

EJERCICIOS LOGARITMOS. SOLUCIONES

1.- Calcula, aplicando la definición, los siguientes logaritmos:

a) $\log_3 27 = y \Leftrightarrow 3^y = 27 \Leftrightarrow 3^y = 3^3 \Leftrightarrow y = 3$

Por tanto, $\log_3 27 = 3$

b) $\log_{\frac{1}{2}} 64 = y \Leftrightarrow \left(\frac{1}{2}\right)^y = 64 \Leftrightarrow 2^{-y} = 2^6 \Leftrightarrow -y = 6 \Leftrightarrow y = -6$

Por tanto, $\log_{\frac{1}{2}} 64 = -6$

c) $\log_2 128 = y \Leftrightarrow 2^y = 128 \Leftrightarrow 2^y = 2^7 \Leftrightarrow y = 7$

Por tanto, $\log_2 128 = 7$

d) $\log_{\sqrt{2}} 32 = y \Leftrightarrow (\sqrt{2})^y = 32 \Leftrightarrow \left(2^{\frac{1}{2}}\right)^y = 2^5 \Leftrightarrow 2^{\frac{y}{2}} = 2^5 \Leftrightarrow \frac{y}{2} = 5 \Leftrightarrow y = 10$

Por tanto, $\log_{\sqrt{2}} 32 = 10$

e) $\log_{\frac{1}{3}} \sqrt[3]{9} = y \Leftrightarrow \left(\frac{1}{3}\right)^y = \sqrt[3]{9} \Leftrightarrow 3^{-y} = \sqrt[3]{3^2} \Leftrightarrow 3^{-y} = 3^{\frac{2}{3}} \Leftrightarrow -y = \frac{2}{3} \Leftrightarrow y = -\frac{2}{3}$

Por tanto, $\log_{\frac{1}{3}} \sqrt[3]{9} = -\frac{2}{3}$

f) $\log_{2\sqrt{2}} 0,25 = y \Leftrightarrow (2\sqrt{2})^y = 0,25 \Leftrightarrow \left(2 \cdot 2^{\frac{1}{2}}\right)^y = \frac{25}{100} \Leftrightarrow \left(2^{\frac{3}{2}}\right)^y = \frac{1}{4} \Leftrightarrow \left(2^{\frac{3}{2}}\right)^y = 2^{-2} \Leftrightarrow 2^{\frac{3y}{2}} = 2^{-2} \Leftrightarrow$

$$\Leftrightarrow \frac{3y}{2} = -2 \Leftrightarrow 3y = -4 \Leftrightarrow y = -\frac{4}{3}$$

Por tanto, $\log_{2\sqrt{2}} 0,25 = -\frac{4}{3}$

g) $\log_{\frac{1}{2} \sqrt{2}} \frac{1}{2\sqrt{8}} = y \Leftrightarrow \left(\frac{1}{2}\right)^y = \frac{1}{2\sqrt{8}} \Leftrightarrow 2^{-y} = \frac{1}{2\sqrt{2^3}} \Leftrightarrow 2^{-y} = \frac{1}{2 \cdot 2^{\frac{3}{2}}} \Leftrightarrow 2^{-y} = \frac{1}{2^{\frac{5}{2}}} \Leftrightarrow$

$$\Leftrightarrow 2^{-y} = 2^{-\frac{5}{2}} \Leftrightarrow -y = -\frac{5}{2} \Leftrightarrow y = \frac{5}{2}$$

Por tanto, $\log_{\frac{1}{2} \sqrt{2}} \frac{1}{2\sqrt{8}} = \frac{5}{2}$

$$\text{h) } \log_{\frac{1}{2}} \sqrt[3]{16} = y \Leftrightarrow \left(\frac{1}{2}\right)^y = \sqrt[3]{16} \Leftrightarrow 2^{-y} = \sqrt[3]{2^4} \Leftrightarrow 2^{-y} = 2^{\frac{4}{3}} \Leftrightarrow -y = \frac{4}{3} \Leftrightarrow y = -\frac{4}{3}$$

$$\text{Por tanto, } \log_{\frac{1}{2}} \sqrt[3]{16} = -\frac{4}{3}$$

$$\text{i) } \ln \sqrt[5]{e^2} = y \Leftrightarrow e^y = \sqrt[5]{e^2} \Leftrightarrow e^y = e^{\frac{2}{5}} \Leftrightarrow y = \frac{2}{5}$$

$$\text{Por tanto, } \ln \sqrt[5]{e^2} = \frac{2}{5}$$

$$\text{j) } \ln \frac{e^2}{\sqrt{e}} = y \Leftrightarrow e^y = \frac{e^2}{\sqrt{e}} \Leftrightarrow e^y = \frac{e^2}{e^{\frac{1}{2}}} \Leftrightarrow e^y = e^{\frac{3}{2}} \Leftrightarrow y = \frac{3}{2}$$

$$\text{Por tanto, } \ln \frac{e^2}{\sqrt{e}} = \frac{3}{2}$$

$$\text{k) } \log 0,0001 = y \Leftrightarrow 10^y = 0,0001 \Leftrightarrow 10^y = 10^{-4} \Leftrightarrow y = -4$$

$$\text{Por tanto, } \log 0,0001 = -4$$

$$\text{l) } \log 0 = \text{no existe (} \log_a x \text{ existe } \Leftrightarrow x > 0)$$

$$\text{m) } \log(-10)^6 = y \Leftrightarrow 10^y = (-10)^6 \Leftrightarrow 10^y = 10^6 \Leftrightarrow y = 6$$

