

1.- 1.- Resolver las siguientes inecuaciones:

a) $x - 1 - \frac{x-2}{x+3} \leq 1$

b) $-x^3 + 3x^2 + x - 3 < 0$

c) $\frac{3+x^2}{x^3-3x^2} \geq 0$

(a) $\frac{x(x+3) - (x+3) - (x-2)}{x+3} \leq 1 \Rightarrow \frac{x^2+3x-x-3-x+2}{x+3} \leq 1 \Rightarrow \frac{x^2+x-1}{x+3} - 1 \leq 0 \Rightarrow$
 $\frac{x^2+x-1-x-3}{x+3} \leq 0 \Rightarrow \frac{x^2-4}{x+3} \leq 0$ $\text{Sig } \frac{(x-2)(x+2)}{x+3}$

Sol: $\forall x \in (-\infty, -3) \cup [-2, 2]$

(b) $-x^3 + 3x^2 + x - 3 = -(x-1)(x+1)(x-3)$

-1	3	1	-3
1	-1	2	3
-1	2	3	0
-1	1	-3	
-1	3	0	

 $\text{Sig } (x-1)(x+1)(x-3)$

Sol: $\forall x \in (-1, 1) \cup (3, \infty)$

(c) $\text{Sig } \frac{3+x^2}{x^2(x-3)}$

$\forall x \in (3, \infty)$

- Resolver las ecuaciones exponenciales:

a. $9 - \frac{28}{3^x} = -3^{1-2x}$

b. $5^{x+1} + 5^x + 5^{x-1} = \frac{62}{5}$

(c) $3^x = z \Rightarrow 3^{2x} = z^2 \Rightarrow 3^{-2x} = z^{-2} = \frac{1}{z^2}$

$$9 - \frac{28}{3^x} = -3 \cdot 3^{-2x} \Rightarrow 9 - \frac{28}{z} = -\frac{3}{z^2} \Rightarrow 9z^2 - 28z = -3$$

$$\Rightarrow 9z^2 - 28z + 3 = 0 \Rightarrow z = \frac{28 \pm \sqrt{784 - 108}}{18}$$

$$z = 3 \Rightarrow 3^x = 3 \Rightarrow x = 1$$

$$z = \frac{1}{9} \Rightarrow 3^x = 3^{-2} \Rightarrow x = -2$$

(b) $\boxed{5^x = z}$

$$5^x \cdot 5 + 5^x + 5^x \cdot 5^{-1} = \frac{62}{5}$$

$$5z + z + \frac{1}{5}z = \frac{62}{5}$$

$$z\left(5 + 1 + \frac{1}{5}\right) = \frac{62}{5}$$

$$z\left(\frac{31}{5}\right) = \frac{62}{5}$$

$$z = \frac{62}{31} = 2 \Rightarrow 5^x = 2$$

$$x = \frac{\ln 2}{\ln 5}$$



$$\ln 5^x = \ln 2 \Rightarrow x \ln 5 = \ln 2 \Rightarrow$$

3.- Resolver las ecuaciones logarítmicas:

a) $\log(5x+4) - \log 2 = \frac{1}{2} \log(x+4)$

b) $(x^2 - x - 3) \cdot \log 4 = 3 \log \frac{1}{4}$

(a) $\log \frac{5x+4}{2} = \log \sqrt{x+4} \Rightarrow \left(\frac{5x+4}{2} \right)^2 = (\sqrt{x+4})^2 \Rightarrow$

$$\frac{25x^2 + 16 + 40x}{4} = x + 4 \Rightarrow 25x^2 + 16 + 40x = 4x + 16 \Rightarrow 25x^2 + 36x = 0$$

$$\Rightarrow x(25x + 36) = 0 \begin{cases} x=0 \text{ válida} \\ x = -\frac{36}{25} \text{ no válida } \cancel{\text{No log}} \end{cases}$$

(b) $\log 4^{x^2-x-3} = \log \left(\frac{1}{4} \right)^3 \Rightarrow 4^{x^2-x-3} = 4^{-3} \Rightarrow x^2-x-3 = -3 \Rightarrow$

$$x^2 - x = 0 \Rightarrow x(x-1) = 0 \begin{cases} x=0 \text{ válida} \\ x=1 \text{ válida} \end{cases}$$

