

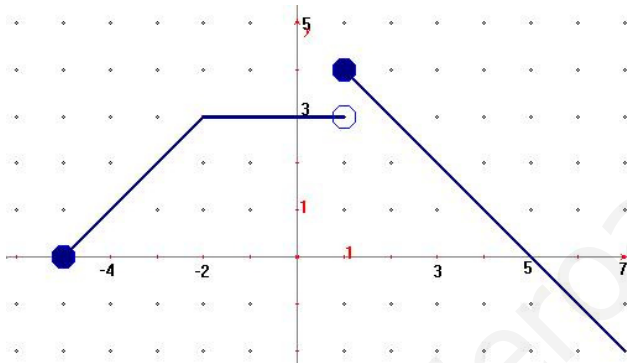
## EXAM 2\_1 (FUNCTIONS)

Name: \_\_\_\_\_

1. Plot the function:  $f(x) = \begin{cases} 1-x & \text{if } x < -2 \\ 3 & \text{if } -2 < x \leq 2 \\ -2x+5 & \text{if } x > 2 \end{cases}$  (1.5 p)

- What is the domain of definition?
- Study its continuity.
- Write the increasing and decreasing intervals.

2. Find the equation of the function (explain your answer): (1.25 p)



3. Find the domain of the following functions: (1.25 p)

$$f(x) = \frac{x+3}{x^2-3x+2}; \quad g(x) = \sqrt[3]{x^4-2x^3+3}; \quad h(x) = \sqrt{9-x^2}$$

4. Given the equation of the parabola  $f(x) = -x^2 + 6x - 5$  (1.5 p)

- Find its vertex and its intersections with the x axis and the y axis.
- Draw the graph of  $f(x)$ .
- Find the range of  $f(x)$ .

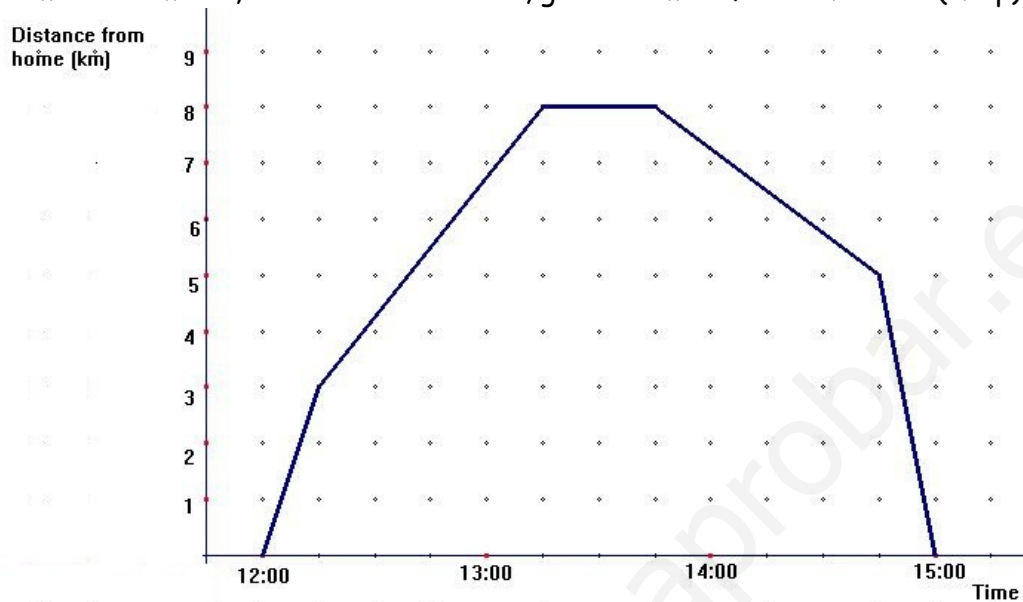
5. Solve by graphing the simultaneous equation:  $\begin{cases} y = x^2 + 4x + 7 \\ y = 3x + 9 \end{cases}$  (1.5 p)

(Write the steps you have taken to reach the solution)

6. A highway tunnel is parabolic in shape. If the curve of the tunnel can be modelled by  $h(x) = 50 - 0.02x^2$ , where  $x$  and  $h(x)$  are in metres.

