

# Relación – Ecuaciones

## Ecuaciones de primer grado

Resolver las siguientes ecuaciones:

$$[1] \frac{5(x+1)}{24} = \frac{x+3}{3} - \frac{5x+9}{8} + \frac{3x+2}{6}$$

$$[2] 3 \cdot \{3 \cdot [3(3x-2) - 2] - 2\} - 2 = 1$$

$$[3] \frac{x+2}{2} - \frac{1-2x}{7} + \frac{3x+5}{4} = \frac{11-x}{14}$$

$$[4] 5x - 3(x-8) = 2(x+4)$$

$$[5] 4 - \frac{x+3}{6} = 2 + \frac{9-2x}{3}$$

$$[6] \frac{5x-3}{6} - \frac{7x-1}{4} = \frac{4x+2}{7} - 5$$

$$[7] 5 - 4\left(x - \frac{1}{2}\right) = 0 \hat{8} \left(x - \frac{7}{4}\right)$$

$$[8] x + 2(x-1) = 4$$

$$[9] \frac{x-2}{8} - \frac{2(2x+6)}{3} + x = -4$$

$$[10] \frac{6-3x}{3} + \frac{(x-1)(x+2)}{4} = \frac{x^2}{4}$$

$$[11] \frac{x-3}{2} - \frac{x-8}{12} = \frac{5-x}{4} - \frac{x}{3}$$

$$[12] \frac{1}{6} \left[ 2x - 1 - 3 \left( \frac{5x}{3} - 1 \right) \right] + 2(x-3) + 6 = \frac{1}{3}$$

$$[13] \frac{3(x+4)}{4} - \left[ \frac{x-1}{4} + \frac{2x-3}{3} - \frac{(x-1)(x+3)}{2} \right] + \frac{(x-3)(2-x)}{2} = 0$$

$$[14] \left(x + \frac{5}{2}\right) \left(x - \frac{3}{2}\right) - (x+5)(x-3) = 3 \left(3 + \frac{1}{4}\right)$$

$$[15] x + \frac{3x-1}{4} - \frac{x+1}{5} + 1 - \frac{x-2}{10} = 2(x+1)$$

$$[16] (x-2)^2 - (x-1)(x+1) - \frac{x-2}{3} = \frac{x+2}{2}$$

$$[17] \frac{x+2}{2} - \frac{1-2x}{7} = \frac{11-x}{14} - \frac{3x+5}{4}$$

Soluciones:

- [1] Identidad [2] 1 [3]  $-\frac{37}{45}$  [4] Sin solución [5] 3  
[6] 3 [7]  $\frac{7}{4}$  [8] 2 [9]  $-\frac{6}{5}$  [10] 2  
[11]  $\frac{25}{12}$  [12] 0 [13]  $\frac{3}{40}$  [14]  $\frac{3}{2}$  [15]  $-\frac{25}{11}$   
[16]  $\frac{28}{29}$  [17]  $-\frac{37}{45}$

## Ecuaciones de segundo grado

1. Resolver las siguientes ecuaciones de segundo grado:

- [1]  $7x^2 = 63$  [2]  $x^2 - 24 = 120$   
[3]  $\frac{x^2}{5} = x$  [4]  $5x^2 - 3 = 42$   
[5]  $3x = 4x^2 - 2x$  [6]  $8x^2 - 56 = 34 - 2x^2$   
[7]  $(x + 1)(x - 3) + 3 = 0$  [8]  $(x + 9)(x - 9) = 3(x - 27)$   
[9]  $(2x + 3)(2x - 3) = 135$  [10]  $3(2x - 3)^2 = 4x(2x - 9) + 43$   
[11]  $12x^2 - 18 = 0$  [12]  $3(x^2 - 2) = 21$   
[13]  $4x^2 + 12 + 9 = 0$  [14]  $x^2 - 7x + 12 = 0$   
[15]  $x^2 - 9x + 18 = 0$  [16]  $x^2 - 9x + 14 = 0$   
[17]  $x^2 - 6x + 8 = 0$  [18]  $x^2 - 6x + 9 = 0$   
[19]  $x^2 + 6x - 27 = 0$  [20]  $x^2 + x + \frac{1}{4} = 0$   
[21]  $x(3x - 2) = 65$  [22]  $x(x - 3) + 1 = 5(x - 3)$   
[23]  $x^2 - \frac{x}{2} = \frac{1}{3} - \frac{2x}{3}$  [24]  $\frac{x^2}{4} + 2 = \frac{3x}{2}$   
[25]  $x + \frac{1}{x} = \frac{65}{8}$  [26]  $x - 2 = \frac{4x - 9}{x}$   
[27]  $\frac{x}{2} + \frac{3}{x} = \frac{x + 13}{3x}$  [28]  $(x + 1)(x - 3) = 8x - 4(x - 1)$   
[29]  $x(x + 1) + \frac{x + \frac{3}{2}}{4} = 0$  [30]  $8x + 11 - \frac{7}{x} = \frac{21 + 63x}{7}$   
[31]  $\frac{x^2}{2} - \frac{3x}{5} = \frac{1}{5} - \frac{x}{6}$  [32]  $2x^2 + \frac{6}{5} = x \left( x + \frac{31}{5} \right)$   
[33]  $\frac{5x^2}{8} - \frac{3x}{5} = \frac{x^2}{8} - \frac{x}{6} + \frac{1}{5}$  [34]  $\frac{5x^2}{6} - \frac{x}{2} + \frac{3}{4} = 8 - \frac{2x}{3} - x^2 + \frac{273}{12}$   
[35]  $2x^2 + 5x + 3 = 0$  [36]  $x^2 = 5x + 6$   
[37]  $x^2 - 5x + 4 = 5(8 - x)$  [38]  $5x^2 - 8x - \frac{3x^2}{4} = 0$   
[39]  $(2x + 7)(2x - 7) = x^2 - 1$  [40]  $3x^2 = (4x + 8)(x - 2)$   
[41]  $\frac{5x^2}{3} = x^2 + 24$  [42]  $8x^2 - 10x - \frac{3x^2}{7} - 43x = 0$

