

EXAM UNIT 5 (EQUATIONS - WORD PROBLEMS)

1) Solve the following equations:

(4 points)

a. $\frac{3x-1}{4} - \frac{x+1}{5} + 1 - \frac{x}{10} = 2 - x$

b. $[4y - 3 + (2y - 1)] - [(4 + 4y) - 3] = 1 + 2y$

c. $\frac{(x-1)^2}{2} - \frac{(x+1)^2}{4} = 4 - x$

d. $\frac{(x-1)(x+1)}{2} - \frac{x-5}{6} = \frac{2(x+1)}{3}$

2) The length of a rectangle is 4 cm longer than its width. If the area is 77 square centimetre, find the length. (1.25 points)

3) Richard's uncle is five times as old as Richard and in 8 years' time he will be three times as old as him. How old is each one? (1.25 points)

4) We mix 32 kg of coffee that sells at €4.75 per kilo with coffee at €4 per kilo. We want to get a mix that sells at €4.50 per kilo. How many kilograms of coffee at 4 euros do we need? (1.25 points)

5) Find three consecutive even numbers. The triple of the second one plus the first one is 322 units less than five times the last number. (1.25 points)

6) Ten students are absent from school because of a flu epidemic. Only half the students, plus four, are present. How many students are normally in the class?(1p)

SOLUTION

1) Solve the following equations:

$$a. \frac{3x-1}{4} - \frac{x+1}{5} + 1 - \frac{x}{10} = 2 - x \rightarrow \frac{15x-5}{20} - \frac{4x+4}{20} + \frac{20}{20} - \frac{2x}{20} = \frac{40-20x}{20}$$

$$15x - 5 - 4x - 4 + 20 - 2x = 40 - 20x \rightarrow 15x - 4x - 2x + 20x = 40 + 5 + 4 - 20$$

$$29x = 29 \Rightarrow x = 1$$

$$b. [4y - 3 + (2y - 1)] - [(4 + 4y) - 3] = 1 + 2y$$

$$[4y - 3 + 2y - 1] - [4 + 4y - 3] = 1 + 2y \rightarrow 4y + 2y - 4 - 4 - 4y + 3 = 1 + 2y$$

$$4y + 2y - 4 - 4 - 4y + 3 = 1 + 2y \rightarrow 4y + 2y - 4y - 2y = 1 + 8 - 3 \rightarrow 0x = 6$$

It does not have any solution

$$c. \frac{(x-1)^2}{2} - \frac{(x+1)^2}{4} = 4 - x \rightarrow \frac{x^2 - 2x + 1}{2} - \frac{x^2 + 2x + 1}{4} = 4 - x$$

$$\frac{2x^2 - 4x + 2}{4} - \frac{x^2 + 2x + 1}{4} = \frac{16 - 4x}{4} \rightarrow 2x^2 - 4x + 2 - x^2 - 2x - 1 = 16 - 4x$$

$$x^2 - 6x + 1 = 16 - 4x \Rightarrow x^2 - 2x - 15 = 0 \rightarrow x = \frac{2 \pm \sqrt{4 + 60}}{2} = \begin{cases} 5 \\ -3 \end{cases}$$

$$d. \frac{(x+1)(x-1)}{2} - \frac{x-5}{6} = \frac{2(x+1)}{3} \rightarrow \frac{x^2 - 1}{2} - \frac{x-5}{6} = \frac{2x+2}{3}$$

$$\frac{3x^2 - 3}{6} - \frac{x-5}{6} = \frac{4x+4}{6} \rightarrow 3x^2 - 3 - x + 5 = 4x + 4$$

$$3x^2 - x - 4x - 3 + 5 - 4 = 0 \rightarrow 3x^2 - 5x - 2 = 0$$

$$x = \frac{5 \pm \sqrt{25 + 24}}{6} = \frac{5 \pm 7}{6} = \begin{cases} 2 \\ -\frac{1}{3} \end{cases}$$

2) The length of a rectangle is 4 cm longer than its width. If the area is 77 square centimetre, find the length.

Rectangle: width x , length $x + 4$

$$\text{Equation: Area} = x \cdot (x + 4) = 77 \rightarrow x^2 + 4x - 77 = 0$$

$$x = \frac{-4 \pm \sqrt{16 + 308}}{2} = \frac{-4 \pm 18}{2} = \begin{cases} 7 \\ -11 \end{cases} \rightarrow \text{No!}$$

Answer: the length is 11 cm (7+4) long

3) Richard's uncle is five times as old as Richard and in 8 years' time he will be three times as old as him. How old is each one?

Age	Now	In 8 years' time
Richard	x	$x + 8$
Richard's uncle	$5x$	$5x + 8$

$$\text{Equation: } 5x + 8 = 3(x + 8) \Rightarrow 5x + 8 = 3x + 24 \Rightarrow 2x = 24 - 8$$

$$2x = 16 \Rightarrow x = 8, \quad 5x = 5 \cdot 8 = 40$$

Answer: Richard is 8 years old and his uncle is 40 years old

4) We mix 32 kg of coffee that sells at €4.75 per kilo with coffee at €4 per kilo. We want to get a mix that sells at €4.50 per kilo. How many kilograms of coffee at 4 euros do we need?

Type of coffee	Number of kilos	Price/kilo(€)
First	32	4.75
Second	x	4
Mixture	$32 + x$	4.50

$$\text{Equation: } 32 \cdot 4.75 + 4x = 4.50 \cdot (32 + x) \Rightarrow 152 + 4x = 144 + 4.50x$$

$$152 - 144 = 4.50x - 4x \Rightarrow 0.5x = 8 \Rightarrow x = 16$$

Answer: We need 16 kilograms of the second type of coffee

5) Find three consecutive even numbers. The triple of the second one plus the first one is 322 units less than five times the last number.

Three consecutive even numbers: $2x$ (first), $2x+2$ (second), $2x+4$ (third)

$$\text{Equation: } 3(2x + 2) + 2x = 5(2x + 4) - 322$$

$$6x + 6 + 2x = 10x + 20 - 322 \Rightarrow 6 - 20 + 322 = 10x - 6x - 2x$$

$$308 = 2x \Rightarrow x = 154$$

First number $2x = 2 \cdot 154 = 308$, second: $308+2=310$, third: $308+4=312$

Answer: Numbers are 308, 310 and 312

6) Ten students are absent from school because of a flu epidemic. Only half the students, plus four, are present. How many students are normally in the class?

Students that are normally in the class x

$$\text{Equation: } x - 10 = \frac{x}{2} + 4$$

$$\frac{2x - 20}{2} = \frac{x}{2} + \frac{8}{2} \rightarrow 2x - 20 = x + 8 \rightarrow 2x - x = 8 + 20 \rightarrow x = 28$$

Answer: there are normally 28 students in the class