Maths 4th ESO

3rd TERM GENERAL EXAM

Name:...

Remember: in each question, write the steps you have taken to reach the solution. (1 point each question)

1. Five years ago a woman's age was the square of her son's age. In ten years' time, her age will be twice that of her son's age. Find:
   a) the age of the son five years ago. b) the present age of the woman.

2. Solve: $\frac{2x-4}{x^2-2x}-\frac{5}{3x+6}=\frac{4}{x^2-4}$

3. Solve by substitution and graphically : $\left.\begin{array}{l}2x-y-1=0\\ y=-x^2+x+1\end{array}\right\}$

4. Solve the system of inequalities: $\left.\begin{array}{l}\frac{x+y}{4}-\frac{x-y}{2}<1\\ 3x-\frac{2y}{3}\leq 2\end{array}\right\}$

5. Find the height of a rectangular box of length 8 cm and width 6 cm, where the length of a diagonal is 11 cm.

6. A ladder 5 m long, leaning against a vertical wall makes an angle of 65°with the ground.
   a) How high on the wall does the ladder reach?
   b) How far is the foot of the ladder from the wall?
   c) What angle does the ladder make with the wall?

7. Suppose that $\cos\alpha=-\frac{3}{5}$ and α lies in quadrant II. Find the other trigonometric ratios for α.

8. Two girls on the same side of a tower notice the angles of elevation to the top of the tower are 45° and 60° respectively. If the height of the tower is 90 m, find the distance between the two girls.

9. A circle is circumscribed about the square ABCD whit vertices A(-1,5), B(-1, 2), C(2,2), D(2,5). Write an equation to the circle.

10. With point A(2,3) and straight line r: 2x −3y + 4 = 0
a) Write the equation of a line parallel to r and joining the point A.
b) Write the equation of a line perpendicular to r and joining the point A.

SOLUTION

1. Five years ago a woman's age was the square of her son's age. In ten years' time, her age will be twice that of her son's age. Find:

	Five years ago	In ten years
Woman	x^2	$x^2 + 15$
Son	x	$x + 15$

$$x^2+15=2(x+15) \to x^2+15=2x+30 \to x^2-2x-15=0$$

$$x=\frac{2\pm\sqrt{4+60}}{2}=\begin{cases}5\\-3\end{cases}$$

a) the age of the son five years ago. He was 5 years old

b) the present age of the woman. $x^2 + 5 = 25+5 = 30$

She is 30 years old

2. Solve: $\dfrac{2x-4}{x^2-2x}-\dfrac{5}{3x+6}=\dfrac{4}{x^2-4} \to mcm=3x(x+2)(x-2)$

$$\frac{(2x-4)(3x+6)}{3x(x^2-4)}-\frac{5x(x-2)}{3x(x^2-4)}=\frac{4\cdot 3x}{3x(x^2-4)} \to 6x^2-24-5x^2+10x=12x$$

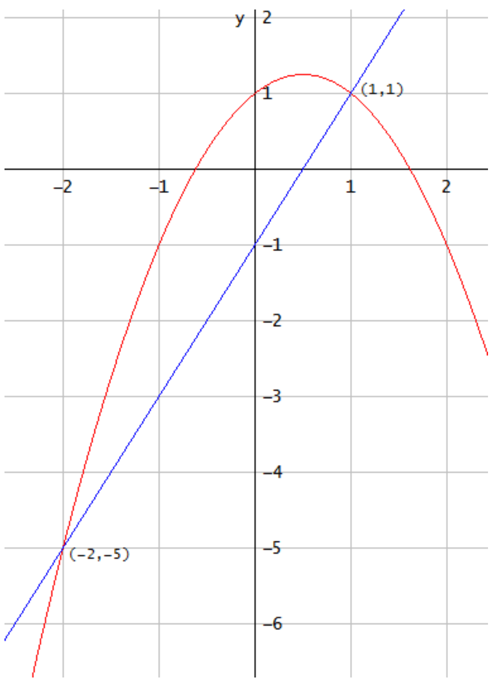
$$x^2-2x-24=0 \to x=\frac{2\pm\sqrt{4+96}}{2}=\begin{cases}6\\-4\end{cases}$$