2. Resolver las siguientes ecuaciones de segundo grado:

$$[43] \frac{4x^2}{5} + x^2 - 5 = \frac{x^2}{5} + 10 + x^2$$

$$[44] x^2 + 6x = -9$$

$$[45] x^2 + 9 = 10x$$

$$[46] 2x^2 + 10x - 48 = 0$$

$$[47] x^2 - x = 20$$

$$[48] x^2 - 5x + 6 = 0$$

$$[49] x^2 + 8x + 15 = 0$$

$$[50] x^2 + 10x + 25 = 0$$

$$[51] 3x^2 - 39x + 108 = 0$$

$$[52] 6x^2 - 37x + 57 = 0$$

$$[53] 3x^2 + 2x = 8$$

$$[54] x^2 - 5x + 4 = 5(8 - x)$$

$$[55] 3x^2 = (4x + 8)(x - 2)$$

$$[56] 6[(x + 2)^2 + (x + 1)^2] = 13(x + 1)(x + 2)$$

$$[57] 9(x - 3) + 3(x + 4)(x - 3) = (4x + 7)(x - 3) + 9(7 - x)$$

$$[58] (2x + 1)(x + 1) = 5(x - 1)(x - 1)$$

$$[59] 76x(x + 3) = 9(x + 3)(4x + 7) - 171(x - 5)$$

$$[60] (x + 1)^2(x - 2) - (x + 12)(x - 1)(x - 2) - (x^2 - 1)(x - 2) + (x + 2)(x^2 - 1) = 0$$

$$[61] (2x - 1)(x - 1)(x + 2) - (x - 7)(x + 1)(x + 2) = 4(x^2 - 1)(x + 2) - (x^2 - 1)(3x - 1)$$

$$[62] (3x - 4)(6x - 11) = (4x + 1)(5x - 16)$$

$$[63] x(3 - x) = 10 - 4x$$

$$[64] 6x^2 + 6(x + 1)^2 = 13x(x + 1)$$

$$[65] 2(2x + 1)^2 - 3(2x - 1)^2 + 5(2x + 1)(2x - 1) = 0$$

$$[66] (x - 3)^2 - (x + 3)^2 = (x - 2)(x - 3)$$

$$[67] 2(6 - x)(6 + x) - 18(x - 4) = 3(x + 6)(x - 2)$$

Soluciones:

- |                                   |                                  |                                  |                               |                                  |                          |                         |
|-----------------------------------|----------------------------------|----------------------------------|-------------------------------|----------------------------------|--------------------------|-------------------------|
| [1] $\pm 3$                       | [2] $\pm 12$                     | [3] $0, 5$                       | [4] $\pm 3$                   | [5] $0, \frac{5}{4}$             | [6] $\pm 3$              | [7] $0, 2$              |
| [8] $0, 3$                        | [9] $\pm 6$                      | [10] $\pm 2$                     | [11] $\pm \frac{\sqrt{6}}{2}$ | [12] $\pm 3$                     | [13] $-\frac{3}{2}$      | [14] $4, 3$             |
| [15] $6, 3$                       | [16] $7, 2$                      | [17] $4, 2$                      | [18] $3$                      | [19] $3, -9$                     | [20] $-\frac{1}{2}$      | [21] $5, -\frac{13}{3}$ |
| [22] $4$                          | [23] $\frac{1}{2}, -\frac{2}{3}$ | [24] $4, 2$                      | [25] $8, \frac{1}{8}$         | [26] $3$                         | [27] $2, -\frac{4}{3}$   | [28] $7, -1$            |
| [29] $-\frac{1}{2}, -\frac{3}{4}$ | [30] $7, 1$                      | [31] $\frac{6}{5}, -\frac{1}{3}$ | [32] $6, \frac{1}{5}$         | [33] $\frac{6}{5}, -\frac{1}{3}$ | [34] $4, -\frac{45}{11}$ | [35] $-\frac{3}{2}, -1$ |
| [36] $6, -1$                      | [37] $\pm 6$                     | [38] $0, \frac{32}{17}$          | [39] $\pm 4$                  | [40] $\pm 4$                     | [41] $\pm 6$             | [42] $0, 7$             |
| [43] $\pm 5$                      | [44] $-3$                        | [45] $9, 1$                      | [46] $3, -8$                  | [47] $5, -4$                     | [48] $3, 2$              | [49] $-3, -5$           |
| [50] $-5$                         | [51] $9, 4$                      | [52] $\frac{19}{6}, 3$           | [53] $\frac{4}{3}, -2$        | [54] $\pm 6$                     | [55] $\pm 4$             | [56] $1, -4$            |
| [57] $21, 5$                      | [58] $4, \frac{1}{3}$            | [59] $3, -\frac{87}{10}$         | [60] $\frac{6}{5}, 5$         | [61] $\frac{3}{2}, 1$            | [62] $6, -5$             | [63] $5, 2$             |
| [64] $2, -3$                      | [65] $\frac{1}{4}, -\frac{3}{2}$ | [66] $-1, -6$                    | [67] $-3 \pm 3\sqrt{5}$       |                                  |                          |                         |