4.- Determinar el valor de: $\log_{\frac{1}{9}}\left(\frac{\sqrt[3]{9}}{3}\right) - \log\sqrt{10^3\sqrt{0.1}} - \ln\sqrt{e} + \log_4\sqrt{2} = (*)$

$$1) \log_{\frac{1}{9}} \frac{\sqrt[3]{9}}{3} = z \Rightarrow \left(\frac{1}{9}\right)^z = \frac{\sqrt[3]{9}}{3} \Rightarrow 3^{-2z} = 3^{\frac{2}{3}-1} \Rightarrow 3^{-2z} = 3^{-\frac{1}{3}} \\ \Rightarrow -2z = -\frac{1}{3} \Rightarrow z = \frac{1}{6} //$$

$$2) \log\sqrt{10^3\sqrt{0.1}} = \log\sqrt[6]{10^3 \cdot 10^{-1}} = \log\sqrt[6]{10^2} = \log\sqrt[3]{10} = \frac{1}{3} //$$

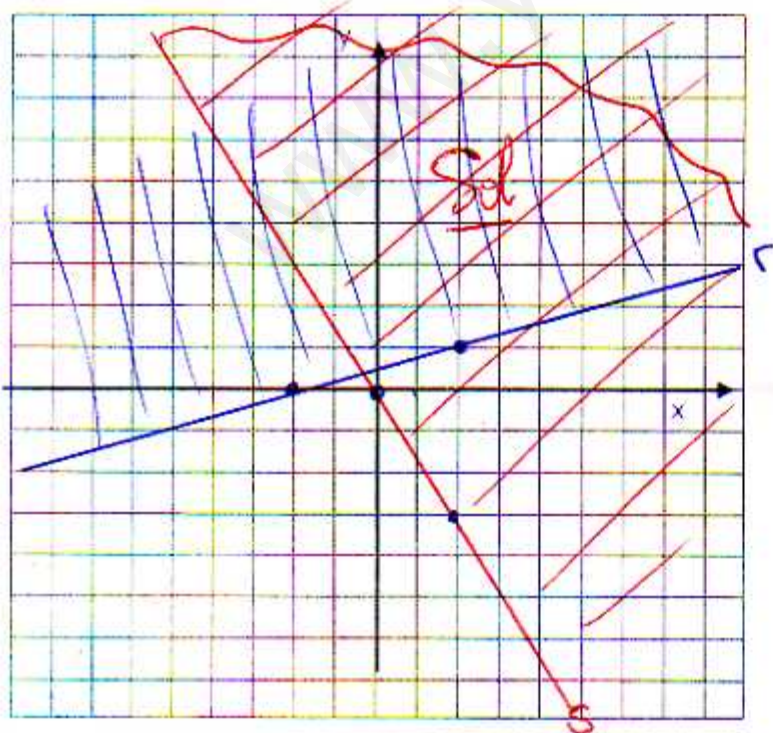
$$3) \ln\sqrt{e} = \ln e^{\frac{1}{2}} = \frac{1}{2} //$$

$$4) \log_4\sqrt{2} = z \Rightarrow 4^z = \sqrt{2} \Rightarrow 2^{2z} = 2^{\frac{1}{2}} \Rightarrow 2z = \frac{1}{2} \Rightarrow z = \frac{1}{4} //$$

$$= (*) \quad \frac{1}{6} - \frac{1}{3} - \frac{1}{2} + \frac{1}{4} = \frac{2 - 2 - 3 + 1}{12} = -\frac{5}{12} //$$

5.- Resuelve el siguiente sistema de inecuaciones

$$\begin{cases} -x + 4y > 2 & r \\ 3x + 2y \geq 0 & s \end{cases}$$



$$r \equiv -x + 4y = 2.$$

x	y
-2	0
2	1

$$O(0,0) \quad 0 > 2 \quad \text{NO}$$

$$s \equiv 3x + 2y = 0.$$

x	y
0	0
2	-3

$$P(-1,0) \quad -3 \geq 0 \quad \text{NO}$$