3. Solve by substitution and graphically: $\left.\begin{matrix}2x-y-1=0\\ y=-x^2+x+1\end{matrix}\right\} \to y=2x-1$

$$2x-1=-x^2+x+1 \to x^2+x-2=0 \to x=\frac{-1\pm\sqrt{1+8}}{2}=\begin{cases}x_1=1 \to y_1=1\\ x_2=-2 \to y_2=-5\end{cases}$$

Graphically: $\begin{cases}2x-y-1=0 \to y=2x-1 \to \text{line, slope}=2 \text{ and } y-\text{intercepts} -1\\ y=-x^2+x+1 \to \text{parabole} \cap \to V=-\dfrac{1}{-2}=\dfrac{1}{2} \to V\left(\dfrac{1}{2},\dfrac{5}{4}\right)\end{cases}$

x- intercepts: $y=0 \to x=1 \to (1,0)$



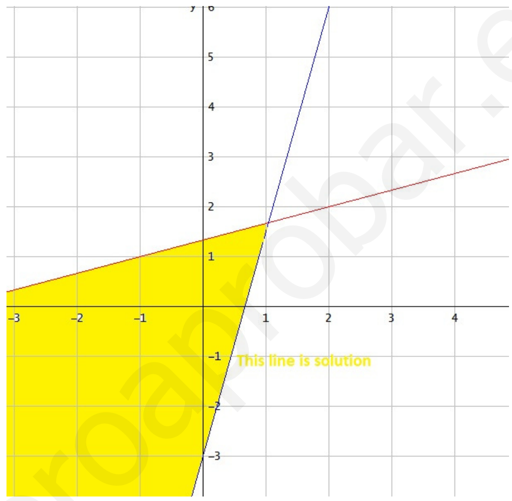
y-intercepts: $x=0 \to -x^2+x+1=0$

$$x=\frac{-1\pm\sqrt{1+4}}{2}=\frac{-1\pm\sqrt{5}}{2}$$

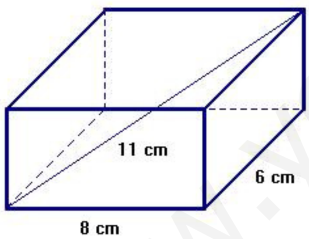
4. Solve the system of inequalities: $\left.\begin{array}{l} \dfrac{x+y}{4}-\dfrac{x-y}{2}<1 \\ 3x-\dfrac{2y}{3}\le 2 \end{array}\right\} \to \left.\begin{array}{l} x+y-2x+2y<4 \\ 9x-2y\le 6 \end{array}\right\}$

$$\left.\begin{array}{l} -x+3y<4 \\ 9x-2y\le 6 \end{array}\right\} \to \begin{cases} -x+3y=4 \\ 9x-2y=6 \end{cases} \to \begin{cases} y=\dfrac{x+4}{3} \\ y=\dfrac{9x-6}{2} \end{cases}$$, graphing these lines:

$$(0,0)\to\begin{cases} -0+0<4 \text{ Yes} \\ 0-0\le 6 \quad \text{Yes} \end{cases}$$



5. Find the height of a rectangular box of length 8 cm and width 6 cm, where the length of a diagonal is 11 cm.



$$d^2=6^2+8^2\to d^2=100\to d=10$$

$$11^2=d^2+h^2\to 121=100+h^2\to h^2=21$$

$$h=\sqrt{21}\,\text{cm}$$

6. A ladder 5 m long, leaning against a vertical wall makes an angle of 65°with the ground.

a) How high on the wall does the ladder reach?