## Ecuaciones bicuadradas

Resolver las siguientes ecuaciones:

- |  |  |                                 |
|--|--|---------------------------------|
| [1] $4x^4 + 8x^2 - 12 = 0$                                   | [2] $5x^4 - 3x^2 + \frac{7}{16} = 0$                 | [3] $3x^4 + x^2 - 4 = 0$        |
| [4] $8x^4 - x^2 - 7 = 0$                                     | [5] $5x^4 - 6x^2 - 351 = 0$                          | [6] $x^4 + 5x^2 - 36 = 0$       |
| [7] $x^4 + 3x^2 + 2 = 0$                                     | [8] $4x^4 - 5x^2 - 9 = 0$                            | [9] $x^4 - 7x^2 + 12 = 0$       |
| [10] $x^4 - 29x^2 + 100 = 0$                                 | [11] $x^4 + 21x^2 - 100 = 0$                         | [12] $9x^4 + 16 = 40x^2$        |
| [13] $x^4 - 7x^2 + 12 = 0$                                   | [14] $34 - x^2 = \frac{225}{x^2}$                    | [15] $x^2 = \frac{12}{x^2 + 1}$ |
| [16] $x^4 - 10x^2 + 9 = 0$                                   | [17] $2x^4 + x^2 - 1 = 0$                            | [18] $x^4 - 16 = 0$             |
| [19] $4x^4 - 9x^2 = 0$                                       | [20] $x^4 - 5x^2 + 4 = 0$                            | [21] $x^4 - 3x^2 + 2 = 0$       |
| [22] $x^2(2x - 5)(2x + 5) = 9(1 - x)(1 + x)$                 |  |                                 |
| [23] $8(x^2 - 1) - 2(x^2 - 5)(x^2 - 1) = (x^2 - 9)(x^2 - 5)$ |  |                                 |
| [24] $x^2(34 - x^2) = 225$                                   | [25] $2x^2(x^2 - 1) + 3x^2(2 - x^2) = -45$           |                                 |
| [26] $2x^4 - 5x^2 + 97 = 0$                                  | [27] $x^4 + 3x^2 + 2 = 0$                            | [28] $2x^4 + x^2 - 1 = 0$       |
| [29] $x^4 - 10x^2 + 9 = 0$                                   | [30] $12[(x^2 + 1)(x^2 - 1) + x^2] = 19x^2(x^2 - 1)$ |                                 |

Soluciones:

[1] $\pm 1$	[2] $\pm \frac{1}{2}, \pm \frac{\sqrt{35}}{10}$	[3] $\pm 1$	[4] $\pm 1$	[5] $\pm 3$
[6] $\pm 2$	[7] Sin solución	[8] $\pm \frac{3}{2}$	[9] $\pm 2, \pm \sqrt{3}$	[10] $\pm 5, \pm 2$
[11] $\pm 2$	[12] $\pm 2, \pm \frac{2}{3}$	[13] $\pm 2, \pm \sqrt{3}$	[14] $\pm 5, \pm 3$	[15] $\pm \sqrt{3}$
[16] $\pm 3, \pm 1$	[17] $\pm \frac{\sqrt{2}}{2}$	[18] $\pm 2$	[19] $0, \pm \frac{3}{2}$	[20] $\pm 1, \pm 2$
[21] $\pm 1, \pm \sqrt{2}$	[22] $\pm \frac{3\sqrt{2}}{2}$	[23] $\pm 3, \pm \frac{\sqrt{21}}{3}$	[24] $\pm 5, \pm 3$	[25] $\pm 3$
[26] Sin solución	[27] Sin solución	[28] $\pm \frac{\sqrt{2}}{2}$	[29] $\pm 3, \pm 1$	[30] $\pm 2, \pm \frac{\sqrt{21}}{7}$

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# Sistemas de dos ecuaciones con dos incógnitas

Resolver los siguientes sistemas:

$$[1] \begin{cases} \frac{3(y+2x+2)}{4} = \frac{4x+y-1}{3} \\ \frac{x+y}{3} - \frac{x-y}{6} = \frac{y-1}{6} \end{cases}$$

$$[2] \begin{cases} \frac{3-2y}{4} - \frac{1}{4} = \frac{1-2x}{6} \\ \frac{25}{8} - 1 = \frac{x+3}{2} - \frac{3(1+y)}{8} \end{cases}$$

$$[3] \begin{cases} \frac{4y-5x}{6} + \frac{3x-2y}{2} = 1 - \frac{2(x+y)}{9} \\ \frac{4y+x-8}{8} - x = \frac{2(y-2x)}{3} \end{cases}$$

