$F = a^2 + b^2 - R^2 \Rightarrow \frac{1}{3} = \frac{25}{36} - R^2 \Rightarrow R = \sqrt{\frac{25}{36} - \frac{1}{3}} \quad \text{SI}$$

$$-x^2 - y^2 + 5x - 2y + 1 = 0 \quad D = -5 \quad E = 2 \quad F = -1$$

$$D = -2a \rightarrow -5 = -2a \rightarrow a = \frac{5}{2}$$

$$E = -2b \rightarrow 2 = -2b \rightarrow b = -1$$

$$F = a^2 + b^2 - R^2 \rightarrow -1 = \frac{25}{4} + 1 - R^2 \Rightarrow R^2 = \frac{25}{4} + 2 \quad \text{SI}$$

$$-2x^2 - 2y^2 + 6x - 4y + 1 = 0 \quad D = -3 \quad E = 2 \quad F = -1/2$$

$$D = -2a \rightarrow -3 = -2a \rightarrow a = 3/2$$

$$E = -2b \rightarrow 2 = -2b \rightarrow b = -1$$

$$F = a^2 + b^2 - R^2 \rightarrow -1/2 = 9/4 + 1 - R^2 \rightarrow R = \sqrt{\frac{11}{4} + 1}$$

(S1)

$$x^2 + y^2 - 2y + 4 = 0 \rightarrow D = 0 \quad E = -2 \quad F = 4$$

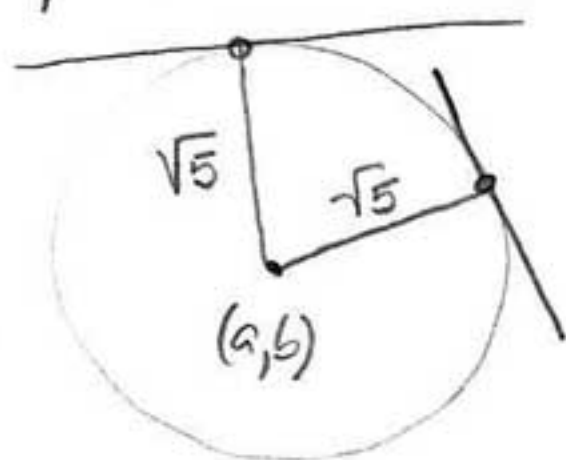
$$-2a = 0 \rightarrow a = 0$$

$$-2 = -2b \rightarrow b = 1$$

$$4 = 0 + 1 - R^2 \rightarrow R = \sqrt{-3}$$

(NO)

Halla una ecuación de la circunferencia de radio $r = \sqrt{5}$ tangente a las dos rectas $2x - y = 0$ $2x + y - 2 = 0$



$$\left\{ \begin{array}{l} \frac{2a - b}{\sqrt{4 + 1}} = \sqrt{5} \\ \frac{2a + b - 2}{\sqrt{5}} = \sqrt{5} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} 2a - b = 5 \\ 2a + b - 2 = 5 \end{array} \right\}$$

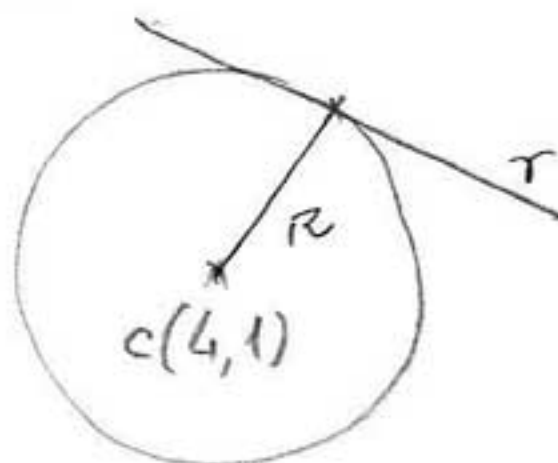
$$\frac{4a - 2}{-2} = 10 \Rightarrow 4a - 2 = -20 \Rightarrow 4a = -18 \Rightarrow a = -\frac{9}{2}$$

$$a = \frac{12}{4} = 3$$

$$b = 2a - 5 = 1$$

$$(x - 3)^2 + (y - 1)^2 = 5$$

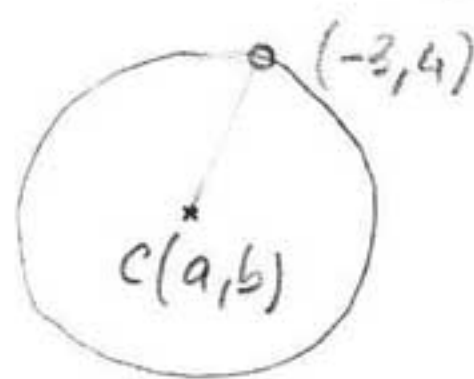
¿Qué circunferencia de centro $C(4, 1)$ es tangente a la recta de ecuación $3x - 4y + 2 = 0$?



$$d(C, r) = \frac{|12 - 4 + 2|}{\sqrt{9 + 16}} = \frac{10}{5} = 2$$

$$(x - 4)^2 + (y - 1)^2 = 4$$

Halla la ecuación de la recta normal a la circunferencia de ecuación $x^2 + y^2 = 25$ en el punto $(-3, 4)$



