

EXAM 1_2 (Algebra - Polynomials)

Name: _____

1) Work out and simplify: (2.5 p)

a) $3(x-2)^2 - 2(x+3)(x-1) - (x+2)(x-2) =$

b) $\left[\frac{2x}{(x+2)^2} \div \frac{4x^2 - 4x}{3x+6} \right] \cdot \frac{x^2 + x - 2}{3x} =$

2) We know the expression $P(x) = x^4 - px^2 + 2px - 7$ has a remainder of 6 when divided by $x + 1$. Find the value of p . (1 p)

3) Factorise and simplify: (3 p)

a) $\frac{x^3 - x^2 - 2x}{x^4 - 5x^2 + 4}$

b) $\frac{12a^2 - 3b^2}{24a^3 - 24a^2b + 6ab^2}$



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(3.5 p)

4) Find LCM and HCF of the polynomials:

a) $P(x) = x^3 - 8x^2 + 20x - 16$ and $Q(x) = 2x^4 - x^3 - 5x^2 - 2x$

b) $P(x) = x^3 + 6x^2 + 9x$ and $Q(x) = 2x^4 - 18x^2$

SOLUTION

1) Work out and simplify:

$$\begin{aligned} \text{a) } 3(x-2)^2 - 2(x+3)(x-1) - (x+2)(x-2) &= 3(x^2 - 4x + 4) - 2(x^2 + 2x - 3) - (x^2 - 4) = \\ &= 3x^2 - 12x + 12 - 2x^2 - 4x + 6 - x^2 + 4 = -16x + 22 \end{aligned}$$

$$\text{b) } \left[\frac{2x}{(x+2)^2} \div \frac{4x^2 - 4x}{3x+6} \right] \cdot \frac{x^2 + x - 2}{3x} = \frac{2x \cdot (3x+6)}{(x+2)^2(4x^2 - 4x)} \cdot \frac{(x+2)(x-1)}{3x} =$$

$$\begin{aligned} &= \frac{2x \cdot 3 \cdot (x+2)(x+2)(x-1)}{4x \cdot (x+2)^2(x-1) \cdot 3x} = \frac{2}{4x} = \frac{1}{2x} \qquad \begin{aligned} x^2 + x - 2 &= 0 \\ x &= \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} = \begin{cases} 1 \\ -2 \end{cases} \end{aligned} \end{aligned}$$

2) We know the expression $P(x) = x^4 - px^2 + 2px - 7$ has a remainder of 6 when divided by $x + 1$. Find the value of p .

$$\begin{aligned} \text{Remainder Theorem: } P(-1) &= (-1)^4 - p(-1)^2 + 2p(-1) - 7 = 1 - p - 2p - 7 = -3p - 6 = 6 \\ -3p - 6 &= 6 \rightarrow -3p = 12 \rightarrow p = -4 \end{aligned}$$

3) Factorise and simplify:

$$\text{a) } \frac{x^3 - x^2 - 2x}{x^4 - 5x^2 + 4} \rightarrow x^3 - x^2 - 2x = x(x^2 - x - 2); \quad x^2 - x - 2 = 0 \rightarrow x = \frac{1 \pm \sqrt{9}}{2} = \begin{cases} 2 \\ -1 \end{cases}$$

$$x^4 - 5x^2 + 4 \rightarrow \text{Factors of 4} = 1, -1, 2, -2, 4, -4$$

$$P(1) = 1^4 - 5 \cdot 1^2 + 4 = 0$$

1	0	-5	0	+4	$Q(x) = x^3 + x^2 - 4x - 4$
1	+1	+1	-4	-4	$Q(1) = 1^3 + 1^2 - 4 - 4 \neq 0$
1	+1	-4	-4	0	$Q(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4 = 0$
-1	-1	0	+4		$x^2 - 4 = (x+2)(x-2)$
1	0	-4	0		

$$\frac{x^3 - x^2 - 2x}{x^4 - 5x^2 + 4} = \frac{x(x-2)(x+1)}{(x-1)(x+1)(x-2)(x+2)} = \frac{x}{(x-1)(x+2)}$$

$$\text{b) } \frac{12a^2 - 3b^2}{24a^3 - 24a^2b + 6ab^2} = \frac{3(4a^2 - b^2)}{6a(4a^2 - 4ab + b^2)} = \frac{3(2a+b)(2a-b)}{6a(2a-b)^2} = \frac{2a+b}{2a(2a-b)}$$

4) Find LCM and HCF of the polynomials:

a) $P(x) = x^3 - 8x^2 + 20x - 16$ and $Q(x) = 2x^4 - x^3 - 5x^2 - 2x$

$P(x) = x^3 - 8x^2 + 20x - 16 \rightarrow \text{Factors}(-16) = \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

$P(1) = 1^3 - 8 \cdot 1^2 + 20 - 16 \neq 0$; $P(-1) = (-1)^3 - 8 \cdot (-1)^2 + 20(-1) - 16 \neq 0$

$P(2) = 2^3 - 8 \cdot 2^2 + 20 \cdot 2 - 16 = 8 - 32 + 40 - 16 = 0$

2	1	-8	+20	-16
		2	-12	+16
	1	-6	+8	0

 $x^2 - 6x + 8 = 0 \rightarrow x = \frac{6 \pm \sqrt{4}}{2} = \left\langle \begin{matrix} 4 \\ 2 \end{matrix} \right.$

$Q(x) = 2x^4 - x^3 - 5x^2 - 2x = x(2x^3 - x^2 - 5x - 2) \rightarrow \text{Factors}(2) = \pm 1, \pm 2$

$Q(1) = 2 \cdot 1^3 - 1^2 - 5 \cdot 1 - 2 \neq 0$; $Q(-1) = 2(-1)^3 - (-1)^2 - 5(-1) - 2 = 0$

-1	2	-1	-5	-2
		-2	+3	+2
	2	-3	-2	0

 $2x^2 - 3x - 2 = 0 \rightarrow x = \frac{3 \pm \sqrt{25}}{4} = \left\langle \begin{matrix} 2 \\ -\frac{1}{2} \end{matrix} \right.$

$P(x) = x^3 - 8x^2 + 20x - 16 = (x - 2)^2(x - 4)$

$Q(x) = 2x^4 - x^3 - 5x^2 - 2x = 2x(x + 1)(x - 2)\left(x + \frac{1}{2}\right)$

$\left. \begin{matrix} \text{LCM} = 2x(x - 2)^2(x + 1)(x - 4)\left(x + \frac{1}{2}\right) \\ \text{HCF} = (x - 2) \end{matrix} \right\}$

b) $P(x) = x^3 + 6x^2 + 9x$ and $Q(x) = 2x^4 - 18x^2$

$P(x) = x^3 + 6x^2 + 9x = x(x^2 + 6x + 9) = x(x + 3)^2$

$Q(x) = 2x^4 - 18x^2 = 2x^2(x^2 - 9) = 2x^2(x + 3)(x - 3)$

$\left. \begin{matrix} \text{LCM} = 2x^2(x + 3)^2(x - 3) \\ \text{HCF} = x(x + 3) \end{matrix} \right\}$