

1. (2 puntos) Resuelve las siguientes ecuaciones:

a.  $\sqrt{2x-1} + \sqrt{x+4} = 6$

b.  $\frac{x}{x^2-9} - \frac{x+3}{x^2-3x} = \frac{9-3x}{3x^2+9x}$

2. (1.5 puntos, 0.75 cada uno) Halla el dominio de las siguientes funciones:

a.  $g(x) = \frac{x-2}{-x^5-x^3+6x}$

b.  $j(x) = \sqrt{\frac{x-2}{-x^2+2x+3}}$

3. (1.5 puntos) Calcula las dimensiones de una habitación rectangular sabiendo que tiene  $12m^2$  de área y su diagonal mide 5m.

4. (2 puntos) Resolver las ecuaciones exponenciales:

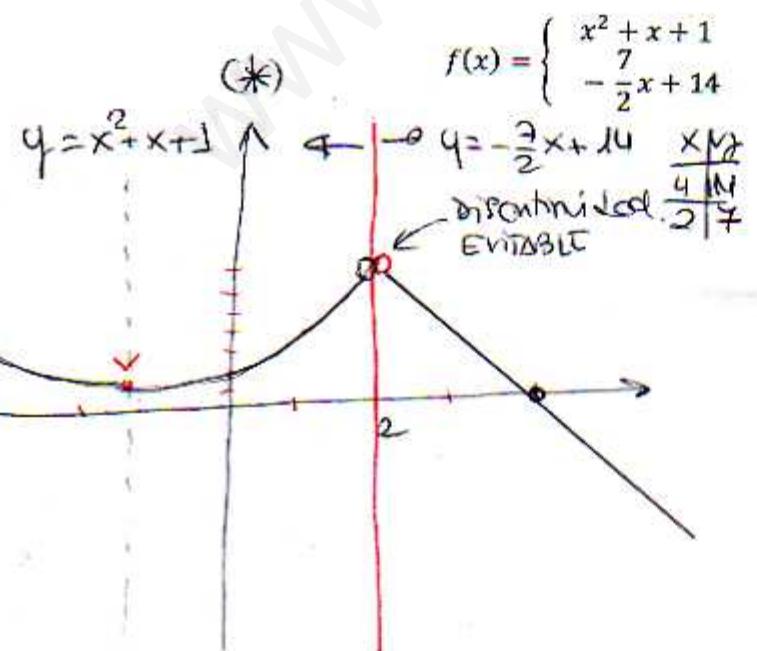
a.  $3^x + 3^{1-x} = 4$

b.  $\frac{8^{x-1}}{2^{2x-1}} = \sqrt[4]{4x-2}$

5. (1,25. puntos) Resolver la ecuaciones logarítmicas:

$$2 \log(2x+3) - \log(x+4) = 1 - \log 2$$

6. (1.75 puntos) Dada la función  $f(x)$ , dibujala y estudia su dominio, imagen, continuidad, intervalos de crecimiento y decrecimiento, máximos y mínimos.



$$f(x) = \begin{cases} x^2 + x + 1 & \text{si } x < 2 \\ -\frac{7}{2}x + 14 & \text{si } x > 2 \end{cases}$$

$$y = x^2 + x + 1$$

$$(*) \quad 1) \quad a = 1 > 0 \quad \cup$$

$$2) \quad x_0 = -\frac{b}{2a} = -\frac{1}{2} \quad y_0 = \frac{3}{4} \quad V(-0.5, 0.75)$$

$$3) \quad \frac{1}{2} \left| \frac{3}{4} \right| \rightarrow \text{Punto Fijo}$$

$$4) \quad \text{calcular } x \in (4, 2) \quad x^2 + x + 1 = \frac{3}{4} \quad \text{NO CORTA}$$

ANÁLISIS

$D(f) = \mathbb{R} - \{-2\}$   $\text{Im}(f) = \mathbb{R}$   
 Crec  $(-\frac{1}{2}, 2)$  Decres  $(-\infty, -\frac{1}{2}) \cup (2, \infty)$   
 MIN  $(-\frac{1}{2}, \frac{3}{4})$  NO MAX NI MO  
 & continua en  $\mathbb{R} - \{-2\}$ , en  $x = 2$   
 presenta una discontinuidad En el valor

(H4) (a)  $\sqrt{2x-1} + \sqrt{x+4} = 6 \Rightarrow (\sqrt{2x-1})^2 = (6 - \sqrt{x+4})^2 \Rightarrow 2x-1 = 36 + x+4 - 12\sqrt{x+4} = 0$   
 $(x-4)^2 = (-12\sqrt{x+4})^2 \Rightarrow x^2 + 16x - 82x = 144x + 576 \Rightarrow x^2 - 226x + 1105 = 0$   
 $x = \frac{226 \pm 216}{2} \quad x_1 = 221 \text{ No valid.} \quad x_2 = 5 \text{ valid.}$