$$\text{Por tanto, } \log(-10)^6 = 6$$

$$\text{n) } \log(-10^6) = \text{no existe (} \log_a x \text{ existe } \Leftrightarrow x > 0)$$

$$\text{o) } \log_5 5\sqrt{5} = y \Leftrightarrow 5^y = 5\sqrt{5} \Leftrightarrow 5^y = 5 \cdot 5^{\frac{1}{2}} \Leftrightarrow 5^y = 5^{\frac{3}{2}} \Leftrightarrow y = \frac{3}{2}$$

$$\text{Por tanto, } \log_5 5\sqrt{5} = \frac{3}{2}$$

$$\text{p) } \log \sqrt{0'01} = y \Leftrightarrow 10^y = \sqrt{10^{-2}} \Leftrightarrow 10^y = 10^{-1} \Leftrightarrow y = -1$$

$$\text{Por tanto, } \log \sqrt{0'01} = -1$$

$$\text{q) } \log_6 \sqrt[5]{216^{-1}} = y \Leftrightarrow 6^y = \sqrt[5]{216^{-1}} \Leftrightarrow 6^y = \sqrt[5]{(6^3)^{-1}} \Leftrightarrow 6^y = \sqrt[5]{6^{-3}} \Leftrightarrow 6^y = 6^{-\frac{3}{5}} \Leftrightarrow y = -\frac{3}{5}$$

$$\text{Por tanto, } \log_6 \sqrt[5]{216^{-1}} = -\frac{3}{5}$$

$$\text{r) } \log_{\sqrt{\frac{1}{5}}} 0,04 = y \Leftrightarrow \left(\sqrt{\frac{1}{5}}\right)^y = 0,04 \Leftrightarrow \left(\sqrt{5^{-1}}\right)^y = \frac{4}{100} \Leftrightarrow \left(5^{-\frac{1}{2}}\right)^y = \frac{1}{25} \Leftrightarrow 5^{-\frac{y}{2}} = 5^{-2} \Leftrightarrow$$

$$\Leftrightarrow -\frac{y}{2} = -2 \Leftrightarrow -y = -4 \Leftrightarrow y = 4$$

Por tanto, $\log_{\sqrt{\frac{1}{5}}} 0,04 = 4$

$$\text{s) } \log_4 \frac{1}{\sqrt[3]{1024}} = y \Leftrightarrow 4^y = \frac{1}{\sqrt[3]{1024}} \Leftrightarrow (2^2)^y = \frac{1}{\sqrt[3]{2^{10}}} \Leftrightarrow 2^{2y} = \frac{1}{2^{\frac{10}{3}}} \Leftrightarrow 2^{2y} = 2^{-\frac{10}{3}} \Leftrightarrow$$

$$\Leftrightarrow 2y = -\frac{10}{3} \Leftrightarrow y = -\frac{5}{3}$$

Por tanto, $\log_4 \frac{1}{\sqrt[3]{1024}} = -\frac{5}{3}$

$$\text{t) } \log_{128} \sqrt[3]{2} = y \Leftrightarrow 128^y = \sqrt[3]{2} \Leftrightarrow (2^7)^y = 2^{\frac{1}{3}} \Leftrightarrow 2^{7y} = 2^{\frac{1}{3}} \Leftrightarrow 7y = \frac{1}{3} \Leftrightarrow y = \frac{1}{21}$$

Por tanto, $\log_{128} \sqrt[3]{2} = \frac{1}{21}$

$$\text{u) } \log_{\frac{1}{9}} \frac{\sqrt[4]{3}}{9} = y \Leftrightarrow \left(\frac{1}{9}\right)^y = \frac{\sqrt[4]{3}}{9} \Leftrightarrow (3^{-2})^y = \frac{3^{\frac{1}{4}}}{3^2} \Leftrightarrow 3^{-2y} = 3^{-\frac{7}{4}} \Leftrightarrow -2y = -\frac{7}{4} \Leftrightarrow y = \frac{7}{8}$$

Por tanto, $\log_{\frac{1}{9}} \frac{\sqrt[4]{3}}{9} = \frac{7}{8}$

$$\text{v) } \log_3 \frac{\sqrt[4]{3}}{\sqrt{27}} = y \Leftrightarrow 3^y = \frac{\sqrt[4]{3}}{\sqrt{27}} \Leftrightarrow 3^y = \frac{3^{\frac{1}{4}}}{3^{\frac{3}{2}}} \Leftrightarrow 3^y = 3^{\frac{1}{4} - \frac{3}{2}} \Leftrightarrow 3^y = 3^{-\frac{5}{4}} \Leftrightarrow y = -\frac{5}{4}$$

Por tanto, $\log_3 \frac{\sqrt[4]{3}}{\sqrt{27}} = -\frac{5}{4}$

w) $\log_2(-16) = \text{no existe}$ ($\log_a x$ existe $\Leftrightarrow x > 0$)

$$\text{x) } \ln \frac{1}{e^3} = y \Leftrightarrow e^y = \frac{1}{e^3} \Leftrightarrow e^y = e^{-3} \Leftrightarrow y = -3$$