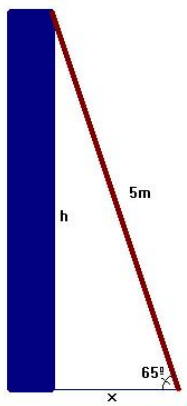
$$\sin 65^\circ=\frac{h}{5}\to h=4.53\text{m}$$

b) How far is the foot of the ladder from the wall?

$$\cos 65^\circ=\frac{x}{5}\to x=2.11\text{m}$$

c) What angle does the ladder make with the wall?

$$\alpha=90^\circ-65^\circ=25^\circ$$



7. Suppose that $\cos\alpha = -\frac{3}{5}$ and α lies in quadrant II. Find the other trigonometric ratios for α.

In quadrant II, $\sin\alpha > 0, \cos\alpha < 0, \tan\alpha < 0$

$$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow \sin^2\alpha = 1 - \cos^2\alpha = 1 - \frac{9}{25} = \frac{16}{25} \rightarrow \sin\alpha = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{4}{5}}{-\frac{3}{5}} = -\frac{4}{3}$$

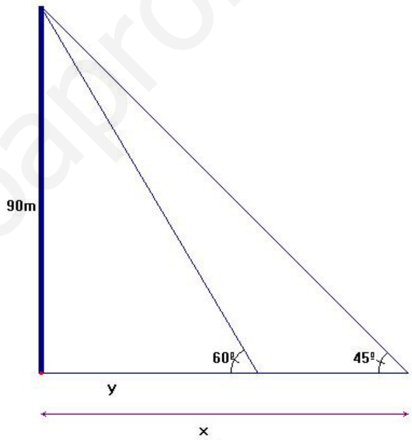
8. Two girls on the same side of a tower notice the angles of elevation to the top of the tower are 45° and 60° respectively. If the height of the tower is 90 m, find the distance between the two girls.

$$\tan 60^o = \frac{90}{y} \rightarrow y = \frac{90}{\sqrt{3}} = \frac{90\sqrt{3}}{3} = 30\sqrt{3}m$$

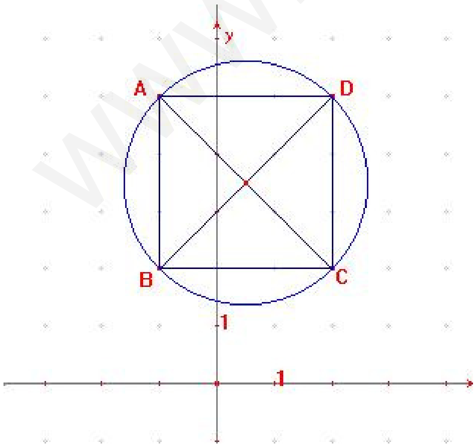
$$\tan 45^o = \frac{90}{x} \rightarrow x = \frac{90}{1} = 90m$$

Distance between the two girls:

$$90 - 30\sqrt{3} = 38.04m$$



10. A circle is circumscribed about the square ABCD whit vertices A(-1,5), B(-1, 2), C(2,2), D(2,5). Write an equation to the circle.



Centre of the circle: Midpoint AC or BD

$$M = \left(\frac{-1+2}{2}, \frac{5+2}{2}\right) = \left(\frac{1}{2}, \frac{7}{2}\right)$$

Radius of de circle: $r = d(A,M)$

$$d(A,M) = \sqrt{\left(\frac{1}{2}+1\right)^2 + \left(\frac{7}{2}-5\right)^2}$$

$$d(A,M) = \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{3}{2}\right)^2} = \sqrt{\frac{18}{4}} = \sqrt{\frac{9}{2}}$$

Equation: $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{9}{2}$

2. With point A(2,3) and straight line $r: 2x-3y+4=0$

a) Write the equation of a line parallel to **r** and joining the point ***A***.

$r: 2x-3y+4=0 \rightarrow y=\frac{2}{3}x+\frac{4}{3} \rightarrow m=\frac{2}{3}$

Parallel line: $y-3=\frac{2}{3}(x-2) \rightarrow y=\frac{2}{3}x+\frac{5}{3}$

b) Write the equation of a line perpendicular to r and joining the point A.

Perpendicular line: $m'=-\frac{3}{2} \rightarrow y-3=-\frac{3}{2}(x-2) \rightarrow y=-\frac{3}{2}x+6$