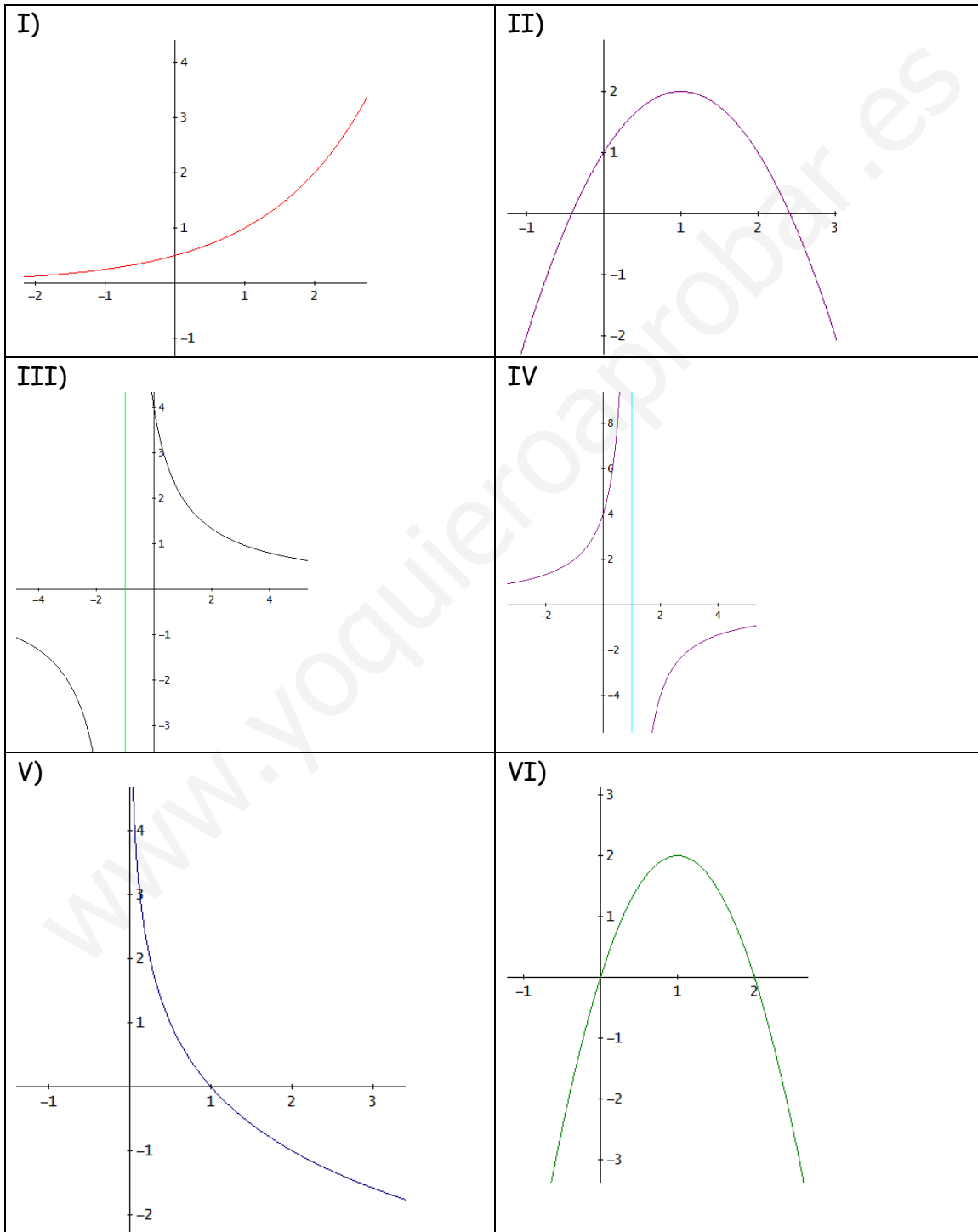EXAM 3_3   (Coordinate Geometry_Functions)

1. A circle has center at (0,4) and passes through the point (3,0). Find an equation to this circle   (1.5 points)

2. A triangle has vertices A(-3,3), B(-3,-2), C(5,-2). Show that it is a right angled triangle. Calculate its perimeter and area.   (1.5 points)

3. Find the equation of the line which passes through the origin and the point of intersection of y = x + 4 and x + y = 6 .   (1.75 points)

4. Find the equation of the perpendicular bisector of the line segment AB, with A(-1,2) and B(1,4). Find a parallel line to AB, passing through the point P(-3,2).