- Sketch the function.
- How high is the highest point of the tunnel and how wide is it? (1.5p)

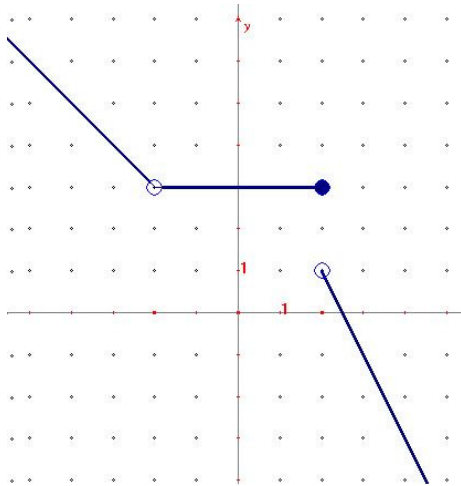
7. Kara and Kerry did a walk. They set off at noon and jogged, then they walked for the next hour. After a rest, they set off to walk back. Kerry's mother drove to meet them and, as it started to rain, gave them a lift back. (1.5 p)



- For how many kilometres did they jog?
- How many minutes did they have for the break?
- What was their average speed from the start until they stopped for the break?
- At what time did Kerry's mother pick them up?
- What was their average speed in the car?
- How far did they jog and walk together?
- Write the domain and range of this function.

**SOLUTION**

1. Plot the function:  $f(x) = \begin{cases} 1-x & \text{if } x < -2 \\ 3 & \text{if } -2 < x \leq 2 \\ -2x+5 & \text{if } x > 2 \end{cases}$



a) What is the domain of definition?

$$\text{Dom}(f) = \mathbb{R} - \{-2\}$$

b) Study its continuity.

Removable discontinuity in  $x = -2$

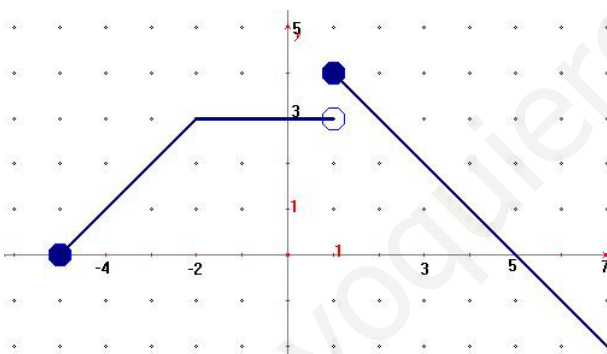
Jump discontinuity in  $x = 2$

c) Increasing and decreasing intervals.

Decreasing in  $(-\infty, -2) \cup (2, +\infty)$

Constant in  $(-2, 2)$

2. Find the equation of the function (explain your answer):



$$f(x) = \begin{cases} x+5 & \text{if } -5 \leq x < -2 \\ 3 & \text{if } -2 < x < 1 \\ -x+5 & \text{if } x \geq 1 \end{cases}$$

1)  $m=1$ , point  $(-5,0)$

2) constant  $y=3$

3)  $m = -1$ , point  $(5,0)$

$$y = -(x-5)$$

3. Find the domain of the following functions:

$$f(x) = \frac{x+3}{x^2-3x+2} \rightarrow x^2-3x+2=0 \Rightarrow x = \left\langle \frac{1}{2} \right\rangle \rightarrow \text{Dom} = \mathbb{R} - \{1,2\}$$

$$g(x) = \sqrt[3]{x^4-2x^3+3}; \text{ Dom} = \mathbb{R}$$

$$h(x) = \sqrt{9-x^2} \rightarrow 9-x^2 \geq 0 \rightarrow 9-x^2=0 \Rightarrow x = \pm 3$$

sign	-	-3	+	3	-
Dom = [-3,3]					

4. Given the equation of the parabola  $f(x) = -x^2 + 6x - 5$

a. Find its vertex and its intersections with the x axis and the y axis.

Vertex  $x = -\frac{b}{2a} = 3 \rightarrow V(3,4)$

Intersection with y axis

$x = 0 \rightarrow y = -5$

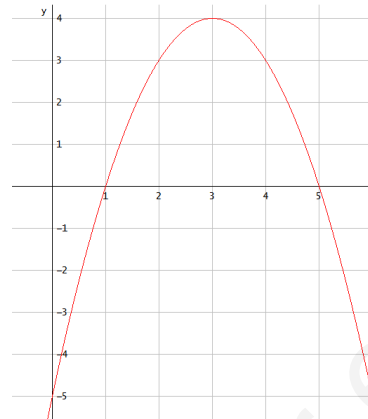
Intersection with x axis

$y = 0 \rightarrow -x^2 + 6x - 5 = 0 \Rightarrow x = \begin{cases} 1 \\ 5 \end{cases}$

b. Draw the graph of  $f(x)$ .

c. Find the range of  $f(x)$ .