(b)  $\frac{x}{x^2-9} - \frac{x+3}{x^2-3x} = \frac{9-3x}{3x^2+9x} \quad \left. \begin{array}{l} x^2-9 = (x-3)(x+3) \\ x^2-3x = x(x-3) \\ 3x^2+9x = 3x(x+3) \end{array} \right\} \text{mcm} = 3x(x+3)(x-3)$   
 $3x^2 - 3(x+3)^2 = (9-3x)(x-3) \Rightarrow 3x^2 - 3(x^2 + 6x + 9) = 9x^2 - 27 - 3x^2 + 9x = 0$   
 $3x^2 - 3x^2 - 27 - 18x = 9x - 27 - 3x^2 + 9x = 0 \Rightarrow -18x - 18x = -3x^2 \Rightarrow 3x^2 - 36x = 0$   
 $\Rightarrow 3x(x-12) = 0 \quad \text{Sd: } x=0 \text{ No valid. (anti domain)} \quad x=12 \text{ valid.}$

(H2) (a)  $g(x) = \frac{x-2}{-x^2-x^3+6x} \Rightarrow -x^5 - x^3 + 6x = 0 \Rightarrow -x(x^4 + x^2 - 6) = 0 \quad \left. \begin{array}{l} x=0 \\ x^4 + x^2 - 6 = 0 \end{array} \right\}$   
 $\left. \begin{array}{l} x=t^2 \\ x=t \end{array} \right\} t^2 + t - 6 = 0 \Rightarrow t = \frac{-1 \pm \sqrt{1+24}}{2} \quad \left. \begin{array}{l} t=2 \Rightarrow x^2=2 \Rightarrow x = \pm \sqrt{2} \\ t=-\frac{6}{2} = -3 \Rightarrow x^2 = -3 \text{ No Sd.} \end{array} \right\} g(x) = 12 - 4t_1 \pm 6t_2$   
(b)  $j(x) = \sqrt{\frac{x-2}{-x^2+2x+3}} \Rightarrow \frac{x-2}{-x^2+2x+3} \geq 0 \quad \left. \begin{array}{l} \text{sgn } \frac{x-2}{-(x-3)(x+1)} \\ -1 \quad 0 \quad 2 \quad 3 \end{array} \right\} -$

$\text{Dom } j(x) = (-\infty, -1) \cup [2, 3]$

(H3)  $\boxed{3w \quad x} \times \quad \left. \begin{array}{l} x \cdot y = 12 \\ x^2 + y^2 = 25 \end{array} \right\} \quad \left. \begin{array}{l} x = \frac{12}{y} \\ y^2 = t \end{array} \right\} \quad \left. \begin{array}{l} \left(\frac{12}{y}\right)^2 + y^2 = 25 \Rightarrow \frac{144}{y^2} + y^2 = 25 \\ 144 + y^4 = 25y^2 \Rightarrow t_1 = 16 \Rightarrow y=4 \quad x=3 \\ y^4 - 25y^2 + 144 = 0 \Rightarrow t_1 = \frac{25 \pm \sqrt{409}}{2} \quad t_2 = 9 \quad \left. \begin{array}{l} y=4 \quad x=3 \\ y=-4 \quad x=-3 \end{array} \right. \end{array} \right\}$   
 $A = 12w^2.$

(H4) (a)  $8^x + 8^{-x} = 4 \Rightarrow 8^x + 3 \cdot 8^{-x} = 4 \Rightarrow \boxed{8^x = t} \quad \boxed{8^{-x} = t^{-1}}$   
 $\Rightarrow t + \frac{3}{t} = 4 \Rightarrow t^2 + 3 = 4t \Rightarrow t^2 - 4t + 3 = 0 \quad \left. \begin{array}{l} t=3 \Rightarrow 8^x = 3 \Rightarrow x=1 \\ t=1 \Rightarrow 8^x = 1 \Rightarrow x=0 \end{array} \right\}$

(b)  $\frac{8^{x+1}}{2^{2x-1}} = \sqrt{4^{x-2}} \Rightarrow \frac{2^{3x-3}}{2^{2x-1}} = 2^{\frac{2x-2}{x+1}} \Rightarrow 2^{x-2} = 2^{\frac{2x-2}{x+1}} \Rightarrow$   
 $x-2 = \frac{2x-2}{x+1} \Rightarrow (x-2)(x+1) = 2x-2 \Rightarrow x^2 - x - 2 = 2x - 2 \Rightarrow x^2 - 3x = 0 \quad \left. \begin{array}{l} x=0 \text{ No valid.} \\ x=3 \text{ valid.} \end{array} \right.$

(H5)  $\log(2x+3)^2 - \log(x+4) = \log \frac{10}{2} \Rightarrow \log \frac{(2x+3)^2}{(x+4)} = \log 5 \Rightarrow$

$\frac{(2x+3)^2}{(x+4)} = 5 \Rightarrow 4x^2 + 9 + 12x = 5x + 20 \Rightarrow 4x^2 + 7x - 11 = 0 \Rightarrow x = \frac{-7 \pm \sqrt{15}}{8} \quad \left. \begin{array}{l} x = -\frac{11}{4} \text{ No valid.} \\ x = -\frac{1}{4} \end{array} \right\}$