(1.75 points)

5. A quadrilateral has vertices A(5,4), B(-2,5), C(-1,-2) and D(6,-3). Show that the quadrilateral is a rhombus and calculate the area of ABCD.   (1.75 points)

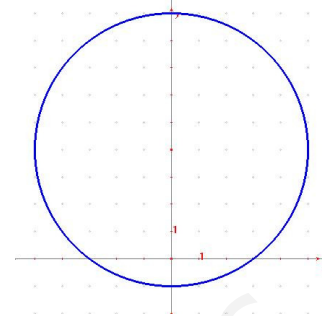6. Match the equations to the corresponding graphs (explaining your answer):

a) $y = -2x^2 + 4x$

b) $y = \dfrac{4}{x+1}$

c) $y = 2^{x-1}$

d) $y = -x^2 + 2x + 1$

e) $y = log_{\frac{1}{2}} x$

f) $y = -\dfrac{4}{x-1}$

I)	II)
III)	IV
V)	VI)

SOLUTION

1. A circle has center at (0,4) and passes through the point (3,0). Find an equation to this circle

$(x-0)^2 + (y-4)^2 = r^2$, It passes through the point (3,0)

$\rightarrow 3^2 + (0-4)^2 = r^2$

$\rightarrow 9 + 16 = r^2 \rightarrow r^2 = 25 \rightarrow$ Equation: $x^2 + (y-4)^2 = 25$

2. A triangle has vertices A(-3,3), B(-3,-2), C(5,-2). Show that it is a right angled triangle. Calculate its perimeter and area.

If it was a right triangle $\rightarrow$ Pythagorean Theorem:

$\rightarrow b^2 + c^2 = h^2$

We are going calculate the distances, to get the legs and hypotenuse:

$d(A,B) = \sqrt{(-3+3)^2 + (-2-3)^2} = \sqrt{0+25} = 5u$

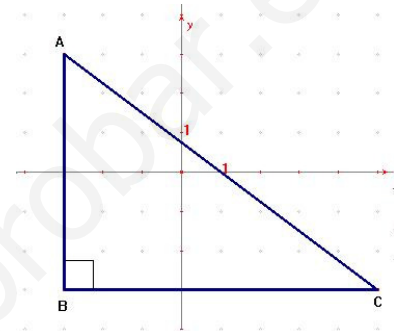$d(A,C) = \sqrt{(5+3)^2 + (-2-3)^2} = \sqrt{64+25} = \sqrt{89}u \rightarrow$ hypotenuse? the biggest

$d(B,C) = \sqrt{(5+3)^2 + (-2+2)^2} = \sqrt{64+0} = 8u$

$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 \Leftrightarrow (\sqrt{89})^2 = 5^2 + 8^2 \rightarrow 89 = 25 + 64 \rightarrow 89 = 89$

So, yes, it is a right angled triangle.

Perimeter: $P = 5 + 8 + \sqrt{89} = 13 + \sqrt{89}$ u

Area: $A = \dfrac{5 \times 8}{2} = 20u^2$

3. Find the equation of the line which passes through the origin and the point of intersection of y = x + 4 and x + y = 6 .