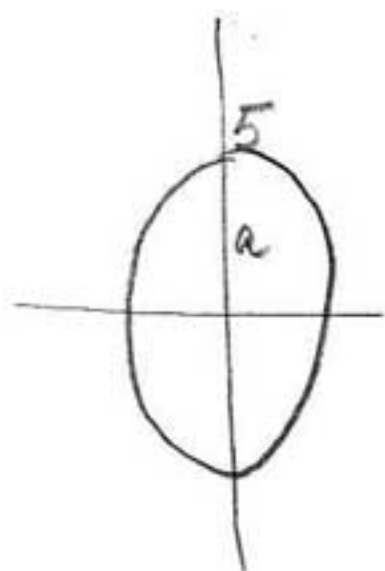
$$D = 0 \quad E = 0 \quad C(0, 0)$$

$$D = -2a \rightarrow a = 0$$

$$E = -2b \rightarrow b = 0$$

$$\frac{x - 0}{-3} = \frac{y - 0}{4} \Rightarrow 4x + 3y = 0$$

Halla la ecuación reducida de la elipse de semieje mayor vertical $a=5$ y excentricidad $e=0.6$



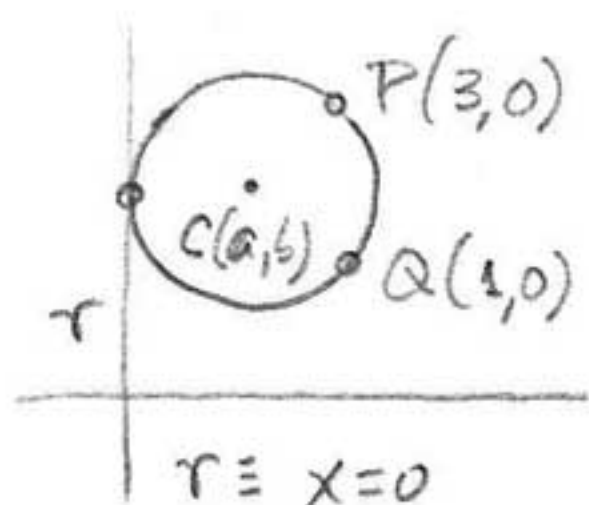
$$e = \frac{c}{a}; \quad 0.6 = \frac{c}{5} \Rightarrow c = 3$$

$$a^2 = b^2 + c^2 \Rightarrow b = \sqrt{5^2 - 3^2} = 4$$

$$\boxed{\frac{x^2}{16} + \frac{y^2}{25} = 1}$$

ya que el eje mayor es vertical.

248. Ecuación de la circunferencia que pasa por los puntos $P(3,0)$ $Q(1,0)$ y es tangente al eje de ordenadas



$$d(C,P) = \sqrt{(3-a)^2 + (0-b)^2} = \sqrt{9 + a^2 - 6a + b^2}$$

$$d(C,Q) = \sqrt{(1-a)^2 + (0-b)^2} = \sqrt{1 + a^2 - 2a + b^2}$$

$$d(C,r) = \frac{1a + 0b + 0}{\sqrt{1^2 + 0^2}} = \frac{a}{1} = a$$

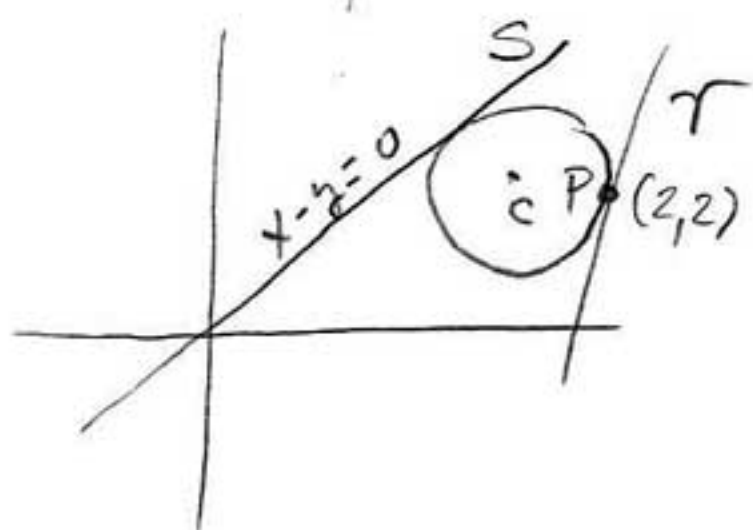
$$\sqrt{9 + a^2 - 6a + b^2} = \sqrt{1 + a^2 - 2a + b^2} \Rightarrow 8 = 4a \Rightarrow a = 2$$

$$\sqrt{9 + a^2 - 6a + b^2} = a \quad \left. \begin{array}{l} \\ 9 + a^2 - 6a + b^2 = a^2 \end{array} \right\} \Rightarrow$$

$$9 - 12 + b^2 = 0 \Rightarrow \underline{b^2 = 3} \quad \text{Radio} = \underline{a = 2}$$

$$\boxed{(x-2)^2 + (y-\sqrt{3})^2 = 4}$$

249. tangente a la bisectriz del I y III cuadrante en el punto $(2,2)$ y tambien tangente a la recta $x+y+1=0$



$$d(C,P) = \sqrt{(2-a)^2 + (2-b)^2}$$

$$d(C,s) = \frac{a-b}{\sqrt{2}}$$

$$d(C,r) = \frac{a+b+1}{\sqrt{2}}$$

$$\frac{a-b}{\sqrt{2}} = \frac{a+b+1}{\sqrt{2}} \Rightarrow \boxed{b = -\frac{1}{2}}$$

Signe $\rightarrow$