$R = (-\infty, 4]$



5. Solve by graphing the simultaneous equation:

$$\left. \begin{matrix} y = x^2 + 4x + 7 \\ y = 3x + 9 \end{matrix} \right\} \rightarrow \begin{cases} \text{parabola} \\ \text{straight line} \end{cases}$$

$y = x^2 + 4x + 7 \rightarrow U,$

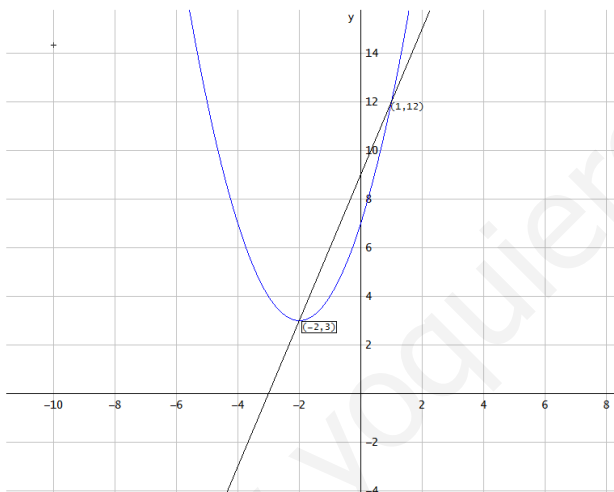
Vertex  $x = -2 \rightarrow V(-2, 3)$

y-Intercepts:  $(0, 7)$

x-Intercepts:  $x^2 + 4x + 7 = 0 \rightarrow$  no solution

Solution:

$x = 1, y = 12$  and  $x = -2, y = 3$



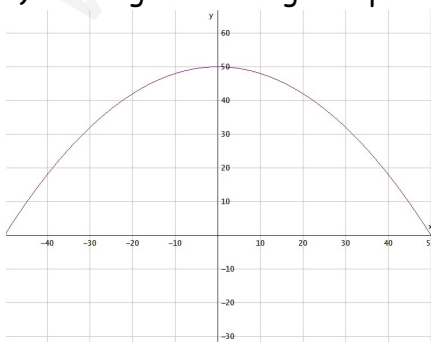
6. A highway tunnel is parabolic in shape. If the curve of the tunnel can be modelled by  $h(x) = 50 - 0.02x^2$ , where  $x$  and  $h(x)$  are in metres.

a) Sketch the function.

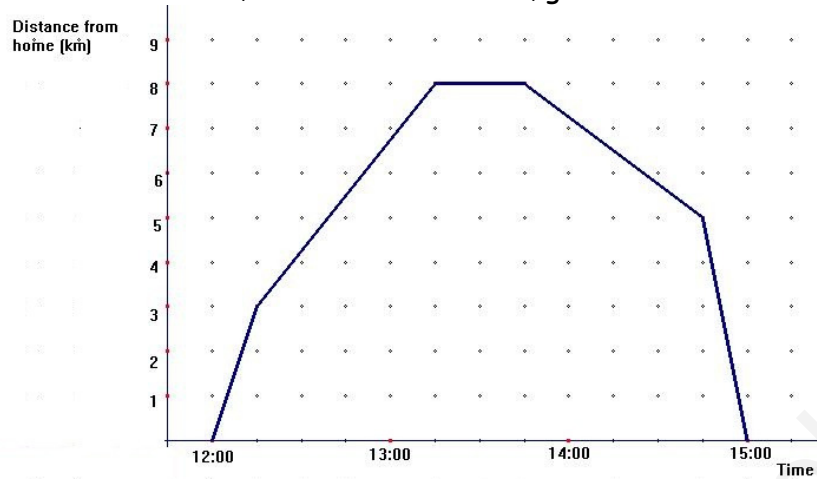
b) How high is the highest point of the tunnel and how wide is it?

The highest point is 50 metres

The wide is 100 metres



7. Kara and Kerry did a walk. They set off at noon and jogged, then they walked for the next hour. After a rest, they set off to walk back. Kerry's mother drove to meet them and, as it started to rain, gave them a lift back. (1.5 p)



- a) For how many kilometres did they jog? They jogged for 3 km
- b) How many minutes did they have for the break? 30 min
- c) What was their average speed from the start until they stopped for the break? 8 km in 1.25 hours  $\rightarrow \frac{8}{1.25} = 6.4$  km/h
- d) At what time did Kerry's mother pick them up? At 14:45 h
- e) What was their average speed in the car? 5 km in 0.25 h  $\rightarrow \frac{5}{0.25} = 20$  km/h
- f) How far did they jog and walk together? 8 km
- g) Write the domain and range of this function. Dom = [12,15], R = [0,8]