$$[4] \begin{cases} \frac{3(x-y)}{4} = \frac{2+y}{4} - \frac{5x-y}{6} \\ 1 + \frac{2y-7x}{12} = \frac{x-y}{2} + \frac{x}{2} \end{cases}$$

$$[5] \begin{cases} \frac{5x}{6} + \frac{3y}{7} = 2 \\ \frac{x}{2} - \frac{y}{7} = 2 \end{cases}$$

$$[6] \begin{cases} x - y = 1 \\ \frac{2x}{5} + \frac{3y}{4} = 5 \end{cases}$$

$$[7] \begin{cases} x + y = 0'3 \\ 0'1x + 0'2y = 0'04 \end{cases}$$

$$[8] \begin{cases} 4x - 3y = 0 \\ 2(x-1) = y + 2 \end{cases}$$

$$[9] \begin{cases} 4(x+1) - y = 0 \\ 5x - y - 1 = 0 \end{cases}$$

$$[10] \begin{cases} 3(x+2) - (x-2) = 2y \\ x + y = 8 \end{cases}$$

$$[11] \begin{cases} 3 - y(x-1) - x(3-y) = 0 \\ xy - x(y-1) = 2 \end{cases}$$

$$[12] \begin{cases} x + 2y = 10 \\ x^2 + y^2 = 25 \end{cases}$$

$$[13] \begin{cases} x - y = 2 \\ xy = 48 \end{cases}$$

$$[14] \begin{cases} 2x^2 - y^2 = 5 \\ 3x^2 + 4y^2 = 57 \end{cases}$$

$$[15] \begin{cases} 5x + 7y = 61 \\ xy = 8 \end{cases}$$

$$[16] \begin{cases} 6x - 5y = 14 \\ xy = 72 \end{cases}$$

$$[17] \begin{cases} x + xy + y = 1 \\ xy = 6 \end{cases}$$

$$[18] \begin{cases} xy + 2y = 4 \\ 3x - y = 5 \end{cases}$$

$$[19] \begin{cases} x^2 + y^2 = 290 \\ x + y = 4 \end{cases}$$

$$[20] \begin{cases} x^2 + y^2 = 9 \\ 2x + y = 3 \end{cases}$$

$$[21] \begin{cases} x - 2y^2 = 0 \\ y + 5 = 3x \end{cases}$$

$$[22] \begin{cases} 2x^2 - 3y^2 = -6 \\ 4x^2 - y^2 = 8 \end{cases}$$

$$[23] \begin{cases} x^2 + y^2 = 40 \\ xy = 12 \end{cases}$$

$$[24] \begin{cases} 3x^2 - 4y^2 = 16 \\ xy = 4 \end{cases}$$

$$[25] \begin{cases} x^2 + y^2 = 61 \\ xy = 30 \end{cases}$$

Soluciones: En los pares que siguen, la primera coordenada es la  $x$  y la segunda la  $y$ .

- [1]  $(39, -20)$       [2]  $(5, 4)$       [3]  $(\frac{4}{7}, -\frac{31}{7})$       [4]  $(\frac{20}{19}, 1)$
- [5]  $(\frac{24}{7}, -2)$       [6]  $(5, 4)$       [7]  $(0'2, 0'1)$       [8]  $(6, 8)$
- [9]  $(5, 24)$       [10]  $(2, 6)$       [11]  $(2, 3)$       [12]  $\{(0, 5), (4, 3)\}$
- [13]  $\{(8, 6), (-6, -8)\}$       [14]  $\{(\sqrt{7}, 3), (-\sqrt{7}, 3), (\sqrt{7}, -3), (-\sqrt{7}, -3)\}$
- [15]  $\{(1, 8), (\frac{56}{5}, \frac{5}{7})\}$       [16]  $\{(9, 8), (-\frac{20}{3}, -\frac{54}{5})\}$       [17]  $\{(-2, -3), (-3, -2)\}$
- [18]  $\{(2, 1), (-\frac{7}{3}, -12)\}$       [19]  $\{(13, 11), (11, 13)\}$       [20]  $\{(0, 3), (\frac{12}{5}, -\frac{9}{5})\}$
- [21]  $\{(2, 1), (\frac{25}{18}, -\frac{5}{6})\}$       [22]  $\{(\sqrt{3}, 2), (-\sqrt{3}, 2), (\sqrt{3}, -2), (-\sqrt{3}, -2)\}$
- [23]  $\{(2, 6), (6, 2), (-6, -2), (-2, -6)\}$
- [24]  $\{(2\sqrt{2}, \sqrt{2}), (2\sqrt{2}, -\sqrt{2}), (-2\sqrt{2}, \sqrt{2}), (-2\sqrt{2}, -\sqrt{2})\}$
- [25]  $\{(6, 5), (5, 6), (-5, -6), (-6, -5)\}$