Por tanto, $\ln \frac{1}{e^3} = -3$

y) $\log_{-3} 81 = \text{no existe}$, la base de un logaritmo debe ser un número real positivo y distinto de 1

z) $\log_a 1 = 0 \quad \forall a > 0, a \neq 1$

2.- Halla el valor de las siguientes expresiones:

$$\text{a) } \log_{25} \frac{1}{\sqrt[5]{5}} - \log_3 243 + \log_{16} \frac{1}{4} = -\frac{1}{10} - 5 + \left(-\frac{1}{2}\right) = -\frac{1}{10} - 5 - \frac{1}{2} = \frac{-1-50-5}{10} = -\frac{56}{10} = -\frac{28}{5}$$

$$(*) \log_{25} \frac{1}{\sqrt[5]{5}} = y \Leftrightarrow 25^y = \frac{1}{\sqrt[5]{5}} \Leftrightarrow (5^2)^y = \frac{1}{5^{\frac{1}{5}}} \Leftrightarrow 5^{2y} = 5^{-\frac{1}{5}} \Leftrightarrow 2y = -\frac{1}{5} \Leftrightarrow y = -\frac{1}{10}$$

$$(*) \log_3 243 = y \Leftrightarrow 3^y = 243 \Leftrightarrow 3^y = 3^5 \Leftrightarrow y = 5$$

$$(*) \log_{16} \frac{1}{4} = y \Leftrightarrow 16^y = \frac{1}{4} \Leftrightarrow (2^4)^y = 2^{-2} \Leftrightarrow 2^{4y} = 2^{-2} \Leftrightarrow 4y = -2 \Leftrightarrow y = -\frac{2}{4} \Leftrightarrow y = -\frac{1}{2}$$

$$\text{b) } \log_2 \sqrt[6]{0,5} - \log_{49} \frac{1}{7} - \log_{216} 6 - \log_4 64 = -\frac{1}{6} - \left(-\frac{1}{2}\right) - \frac{1}{3} - 3 = -\frac{1}{6} + \frac{1}{2} - \frac{1}{3} - 3 =$$

$$= \frac{-1+3-2-18}{6} = -\frac{18}{6} = -3$$

$$(*) \log_2 \sqrt[6]{0,5} = y \Leftrightarrow 2^y = \sqrt[6]{0,5} \Leftrightarrow 2^y = \sqrt[6]{\frac{1}{2}} \Leftrightarrow 2^y = \sqrt[6]{2^{-1}} \Leftrightarrow 2^y = 2^{-\frac{1}{6}} \Leftrightarrow y = -\frac{1}{6}$$

$$(*) \log_{49} \frac{1}{7} = y \Leftrightarrow 49^y = \frac{1}{7} \Leftrightarrow (7^2)^y = 7^{-1} \Leftrightarrow 7^{2y} = 7^{-1} \Leftrightarrow 2y = -1 \Leftrightarrow y = -\frac{1}{2}$$

$$(*) \log_{216} 6 = y \Leftrightarrow 216^y = 6 \Leftrightarrow (6^3)^y = 6 \Leftrightarrow 6^{3y} = 6^1 \Leftrightarrow 3y = 1 \Leftrightarrow y = \frac{1}{3}$$

$$(*) \log_4 64 = y \Leftrightarrow 4^y = 64 \Leftrightarrow 4^y = 4^3 \Leftrightarrow y = 3$$

$$\text{c) } \log_5 (25^5 \cdot 0,008^2) = y \Leftrightarrow 5^y = (25^5 \cdot 0,008^2) \Leftrightarrow 5^y = (5^2)^5 \cdot \left(\frac{8}{1000}\right)^2 \Leftrightarrow 5^y = 5^{10} \cdot \left(\frac{1}{125}\right)^2 \Leftrightarrow$$

$$\Leftrightarrow 5^y = 5^{10} \cdot (5^{-3})^2 \Leftrightarrow 5^y = 5^{10} \cdot 5^{-6} \Leftrightarrow 5^y = 5^4 \Leftrightarrow y = 4$$

Por tanto, $\log_5 (25^5 \cdot 0,008^2) = 4$

Otra forma (aplicando propiedades)

$$\log_5 (25^5 \cdot 0,008^2) \stackrel{\text{Prop.1}}{=} \log_5 25^5 + \log_5 0,008^2 = \log_5 (5^2)^5 + \log_5 \left(\frac{8}{1000}\right)^2 = \log_5 5^{10} + \log_5 \left(\frac{1}{125}\right)^2 =$$

$$= \log_5 5^{10} + \log_5 (5^{-3})^2 \stackrel{\text{Prop.3}}{=} \log_5 5^{10} + \log_5 5^{-6} \stackrel{\log_a a=1}{=} 10 \cdot \log_5 5 - 6 \cdot \log_5 5 = 4 \cdot \log_5 5 \stackrel{\log_a a=1}{=} 4 \cdot 1 = 4$$

$$\text{d) } \log_2 \left(\frac{4 \cdot 0,125^{\frac{3}{2}}}{\sqrt{2}} \right) = y \Leftrightarrow 2^y = \left(\frac{4 \cdot 0,125^{\frac{3}{2}}}{\sqrt{2}} \right) \Leftrightarrow 2^y = \left(\frac{2^2 \cdot \left(\frac{125}{1000}\right)^{\frac{3}{2}}}{2^{\frac{1}{2}}} \right) \Leftrightarrow 2^y = \left(\frac{2^2 \cdot \left(\frac{1}{8}\right)^{\frac{3}{2}}}{2^{\frac{1}{2}}} \right) \Leftrightarrow$$

$$\Leftrightarrow 2^y = \left(\frac{2^2 \cdot (2^{-3})^{\frac{3}{2}}}{2^{\frac{1}{2}}} \right) \Leftrightarrow 2^y = \left(\frac{2^2 \cdot 2^{-\frac{9}{2}}}{2^{\frac{1}{2}}} \right) \Leftrightarrow 2^y = \left(\frac{2^{-\frac{5}{2}}}{2^{\frac{1}{2}}} \right) \Leftrightarrow 2^y = 2^{-3} \Leftrightarrow y = -3$$