Point A (0,0), point B:

$\left. \begin{array}{l} y = x+4 \\ x+y = 6 \end{array} \right\} x + x + 4 = 6 \rightarrow 2x = 2 \rightarrow x = 1$

$y = 1 + 4 = 5 \rightarrow B(1,5)$

Equation $\overline{AB}: \dfrac{x-0}{1-0} = \dfrac{y-0}{5-0} \rightarrow 5x = y \rightarrow y = 5x$

4. Find the equation of the perpendicular bisector of the line segment AB, with A(-1,2) and B(1,4). Find a parallel line to AB, passing through the point P(-3,2).
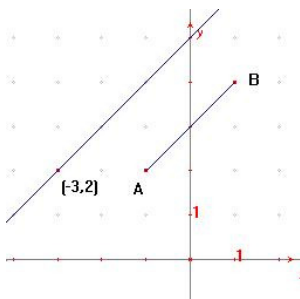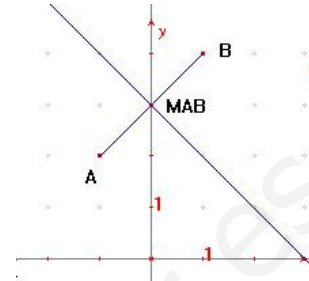
Equation of $\overline{AB}$: $\dfrac{x+1}{1+1} = \dfrac{y-2}{4-2} \rightarrow 2(x+1) = 2(y-2) \rightarrow x+1 = y+2 \rightarrow y = x-1$

Perpendicular bisector(It passes through the midpoint of AB, perpendicular):

$M_{AB} = \left(\dfrac{-1+1}{2}, \dfrac{2+4}{2}\right) = (0,3) \rightarrow$ slope:
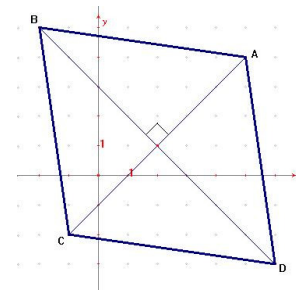
$m' = -\dfrac{1}{m} = -\dfrac{1}{1} = -1$

$y - 3 = -1(x - 0) \rightarrow y = -x + 3$ Perpendicular bisector

Parallel to AB passing through (-3,2): slope m = 1

$y - 2 = 1(x + 3) \rightarrow y = x + 5$ parallel line to AB

5. A quadrilateral has vertices A(5,4), B(-2,5), C(-1,-2) and D(6,-3). Show that the quadrilateral is a rhombus and calculate the area of ABCD.

If the quadrilateral is a rhombus, its diagonals are perpendicular lines. We are going to find the equations of the diagonals $\overline{AC}$ and $\overline{BD}$:

$\overline{AC}$: $\dfrac{x-5}{-1-5} = \dfrac{y-4}{-2-4} \rightarrow -6(x-5) = -6(y-4) \rightarrow x-5 = y-4 \rightarrow y = x-1$

$\overline{BD}$: $\dfrac{x+2}{6+2} = \dfrac{y-5}{-3-5} \rightarrow -8(x+2) = 8(y-5) \rightarrow -x-2 = y-5 \rightarrow y = -x+3$

We study the slopes: $m_{AC} = 1$, $m_{BD} = -1 \rightarrow$ perpendicular, so it is a rhombus.

Area: $A = \dfrac{D \times d}{2}$

$d = d(A,C) = \sqrt{(-1-5)^2 + (-2-4)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$ u

$D = d(B,D) = \sqrt{(6+2)^2 + (-3-5)^2} = \sqrt{64+64} = \sqrt{128} = 8\sqrt{2}$ u

$A = \dfrac{D \times d}{2} = \dfrac{8\sqrt{2} \times 6\sqrt{2}}{2} = 48u^2$

6. Match the equations to the corresponding graphs (explaining your answer):

a) $y = -2x^2 + 4x$ Parabola $\cap$ passing through (0,0) $\rightarrow$ Graph VI

b) $y = \dfrac{4}{x+1}$ Hyperbole, asymptote x= -1 $\rightarrow$ Graph III

c) $y = 2^{x-1}$ Exponential , asymptote x-axis, passing (0,1/2) $\rightarrow$ Graph I

d) $y = -x^2 + 2x + 1$ Parabola $\cap$ passing through (0,1) $\rightarrow$ Graph II

e) $y = log_{\frac{1}{2}} x$ Logarithmic, asymptote y-axis, passing (1,0) $\rightarrow$ Graph V

f) $y = -\dfrac{4}{x-1}$ Hyperbole, asymptote x = 1 $\rightarrow$ Graph IV