## Sistemas de tres ecuaciones con tres incógnitas

Resolver los siguientes sistemas:

$$\begin{array}{l}
 [1] \begin{cases} x + y + z = 31 \\ x + y - z = 25 \\ x - y - z = 9 \end{cases} \quad [2] \begin{cases} 2x + y - z = 11 \\ 3x - 4y - 2z = 2 \\ x + 5y + 3z = 5 \end{cases} \quad [3] \begin{cases} x + y + z = 4 \\ x - 2y + 3z = 13 \\ x + 3y + 4z = 11 \end{cases} \\
 [4] \begin{cases} x + y - 2z = 13 \\ x - 3y - z = -3 \\ x - y + 4z = -17 \end{cases} \quad [5] \begin{cases} x - y - z = 1 \\ 2x - 5y + 10z = 5 \\ 3x + 2y - 11z = 10 \end{cases} \quad [6] \begin{cases} 4x + 2y - z = 5 \\ 5x - 3y + z = 2 \\ 2x - y + z = 3 \end{cases} \\
 [7] \begin{cases} 4x - 3y + 2z = 8 \\ 5x + y - z = 16 \\ 6x - 2y - 3z = 11 \end{cases} \quad [8] \begin{cases} 2x - y + z = 12 \\ 4y - 3x - z = -18 \\ x + 3y - 4z = -20 \end{cases} \quad [9] \begin{cases} 2x + 4y - 3z = 22 \\ 4x - 2y + 5z = 18 \\ 6x + 7y - z = 63 \end{cases}
 \end{array}$$

Soluciones: En las ternas que siguen, la primera coordenada es la  $x$ , la segunda la  $y$ , y la tercera es la  $z$

- [1]  $(20, 8, 3)$       [2]  $(2, 3, -4)$       [3]  $(2, -1, 3)$       [4]  $(2, 3, -4)$       [5]  $(5, 3, 1)$   
 [6]  $(1, 2, 3)$       [7]  $(3, 2, 1)$       [8]  $(3, -1, 5)$       [9]  $(3, 7, 4)$

## Problemas

- La suma de los catetos de un triángulo rectángulo es de 49 cm. Determinar los lados del triángulo y el área sabiendo que la hipotenusa es 32 cm. mayor que el cateto

menor.

Solución: lados = 9, 40, 41 cm., Área =  $180 \text{ cm}^2$ .

2. ¿Cuál es el número cuyos  $\frac{3}{4}$  más 9 multiplicados por los  $\frac{3}{4}$  menos 9 dan por resultado 1 008.

Solución:  $\pm 44$

3. Hallar un número cuyo cuadrado lo sobrepase en 13 110 unidades.

Solución: 115, -114

4. La suma de dos números es 42 y la diferencia de sus cuadrados es 504. ¿Cuáles son estos números?

Solución: 27, 15

5. El perímetro de un triángulo rectángulo mide 60 cm. y la hipotenusa 26 cm. Calcular las longitudes de los dos catetos.

Solución: 24, 10

6. Un librero vende 84 libros a dos precios distintos: unos a 45 pts. y otros a 36 pts., obteniendo por la venta 3 105 pts. ¿Cuántos libros vendió de cada clase?

Solución: 9 libros de 45 pts., 75 libros de 36 pts.

7. Una habitación rectangular tiene una superficie de  $28 \text{ m}^2$  y su perímetro es de 22 m. Hallar las dimensiones de la habitación.

Solución: 4, 7 m.

8. La diagonal de un rectángulo mide 55 cm. ¿Cuánto miden sus lados, si la base es los  $\frac{3}{4}$  de la altura?

Solución: 44, 33 m.

9. El perímetro de un rectángulo es de 30 m. y su área  $54 \text{ m}^2$ . ¿Cuánto miden sus lados?

Solución: 9, 6 m.

10. El área de un rectángulo es de  $12 \text{ m}^2$  y su perímetro es 16 m. Hallar las dimensiones.

Solución: 6, 2 m.

11. El área de un rectángulo es  $620 \text{ m}^2$ . Si cada dimensión aumenta en 1 m., el área aumenta en  $52 \text{ m}^2$ . Hallar las dimensiones.

Solución: 20, 31 m.

12. Hallar la altura de un cono sabiendo que su generatriz mide 13 cm. y el área de su base  $25\pi \text{ cm}^2$ .

Solución: 12 cm.

13. Hallar la altura y el radio de la base de un cono, sabiendo que la generatriz mide 50 m., y que la altura es 10 m. más larga que el radio de la base.

Solución: altura = 40 m., radio = 30 m.



14. La suma de dos números es 8 y su producto es 15. Calcular dichos números.  
Solución: 3, 5
15. La hipotenusa de un triángulo mide 5 cm. ¿Cuánto miden los catetos sabiendo que uno es 1 cm. mayor que el otro?.  
Solución: 3, 4 cm.
16. La hipotenusa de un triángulo rectángulo vale 34 cm. Hallar las longitudes de los catetos sabiendo que uno de ellos es 14 cm. mayor que el otro.  
Solución: 16, 30 cm.
17. Hallar las dimensiones de un rectángulo cuyo perímetro es 50 cm. y su área 150 cm<sup>2</sup>.  
Solución: 10, 15 cm.
18. Hallar un número sabiendo que la suma del triple del mismo con el doble de su recíproco es igual a 5.  
Solución: 1 ó  $\frac{2}{3}$ .
19. Hallar dos números positivos sabiendo que uno de ellos es igual al triple del otro más 5 y que el producto de ambos es 68.  
Solución: 4, 17