Por tanto, $\log_2 \left(\frac{4 \cdot 0,125^{\frac{3}{2}}}{\sqrt{2}} \right) = -3$

Otra forma (aplicando propiedades)

$$\begin{aligned} \log_2 \left(\frac{4 \cdot 0,125^{\frac{3}{2}}}{\sqrt{2}} \right) &\stackrel{\text{Prop. 2}}{=} \log_2 \left(4 \cdot 0,125^{\frac{3}{2}} \right) - \log_2 \sqrt{2} && \stackrel{4=2^2}{=} \log_2 \left(2^2 \cdot (2^{-3})^{\frac{3}{2}} \right) - \log_2 2^{\frac{1}{2}} = \\ & && 0,125 = \frac{125}{1000} = \frac{1}{8} = \frac{1}{2^3} = 2^{-3} \\ & && \sqrt{2} = 2^{\frac{1}{2}} \\ &= \log_2 \left(2^2 \cdot 2^{-\frac{9}{2}} \right) - \log_2 2^{\frac{1}{2}} = \log_2 2^{-\frac{5}{2}} - \log_2 2^{\frac{1}{2}} = -\frac{5}{2} \cdot \log_2 2 - \frac{1}{2} \cdot \log_2 2 = -\frac{6}{2} \cdot \log_2 2 = -3 \cdot \log_2 2 \stackrel{\log_a a=1}{=} \\ &= -3 \cdot 1 = -3 \end{aligned}$$

e) $\log_2 \sqrt[5]{\frac{16^2}{0,5 \cdot \sqrt{2}}} = y \Leftrightarrow 2^y = \sqrt[5]{\frac{16^2}{0,5 \cdot \sqrt{2}}} \Leftrightarrow 2^y = \sqrt[5]{\frac{(2^4)^2}{\frac{1}{2} \cdot 2^{\frac{1}{2}}}} \Leftrightarrow 2^y = \sqrt[5]{\frac{2^8}{2^{-1} \cdot 2^{\frac{1}{2}}}} \Leftrightarrow 2^y = \sqrt[5]{\frac{2^8}{2^{-\frac{1}{2}}}} \Leftrightarrow$

$\Leftrightarrow 2^y = \sqrt[5]{2^{\frac{17}{2}}} \Leftrightarrow 2^y = 2^{\frac{17}{10}} \Leftrightarrow y = \frac{17}{10}$

Por tanto, $\log_2 \sqrt[5]{\frac{16^2}{0,5 \cdot \sqrt{2}}} = \frac{17}{10}$

Otra forma (aplicando propiedades)

$$\begin{aligned} \log_2 \sqrt[5]{\frac{16^2}{0,5 \cdot \sqrt{2}}} &= \log_2 \left(\frac{(2^4)^2}{\frac{1}{2} \cdot 2^{\frac{1}{2}}} \right)^{\frac{1}{5}} = \log_2 \left(\frac{2^8}{2^{-1} \cdot 2^{\frac{1}{2}}} \right)^{\frac{1}{5}} = \log_2 \left(\frac{2^8}{2^{-\frac{1}{2}}} \right)^{\frac{1}{5}} = \log_2 \left(2^{\frac{17}{2}} \right)^{\frac{1}{5}} = \log_2 2^{\frac{17}{10}} \stackrel{\text{Prop. 3}}{=} \\ &= \frac{17}{10} \cdot \log_2 2 \stackrel{\log_a a=1}{=} \frac{17}{10} \cdot 1 = \frac{17}{10} \end{aligned}$$

3.- Halla el valor de x en cada caso:

En todos los apartados aplicamos la definición de logaritmo y luego desarrollamos

$$\text{a) } \log_x 7 = -2 \Leftrightarrow x^{-2} = 7 \Leftrightarrow \frac{1}{x^2} = 7 \Leftrightarrow 1 = 7x^2 \Leftrightarrow x^2 = \frac{1}{7} \Leftrightarrow x = \sqrt{\frac{1}{7}} \Leftrightarrow x = \frac{1}{\sqrt{7}} \Leftrightarrow x = \frac{\sqrt{7}}{7} \text{ racionalizar}$$

$$\text{b) } \log_x 7 = \frac{1}{2} \Leftrightarrow x^{\frac{1}{2}} = 7 \Leftrightarrow \sqrt{x} = 7 \Leftrightarrow (\sqrt{x})^2 = 7^2 \Leftrightarrow x = 49$$

$$\text{c) } \log_7 x^4 = 2 \Leftrightarrow 7^2 = x^4 \Leftrightarrow x = \pm \sqrt[4]{7^2} \Leftrightarrow x = \pm \sqrt{7} \text{ simplificar}$$

$$\text{d) } \log_x \left(\frac{1}{49} \right) = \frac{1}{4} \Leftrightarrow x^{\frac{1}{4}} = \frac{1}{49} \Leftrightarrow \sqrt[4]{x} = \frac{1}{49} \Leftrightarrow (\sqrt[4]{x})^4 = \left(\frac{1}{49} \right)^4 \Leftrightarrow x = \frac{1}{7^8} \Leftrightarrow x = 7^{-8}$$