## Ecuaciones de grado superior e irracionales

1. Resolver las siguientes ecuaciones:

[1]  $x^3 + 2x^2 + 2x + 1 = 0$

[2]  $x^3 + 3x^2 - x - 3 = 0$

[3]  $x^5 - 32 = 0$

[4]  $x^4 - x^3 - 16x^2 - 20x = 0$

[5]  $x^4 - 6x^3 - 11x^2 + 96x - 80 = 0$

[6]  $6x^3 + x^2 - 26x - 21 = 0$

[7]  $x^3 + 6x^2 + 3x - 10 = 0$

[8]  $2x^4 - 5x^3 + 5x - 2 = 0$

[9]  $6x^3 + 7x^2 - 9x + 2 = 0$

[10]  $x^3 + 3x^2 - 4x - 12 = 0$

[11]  $x^3 + 2x^2 - x - 2 = 0$

[12]  $x^3 - 5x^2 + 6 = 0$

[13]  $8x^4 - 14x^3 - 9x^2 + 11x - 2 = 0$

[14]  $4x^5 + 4x^4 + 3x^3 + 3x^2 - x - 1 = 0$

[15]  $x^4 + x^3 - 16x^2 - 4x + 48 = 0$

Soluciones:

[1] -1

[2] {1, -1, -3}

[3] 2

[4] {0, 5, -2}

[5] {1, 5, 4, -4}

[6]  $\{-1, \frac{7}{3}, -\frac{3}{2}\}$

[7] {1, -2, -5}

[8]  $\{1, -1, 2, \frac{1}{2}\}$

[9]  $\{-2, \frac{1}{2}, \frac{1}{3}\}$

[10] {2, -2, -3}

[11] {1, -1, -2}

[12]  $\{-1, 3 \pm \sqrt{3}\}$

[13]  $\{\frac{1}{4}, -1, 2, \frac{1}{2}\}$

[14]  $\{-1, \frac{1}{2}, -\frac{1}{2}\}$

[15] {2, -2, 3, -4}

2. Resolver las siguientes ecuaciones:

- |  |  |
|--|--|
| [1] $\sqrt{3x-2} = 0$  | [2] $\sqrt{2x+1} = x-1$                              |
| [3] $\sqrt{7-3x} - x = 7$  | [4] $3\sqrt{6x+1} - 5 = 2x$                          |
| [5] $\sqrt{x+4} = 3 - \sqrt{x-1}$                                    | [6] $\sqrt{2x-1} + \sqrt{x+4} = 6$                   |
| [7] $\sqrt{x^2+3x+7} = 5$  | [8] $\sqrt{x^2-x+2} = 2$                             |
| [9] $2\sqrt{2x-1} = \sqrt{6x-5} + \sqrt{2x-9}$                       | [10] $\sqrt{2x-2} = x-1$                             |
| [11] $\sqrt{4x+1} = 3-3x$  | [12] $\sqrt{2x+7} = \sqrt{x}+2$                      |
| [13] $\sqrt{2x^2-7} - x = 3$   | [14] $3-2\sqrt{x} = x$                               |
| [15] $4 + \sqrt{x+2} = \frac{4x}{7}$                                 | [16] $5\sqrt{x} - 2 = 2x$                            |
| [17] $\sqrt{2x-5} + 6 = x+2$   | [18] $\sqrt{x} + 1 = \sqrt{x-3} + 2$                 |
| [19] $\sqrt{2x+10} = 1 + \sqrt{2x+3}$                                | [20] $\sqrt{2x+5} - \sqrt{3x+4} = 1$                 |
| [21] $\sqrt{2(x-4)} - \sqrt{x-3} = \sqrt{3(x-5)}$                    | [22] $\sqrt{x-4} + \sqrt{x+4} = \sqrt{2x}$           |
| [23] $\sqrt{x+6} + \sqrt{x+1} = \sqrt{7x+4}$                         | [24] $\sqrt{3\sqrt{15-x}} = \sqrt{2x-3}$             |
| [25] $\sqrt{5+2x} = x+1$   | [26] $\sqrt{2x+1} - \sqrt{x} = 1$                    |
| [27] $\sqrt{4x-1} + \sqrt{2x-3} = 1$                                 | [28] $2\sqrt{2x-1} = \sqrt{6x-5} + \sqrt{2x-9}$      |
| [29] $\frac{\sqrt{4x+20}}{4+\sqrt{x}} = \frac{4-\sqrt{x}}{\sqrt{x}}$ | [30] $\sqrt{\sqrt{x+16} - \sqrt{x}} = 2$             |
| [31] $\sqrt{x^2-6x+9} - 8 = 0$                                       | [32] $\sqrt{x} + 1 = \sqrt{3(x-1)}$                  |
| [33] $3-x = \sqrt{x}+1$  | [34] $\sqrt{4x+1} = 5\sqrt{3x-2} - 7$                |
| [35] $x + \sqrt{x} = 6$  | [36] $x + 5 = \sqrt{x^2 + 10x + 5}\sqrt{5(x^2 + 4)}$ |
| [37] $(\sqrt{x-1} - 1)^2 + 2 = 2\sqrt{x-1}$                          | [38] $\sqrt{x-85} = \sqrt[4]{3x-47}$                 |

Soluciones:

- |                   |                       |                   |                           |                    |                  |
|-------------------|-----------------------|-------------------|---------------------------|--------------------|------------------|
| [1] $\frac{2}{3}$ | [2] 4                 | [3] -3            | [4] $\{8, \frac{1}{2}\}$  | [5] $\frac{13}{9}$ | [6] 5            |
| [7] $\{3, -6\}$   | [8] $\{2, -1\}$       | [9] 5             | [10] $\{3, 1\}$           | [11] $\frac{4}{9}$ | [12] $\{9, 1\}$  |
| [13] $\{8, -2\}$  | [14] 1                | [15] 14           | [16] $\{4, \frac{1}{4}\}$ | [17] 7             | [18] 4           |
| [19] 3            | [20] $6 - 2\sqrt{13}$ | [21] 5            | [22] 4                    | [23] 3             | [24] 6           |
| [25] 2            | [26] $\{0, 4\}$       | [27] Sin solución | [28] 5                    | [29] 4             | [30] 0           |
| [31] $\{11, -5\}$ | [32] 4                | [33] 1            | [34] 2                    | [35] 4             | [36] $\{1, -1\}$ |
| [37] $\{10, 2\}$  | [38] 101              |                   |                           |                    |                  |

# Inecuaciones

1. Resolver las siguientes desigualdades:

$$[1] x - 2 > 0$$

$$[2] 3 - x > 0$$

$$[3] x + 3 > 4$$

$$[4] 3 - x \leq 6$$

$$[5] 3(x - 2) < 6$$

$$[6] 2(x + 3) > 3(x + 2)$$

$$[7] \frac{x-1}{4} - \frac{x+2}{3} > \frac{3x-1}{6} - x$$

$$[8] (x-3)^2 - (x+2)^2 < 5$$

$$[9] (4x-3)(2+x) > (3-2x)^2$$

$$[10] 3 \left[ x - 2 \left( \frac{x(x-1)}{4} - 5 \right) \right] < \frac{3}{2}x(4-x)$$

$$[11] \frac{3 - \left[ \frac{x-2}{4} + x \left( \frac{x-3}{2} - x \right) \right]}{2 - \frac{3}{2}} \leq (x-2)(x-3)$$

$$[12] 3x + 2 > 6x - 3$$

$$[13] 2(3x - 7) \leq -3(2x + 4) + 1$$

$$[14] 7x - 6 \geq -2x + 5(x + 3) - 1$$

$$[15] 4x^2 - (2x - 2) < (2x + 3)^2$$

$$[16] \frac{3 - \frac{x}{3}}{3 + \frac{1}{2}} - x \geq \frac{3x - \frac{5}{2}}{1 - \frac{2}{3}}$$

Soluciones:

$$[1] x > 2 \quad [2] x < 3 \quad [3] x > 1 \quad [4] x \geq -3$$

$$[5] x < 4 \quad [6] x < 0 \quad [7] x > \frac{9}{5} \quad [8] x > 0$$

$$[9] x > \frac{15}{17} \quad [10] x > 20 \quad [11] x \leq -\frac{2}{15} \quad [12] x < \frac{5}{3}$$

$$[13] x \leq \frac{1}{4} \quad [14] x \geq 5 \quad [15] x > -\frac{1}{2} \quad [16] x \leq \frac{351}{424}$$

2. Resolver las siguientes desigualdades:

$$\begin{aligned}
 [1] \quad & \begin{cases} 2x - 3 > x - 2 \\ 3x - 7 < x - 1 \end{cases} & [2] \quad & \begin{cases} \frac{x}{3} + \frac{x}{5} < 8 \\ \frac{x}{2} - \frac{4x}{9} < 5 \end{cases} \\
 [3] \quad & \begin{cases} (x-1)^2 + (x+2)^2 > \frac{(2x-3)^2}{2} \\ (2x+1)^2 - (x-3)^2 < 3(x+2)^2 \\ \frac{x-1}{3} + 1 > x \end{cases} & [4] \quad & \begin{cases} 2x - 1 < 3x + 7 \\ x + 2 \geq 2x - 5 \end{cases} \\
 [5] \quad & \begin{cases} 5(2x+1) < 3x + 19 \\ 2x(x+1) < 3 + 2(x-1)^2 \end{cases} & [6] \quad & \begin{cases} 6x + 5(2-x) > 3x - 8(x+4) \\ x(7-2x) > 2x(5-x) + 10x \end{cases} \\
 [7] \quad & \begin{cases} (3x+4)(2x-1) + 11 \leq (2x-1)^2 + x(2x+3) \\ 7(x-2) + 7 > 2(4x+9) - 3(5x+5) \end{cases} \\
 [8] \quad & \begin{cases} \frac{2x-2}{5} + \frac{5-2x}{3} < 1 \\ \frac{x+2}{3} - \frac{2x-3}{4} > \frac{3}{4} \end{cases} & [9] \quad & \begin{cases} \frac{13x-2}{12} - 1 < \frac{3x-2}{10} + \frac{x+1}{5} \\ (2x+1)^2 - 8 \leq (2x-1)^2 \\ (x+1)(x-1) > (x-2)^2 - 3 \end{cases} \\
 [10] \quad & \begin{cases} \frac{3x}{2} + \frac{1}{2} - x + 3 > \frac{1-3x}{5} \\ \frac{3x+1}{4} + \frac{1-3x}{2} + \frac{1-x}{3} > 0 \end{cases}
 \end{aligned}$$

Soluciones:

$$\begin{aligned}
 [1] \quad & 1 < x < 3 & [2] \quad & x < 15 & [3] \quad & -\frac{1}{16} < x < 1 & [4] \quad & -8 < x \leq 7 & [5] \quad & x < \frac{5}{6} \\
 [6] \quad & -7 < x < 0 & [7] \quad & \text{Sin solución} & [8] \quad & 1 < x < 4 & [9] \quad & \frac{1}{2} < x \leq 1 & [10] \quad & -3 < x < 1
 \end{aligned}$$

3. Resolver las siguientes desigualdades:

$$\begin{aligned}
 [1] \quad & x^2 - 5x + 6 > 0 & [2] \quad & (x+1)^2 + 6x + 2 \geq 2(x+3)(x-2) + 4x \\
 [3] \quad & x^2 + 5x + 6 > 0 & [4] \quad & x^2 - 3x + 2 < 0 \\
 [5] \quad & x^2 - 2x - 8 \geq 0 & [6] \quad & x^2 - x - 20 \leq 0 \\
 [7] \quad & x^2 + 2x + 1 > 0 & [8] \quad & x^2 - 6x + 9 < 0 \\
 [9] \quad & x^2 - 4x + 4 \geq 0 & [10] \quad & x^2 + 10x + 25 \leq 0 \\
 [11] \quad & x^2 + 2x + 2 > 0 & [12] \quad & x^2 - 4x + 9 \geq 0 \\
 [13] \quad & x^2 - 4x + 9 < 0 & [14] \quad & x^2 - 8x + 25 \leq 0 \\
 [15] \quad & 2(x+1)^2 - 2x + 5 > x(x+4) + 10 & [16] \quad & (x+3)(x-1) + x \leq 5x - 4 \\
 [17] \quad & (3x-4)^2 - (5x+3)(x+1) \geq 4x + 13 & [18] \quad & (2x+3)^2 + 4x - 5 < 8(x+2)^2 + 10
 \end{aligned}$$

Soluciones:

- [1]  $x < 2$  ó  $x > 3$     [2]  $-3 \leq x \leq 5$     [3]  $x < -3$  ó  $x > -2$     [4]  $1 < x < 2$   
[5]  $x \leq -2$  ó  $x \geq 4$     [6]  $-4 \leq x \leq 5$     [7]  $\mathbb{R} - \{-1\}$     [8] Sin solución  
[9]  $\mathbb{R}$     [10]  $x = -5$     [11]  $\mathbb{R}$     [12]  $\mathbb{R}$   
[13] Sin solución    [14] Sin solución    [15]  $x < -1$  ó  $x > 3$     [16]  $x = 1$   
[17]  $x \leq 0$  ó  $x \geq 9$     [18]  $\mathbb{R}$

4. Resolver las siguientes desigualdades:

- [1]  $\frac{x+3}{x-2} \leq 0$     [2]  $\frac{3x+1}{2x-3} \geq 0$   
[3]  $\frac{6x+5}{3x+2} > 0$     [4]  $\frac{9x-7}{5-2x} \leq 0$   
[5]  $\frac{3-4x}{2x+5} \geq 0$     [6]  $\frac{x}{(x+3)(x-2)} < 0$   
[7]  $2x^3 - 7x^2 + 7x - 2 < 0$     [8]  $\frac{2x^2+x-3}{x+1} \geq 0$   
[9]  $2x - \frac{3}{5} > \frac{x}{3} + \frac{1}{6}$     [10]  $\frac{1}{2} \left( x - \frac{1}{3} \right) + x < \frac{x+2}{4}$   
[11]  $\frac{3(x-1)}{5} + \frac{x}{2} > 2x - \frac{5}{8}$     [12]  $\frac{5(6x-2)}{8} - \frac{7}{3} \left( 1 - \frac{2x}{3} \right) < 4x + \frac{2}{3} \left( \frac{x}{2} - \frac{5}{12} \right)$   
[13]  $\frac{x-2}{x^3-x} \geq 0$     [14]  $4^{-\frac{1}{2}}(x-1) - 8^{-\frac{2}{3}}(x+1) \leq -2^{-2}(1-3x)$

Soluciones:

- [1]  $x \in [-3, 2]$     [2]  $x \in ]-\infty, -\frac{1}{3}] \cup ]\frac{3}{2}, +\infty[$   
[3]  $x \in ]-\infty, -\frac{5}{6}] \cup ]-\frac{2}{3}, +\infty[$     [4]  $x \in ]-\infty, \frac{7}{9}] \cup ]\frac{5}{2}, +\infty[$   
[5]  $x \in ]-\frac{5}{2}, \frac{3}{4}]$     [6]  $x \in ]-\infty, -3[ \cup ]0, 2[$   
[7]  $x \in ]-\infty, \frac{1}{2}[ \cup ]1, 2[$     [8]  $x \in ]-\frac{3}{2}, -1] \cup ]1, +\infty[$   
[9]  $x \in ]\frac{23}{50}, +\infty[$     [10]  $x \in ]-\infty, \frac{8}{15}[$   
[11]  $x \in ]-\infty, \frac{1}{36}[$     [12]  $x \in ]-\infty, \frac{17}{5}[$   
[13]  $x \in ]-\infty, -1[ \cup ]0, 1[ \cup ]2, +\infty[$     [14]  $x \in [-1, +\infty[$