$$\text{e) } \log_2 x = -\frac{1}{2} \Leftrightarrow 2^{-\frac{1}{2}} = x \Leftrightarrow x = \frac{1}{2^{\frac{1}{2}}} \Leftrightarrow x = \frac{1}{\sqrt{2}} \Leftrightarrow x = \frac{\sqrt{2}}{2} \text{ racionalizar}$$

$$\text{f) } \log_{\frac{1}{8}} x = \frac{1}{3} \Leftrightarrow \left(\frac{1}{8} \right)^{\frac{1}{3}} = x \Leftrightarrow x = \sqrt[3]{\frac{1}{8}} \Leftrightarrow x = \frac{1}{2}$$

$$\text{g) } \log_7 (7x) = 2 \Leftrightarrow 7^2 = 7x \Leftrightarrow x = \frac{7^2}{7} \Leftrightarrow x = 7$$

$$\text{h) } \log_x \frac{1}{3} = -\frac{1}{2} \Leftrightarrow x^{-\frac{1}{2}} = \frac{1}{3} \Leftrightarrow \frac{1}{x^{\frac{1}{2}}} = \frac{1}{3} \Leftrightarrow x^{\frac{1}{2}} = 3 \Leftrightarrow \sqrt{x} = 3 \Leftrightarrow x = 9$$

$$\text{i) } \log_x 0,001 = -3 \Leftrightarrow x^{-3} = 0,001 \Leftrightarrow \frac{1}{x^3} = \frac{1}{1000} \Leftrightarrow x^3 = 1000 \Leftrightarrow x = \sqrt[3]{1000} \Leftrightarrow x = 10$$

$$\text{j) } \log_x 27 = -\frac{1}{3} \Leftrightarrow x^{-\frac{1}{3}} = 27 \Leftrightarrow \frac{1}{x^{\frac{1}{3}}} = 27 \Leftrightarrow x^{\frac{1}{3}} = \frac{1}{27} \Leftrightarrow \sqrt[3]{x} = 3^{-3} \Leftrightarrow (\sqrt[3]{x})^3 = (3^{-3})^3 \Leftrightarrow \\ \Leftrightarrow x = 3^{-9} \Leftrightarrow x = \frac{1}{3^9} \Leftrightarrow x = \frac{1}{19683}$$

$$\text{k) } \log_x e = -3 \Leftrightarrow x^{-3} = e \Leftrightarrow \frac{1}{x^3} = e \Leftrightarrow x^3 = \frac{1}{e} \Leftrightarrow x = \sqrt[3]{\frac{1}{e}} \Leftrightarrow x = \frac{1}{\sqrt[3]{e}}$$

$$l) \log_x 0,015625 = -3 \Leftrightarrow x^{-3} = 0,015625 \Leftrightarrow \frac{1}{x^3} = \frac{15625}{1000000} \Leftrightarrow \frac{1}{x^3} = \frac{1}{64} \Leftrightarrow x^3 = 64 \Leftrightarrow x = 4$$

4.- Sabiendo que $\log 2 = 0,301$ y $\log 3 = 0,477$ calcula:

$$a) \log 12 = \log(2^2 \cdot 3) \underset{\text{Prop.1}}{=} \log 2^2 + \log 3 \underset{\text{Prop.3}}{=} 2\log 2 + \log 3 = 2 \cdot (0,301) + 0,477 = 0,602 + 0,477 = 1,079$$

$$b) \log 0,0002 = \log\left(\frac{2}{10000}\right) \underset{\text{Prop.2}}{=} \log 2 - \log 10000 = \log 2 - \log 10^4 \underset{\text{Prop.3}}{=} \log 2 - 4\log 10 \underset{\log_a a=1}{=} \\ = 0,301 - 4 \cdot 1 = 0,301 - 4 = -3,699$$

$$c) \log \sqrt[5]{6} = \log 6^{\frac{1}{5}} \underset{\text{Prop.3}}{=} \frac{1}{5} \log 6 = \frac{1}{5} \log(2 \cdot 3) \underset{\text{Prop.1}}{=} \frac{1}{5} (\log 2 + \log 3) = \frac{1}{5} (0,301 + 0,477) = 0,1556$$

$$d) \log 27000 = \log(27 \cdot 1000) = \log(3^3 \cdot 10^3) \underset{\text{Prop.1}}{=} \log 3^3 + \log 10^3 \underset{\text{Prop.3}}{=} 3\log 3 + 3\log 10 = \\ 3 \cdot 0,477 + 3 \cdot 1 = 1,431 + 3 = 4,431$$

$$e) \log \frac{\sqrt{32}}{6} \underset{\text{Prop.2}}{=} \log \sqrt{32} - \log 6 = \log \sqrt{2^5} - \log(2 \cdot 3) \underset{\text{Prop.1}}{=} \log 2^{\frac{5}{2}} - (\log 2 + \log 3) \underset{\text{Prop.3}}{=} \\ \text{Quitar paréntesis} \\ = \frac{5}{2} \log 2 - \log 2 - \log 3 = \underset{\substack{\uparrow \\ \frac{5}{2} - 1 = \frac{3}{2}}}{=} \frac{3}{2} \log 2 - \log 3 = \frac{3}{2} \cdot 0,301 - 0,477 = -0,0255$$

$$f) \log 0,0125 = \log\left(\frac{125}{10000}\right) \underset{\text{simplificar}}{=} \log\left(\frac{1}{80}\right) \underset{\text{Prop.2}}{=} \log 1 - \log 80 = 0 - \log(8 \cdot 10) = -\log(2^3 \cdot 10) \underset{\text{Prop.1}}{=} \\ = -(\log 2^3 + \log 10) \underset{\text{Quitar paréntesis y Prop.3}}{=} = -3\log 2 - \log 10 = -3 \cdot 0,301 - 1 = -1,903$$

$$g) \log \sqrt[5]{0,48} = \log \sqrt[5]{\frac{48}{100}} = \log\left(\frac{2^4 \cdot 3}{10^2}\right)^{\frac{1}{5}} = \log\left(\frac{2^{\frac{4}{5}} \cdot 3^{\frac{1}{5}}}{10^{\frac{2}{5}}}\right) \underset{\text{Prop.2}}{=} \log\left(2^{\frac{4}{5}} \cdot 3^{\frac{1}{5}}\right) - \log 10^{\frac{2}{5}} \underset{\text{Prop.1}}{=} \\ = \log 2^{\frac{4}{5}} + \log 3^{\frac{1}{5}} - \log 10^{\frac{2}{5}} \underset{\text{Prop.3}}{=} = \frac{4}{5} \log 2 + \frac{1}{5} \log 3 - \frac{2}{5} \log 10 = \frac{4}{5} \cdot 0,301 + \frac{1}{5} \cdot 0,477 - \frac{2}{5} \cdot 1 = \\ = 0,2408 + 0,0954 - 0,4 = -0,0638$$

$$h) \log \frac{1}{\sqrt[4]{0,6}} \underset{\text{Prop.2}}{=} \log 1 - \log \sqrt[4]{0,6} = 0 - \log \sqrt[4]{0,6} = -\log \sqrt[4]{\frac{6}{10}} = -\log \sqrt[4]{\frac{2 \cdot 3}{10}} = -\log\left(\frac{2 \cdot 3}{10}\right)^{\frac{1}{4}} \underset{\text{Prop.3}}{=}$$

$$= -\frac{1}{4} \cdot \log\left(\frac{2 \cdot 3}{10}\right) \stackrel{\text{Prop.2}}{=} -\frac{1}{4} \cdot (\log 2 + \log 3 - \log 10) \stackrel{\text{Prop.1}}{=} -\frac{1}{4} \cdot (0,301 + 0,477 - 1) = -\frac{1}{4} \cdot (-0,222) = 0,0555$$

$$\text{i) } \log 3,6 = \log\left(\frac{36}{10}\right) \stackrel{\text{Prop.1}}{=} \log\left(\frac{2^2 \cdot 3^2}{10}\right) \stackrel{\text{Prop.2}}{=} \log 2^2 + \log 3^2 - \log 10 \stackrel{\text{Prop.3}}{=} 2\log 2 + 2\log 3 - 1 = \\ = 2 \cdot 0,301 + 2 \cdot 0,477 - 1 = 0,556$$

$$\text{j) } \log 360 = \log(36 \cdot 10) \stackrel{\text{Prop.1}}{=} \log 36 + \log 10 = \log(2^2 \cdot 3^2) + 1 \stackrel{\text{Prop.1}}{=} \log 2^2 + \log 3^2 + 1 \stackrel{\text{Prop.3}}{=} \\ = 2\log 2 + 2\log 3 + 1 = 2 \cdot 0,301 + 2 \cdot 0,477 + 1 = 2,556$$

$$\text{k) } \log(5 \cdot \sqrt[3]{9}) \stackrel{\text{Prop.1}}{=} \log 5 + \log \sqrt[3]{9} = \log\left(\frac{10}{2}\right) + \log \sqrt[3]{3^2} \stackrel{\text{Prop.2}}{=} \log 10 - \log 2 + \log 3^{\frac{2}{3}} \stackrel{\text{Prop.3}}{=} \\ = \log 10 - \log 2 + \frac{2}{3}\log 3 = 1 - 0,301 + \frac{2}{3} \cdot 0,477 = 1,017$$

$$\text{l) } \log(3,2 \cdot 2,7^3) \stackrel{\text{Prop.1}}{=} \log 3,2 + \log 2,7^3 \stackrel{\text{Prop.3}}{=} \log 3,2 + 3\log 2,7 = \log\left(\frac{32}{10}\right) + 3\log\left(\frac{27}{10}\right) = \\ = \log\left(\frac{2^5}{10}\right) + 3\log\left(\frac{3^3}{10}\right) \stackrel{\text{Prop.2}}{=} \log 2^5 - \log 10 + 3(\log 3^3 - \log 10) \stackrel{\text{Prop.3}}{=} 5\log 2 - \log 10 + 3(3\log 3 - \log 10) = \\ = 5\log 2 - \log 10 + 9\log 3 - 3\log 10 = 5\log 2 + 9\log 3 - 4\log 10 = 5 \cdot 0,301 + 9 \cdot 0,477 - 4 \cdot 1 = 1,798$$

quitar
paréntesis

5.- Pasa a forma algebraica:

$$\text{a) } \frac{1}{2}\log C = 3\log A - \log 2 + 2\log B$$

$$\log C^{\frac{1}{2}} = \log A^3 - \log 2 + \log B^2$$

$$\log \sqrt{C} = \log\left(\frac{A^3}{2}\right) + \log B^2$$

$$\log \sqrt{C} = \log\left(\frac{A^3}{2} \cdot B^2\right)$$

$$\sqrt{C} = \frac{A^3 \cdot B^2}{2}$$

$$\text{b) } \frac{1}{3}\log z = \frac{2}{3}\log x - \log y + 3\log s$$

$$\log z^{\frac{1}{3}} = \log x^{\frac{2}{3}} - \log y + \log s^3$$

$$\log \sqrt[3]{z} = \log \left(\frac{\sqrt[3]{x^2}}{y} \right) + \log s^3$$

$$\log \sqrt[3]{z} = \log \left(\frac{\sqrt[3]{x^2}}{y} \cdot s^3 \right)$$

$$\sqrt[3]{z} = \frac{\sqrt[3]{x^2} \cdot s^3}{y}$$

$$z = \left(\frac{\sqrt[3]{x^2} \cdot s^3}{y} \right)^3$$

$$z = \frac{x^2 \cdot s^9}{y^3}$$

c) $2 - \log D = 2 \log A - 3 \log B - 4 \log C$

$$\log 100 - \log D = \log A^2 - \log B^3 - \log C^4$$

$$\log \left(\frac{100}{D} \right) = \log \left(\frac{A^2}{B^3} \right) - \log C^4$$

$$\log \left(\frac{100}{D} \right) = \log \left(\frac{A^2}{B^3} : C^4 \right)$$

$$\log \left(\frac{100}{D} \right) = \log \left(\frac{A^2}{B^3 \cdot C^4} \right)$$

$$\frac{100}{D} = \frac{A^2}{B^3 \cdot C^4}$$

$$100 = \frac{A^2 \cdot D}{B^3 \cdot C^4}$$

d) $\log A = \frac{1}{2} - \frac{1}{3} \log B + \log C - \frac{2}{5} \log D$

$$\log A = \log 10^{\frac{1}{2}} - \log B^{\frac{1}{3}} + \log C - \log D^{\frac{2}{5}}$$

$$\log A = \log \sqrt{10} - \log \sqrt[3]{B} + \log C - \log \sqrt[5]{D^2}$$

$$\log A = \log \left(\frac{\sqrt{10}}{\sqrt[3]{B}} \right) + \log C - \log \sqrt[5]{D^2}$$

$$\log A = \log \left(\frac{\sqrt{10}}{\sqrt[3]{B}} \cdot C \right) - \log \sqrt[5]{D^2}$$

$$\log A = \log \left(\frac{\sqrt{10} \cdot C}{\sqrt[3]{B}} : \sqrt[5]{D^2} \right)$$

$$\log A = \log\left(\frac{\sqrt{10} \cdot C}{\sqrt[3]{B} \cdot \sqrt[5]{D^2}}\right) \Rightarrow A = \frac{\sqrt{10} \cdot C}{\sqrt[3]{B} \cdot \sqrt[5]{D^2}}$$

6.- Toma logaritmos en las siguientes expresiones y desarrolla:

a) $A = \frac{x^3 \cdot y}{z^5} \Rightarrow \log A = \log\left(\frac{x^3 \cdot y}{z^5}\right) \xrightarrow{\text{Prop.2}} \log A = \log(x^3 \cdot y) - \log z^5 \xrightarrow{\text{Prop.1}} \log A = \log x^3 + \log y - \log z^5 \xrightarrow{\text{Prop.3}}$
 $\Rightarrow \log A = 3 \log x + \log y - 5 \log z$

b) $B = \sqrt{x^3 \cdot y^5 \cdot z^2} \Rightarrow \log B = \log \sqrt{x^3 \cdot y^5 \cdot z^2} \Rightarrow \log B = \log\left(x^{\frac{3}{2}} \cdot y^{\frac{5}{2}} \cdot z\right) \xrightarrow{\text{Prop.1}} \log B = \log x^{\frac{3}{2}} + \log y^{\frac{5}{2}} + \log z \xrightarrow{\text{Prop.3}}$
 $\Rightarrow \log B = \frac{3}{2} \log x + \frac{5}{2} \log y + \log z$

c) $C = \frac{X^2}{D \cdot \sqrt{A}} \Rightarrow \log C = \log\left(\frac{X^2}{D \cdot \sqrt{A}}\right) \xrightarrow{\text{Prop.2}} \log C = \log X^2 - \log(D \cdot \sqrt{A}) \xrightarrow{\text{Prop.1}} \log C = \log X^2 - \left(\log D + \log A^{\frac{1}{2}}\right) \xrightarrow{\text{Prop.3}}$
 $\Rightarrow \log C = 2 \log X - \left(\log D + \frac{1}{2} \log A\right) \xrightarrow{\text{quitar paréntesis}} \log C = 2 \log X - \log D - \frac{1}{2} \log A$

d) $D = \frac{A^5 \cdot \sqrt{B}}{C^4} \Rightarrow \log D = \log\left(\frac{A^5 \cdot \sqrt{B}}{C^4}\right) \xrightarrow{\text{Prop.2}} \log D = \log(A^5 \cdot \sqrt{B}) - \log C^4 \xrightarrow{\text{Prop.1}} \log D = \log A^5 + \log B^{\frac{1}{2}} - \log C^4 \xrightarrow{\text{Prop.3}}$
 $\Rightarrow \log D = 5 \log A + \frac{1}{2} \log B - 4 \log C$

e) $E = \sqrt{\frac{A}{B \cdot \sqrt{C}}} \Rightarrow \log E = \log \sqrt{\frac{A}{B \cdot \sqrt{C}}} \Rightarrow \log E = \log \sqrt{\frac{A}{B \cdot C^{\frac{1}{2}}}} \Rightarrow \log E = \log\left(\frac{A^{\frac{1}{2}}}{B^{\frac{1}{2}} \cdot C^{\frac{1}{4}}}\right) \xrightarrow{\text{Prop.2}}$
 $\Rightarrow \log E = \log A^{\frac{1}{2}} - \log\left(B^{\frac{1}{2}} \cdot C^{\frac{1}{4}}\right) \xrightarrow{\text{Prop.1}} \log E = \log A^{\frac{1}{2}} - \left(\log B^{\frac{1}{2}} + \log C^{\frac{1}{4}}\right) \xrightarrow{\text{Prop.3}} \xrightarrow{\text{Quitar paréntesis}}$
 $\Rightarrow \log E = \frac{1}{2} \log A - \frac{1}{2} \log B - \frac{1}{4} \log C$

f) $F = \sqrt[3]{\frac{A^2}{B \cdot \sqrt{C}}} \Rightarrow \log F = \log \sqrt[3]{\frac{A^2}{B \cdot \sqrt{C}}} \Rightarrow \log F = \log \sqrt[3]{\frac{A^2}{B \cdot C^{\frac{1}{2}}}} \Rightarrow \log F = \log\left(\frac{A^{\frac{2}{3}}}{B^{\frac{1}{3}} \cdot C^{\frac{1}{6}}}\right) \xrightarrow{\text{Prop.2}}$
 $\Rightarrow \log F = \log A^{\frac{2}{3}} - \log\left(B^{\frac{1}{3}} \cdot C^{\frac{1}{6}}\right) \xrightarrow{\text{Prop.1}} \log F = \log A^{\frac{2}{3}} - \left(\log B^{\frac{1}{3}} + \log C^{\frac{1}{6}}\right) \xrightarrow{\text{Prop.3}} \xrightarrow{\text{Quitar paréntesis}}$
 $\Rightarrow \log F = \frac{2}{3} \log A - \frac{1}{3} \log B - \frac{1}{6} \log C$

7.- Sabiendo que $\log 2 = 0,301$, $\log 3 = 0,477$ y utilizando el cambio de base calcula:

$$\text{CAMBIODE BASE} \rightarrow \log_a x = \frac{\log_b x}{\log_b a}$$

$$\text{a) } \log_3 32 = \frac{\log 32}{\log 3} = \frac{\log 2^5}{\log 3} = \frac{5 \log 2}{\log 3} = \frac{5 \cdot 0,301}{0,477} = 3,155$$

$$\text{b) } \log_4 0,3 = \frac{\log 0,3}{\log 4} = \frac{\log\left(\frac{3}{10}\right)}{\log 2^2} = \frac{\log 3 - \log 10}{2 \log 2} = \frac{0,477 - 1}{2 \cdot 0,301} = \frac{-0,523}{0,602} = -0,869$$

$$\text{c) } \log_{\sqrt{2}} 27 = \frac{\log 27}{\log \sqrt{2}} = \frac{\log 3^3}{\log 2^{\frac{1}{2}}} = \frac{3 \log 3}{\frac{1}{2} \log 2} = \frac{3 \cdot 0,477}{\frac{1}{2} \cdot 0,301} = \frac{1,431}{0,1505} = 9,508$$

$$\text{d) } \log_8 2 = \frac{\log 2}{\log 8} = \frac{\log 2}{\log 2^3} = \frac{\log 2}{3 \log 2} = \frac{1}{3}$$

$$\text{e) } \log_{\sqrt{3}} 8 = \frac{\log 8}{\log \sqrt{3}} = \frac{\log 2^3}{\log 3^{\frac{1}{2}}} = \frac{3 \log 2}{\frac{1}{2} \log 3} = \frac{3 \cdot 0,301}{\frac{1}{2} \cdot 0,477} = \frac{0,903}{0,2385} = 3,786$$

$$\text{f) } \log_{0,5} \sqrt[5]{3} = \frac{\log \sqrt[5]{3}}{\log 0,5} = \frac{\log 3^{\frac{1}{5}}}{\log \frac{1}{2}} = \frac{\frac{1}{5} \cdot \log 3}{\log 1 - \log 2} = \frac{\frac{1}{5} \cdot 0,477}{0 - 0,301} = \frac{0,0954}{-0,301} = -0,317$$

$$\begin{aligned} \text{g) } \log_{\frac{1}{\sqrt{2}}} \sqrt[3]{0,03} &= \frac{\log \sqrt[3]{0,03}}{\log \frac{1}{\sqrt{2}}} = \frac{\log \sqrt[3]{\frac{3}{100}}}{\log 2^{-\frac{1}{2}}} = \frac{\log\left(\frac{3}{100}\right)^{\frac{1}{3}}}{-\frac{1}{2} \log 2} = \frac{\frac{1}{3} \log\left(\frac{3}{10^2}\right)}{-\frac{1}{2} \log 2} = \frac{\frac{1}{3} \cdot (\log 3 - \log 10^2)}{-\frac{1}{2} \log 2} = \\ &= \frac{\frac{1}{3} \cdot (\log 3 - 2 \log 10)}{-\frac{1}{2} \log 2} = \frac{\frac{1}{3} \cdot (0,477 - 2)}{-\frac{1}{2} \cdot 0,301} = \frac{\frac{1}{3} \cdot (-1,523)}{-0,1505} = 3,373 \end{aligned}$$