

EXAM 2_3 (Simultaneous equations)

Name: _____

1. Solve the following equations: (2.5 points)
 - a) $x + 3(x + 3) = 5[(2 - 3x) - (x - 3)] + 3$
 - b) $\frac{3(x^2 - 11)}{5} - \frac{2(x^2 - 60)}{7} = 36$
 - c) $2(x - 1)^2 + 3x - (x + 1)^2 - x^2 = 2x - 11$
2. A hunter and his dog, 230 km away from each other, are going to meet. The speed of the hunter is 5km/h and the speed of the dog is 12km/h. If the dog left 5 hours before, where and when will they meet? (1.5 points)
3. In a parking lot, there are 458 vehicles; cars and motorcycles. Find the number of each type of vehicle knowing that in total there are 1764 wheels. (1.5 points)
4. Solve the simultaneous equation: (1.5 points)
$$\left. \begin{array}{l} 2x + y = 1 \\ 2x^2 + y^2 = 3 \end{array} \right\}$$
5. Classify these systems depending on the number of solutions. In order to solve them use the most convenient method in each one (and graphing): (3 points)
 - a) $\left. \begin{array}{l} x - 2y = 7(1 - y) \\ 2(x + y) - 3(y - 2) = x - 6y \end{array} \right\}$
 - b) $\left. \begin{array}{l} 2x + \frac{3y}{4} = 5 \\ 5x - \frac{y}{2} = 3 \end{array} \right\}$

SOLUTION

1.

$$a) \quad x + 3(x + 3) = 5[(2 - 3x) - (x - 3)] + 3 \rightarrow x + 3x + 9 = 5(2 - 3x - x + 3) + 3$$

$$x + 3x + 9 = 10 - 15x - 5x + 15 + 3 \rightarrow 24x = 19 \rightarrow x = \frac{19}{24}$$

$$b) \quad \frac{3(x^2 - 11)}{5} - \frac{2(x^2 - 60)}{7} = 36 \rightarrow \frac{21(x^2 - 11)}{35} - \frac{10(x^2 - 60)}{35} = \frac{1260}{35}$$

$$21x^2 - 231 - 10x^2 + 600 = 1260 \rightarrow 11x^2 - 891 = 0 \rightarrow 11x^2 = 891$$

$$x^2 = \frac{891}{11} = 81 \rightarrow x = \pm\sqrt{81} \rightarrow x = \pm 9$$

$$c) \quad 2(x - 1)^2 + 3x - (x + 1)^2 - x^2 = 2x - 11$$

$$2(x^2 - 2x + 1) + 3x - (x^2 + 2x + 1) - x^2 = 2x - 11$$

$$2x^2 - 4x + 2 + 3x - x^2 - 2x - 1 - x^2 = 2x - 11 \rightarrow -3x + 1 = 2x - 11 \rightarrow x = \frac{12}{5}$$

2. A hunter and his dog, 230 km away from each other, are going to meet. The speed of the hunter is 5km/h and the speed of the dog is 12km/h. If the dog left 5 hours before, where and when will they meet?

	speed	distance	time
hunter	5 km/h	230 - x	y
dog	12 km/h	x	y + 5

$$d = s \cdot t \rightarrow \begin{cases} 230 - x = 5y \\ x = 12(5 + y) \end{cases} \rightarrow \begin{cases} x = 230 - 5y \\ x = 60 + 12y \end{cases} \rightarrow 230 - 5y = 60 + 12y$$

$$230 - 60 = 5y + 12y \rightarrow 170 = 17y \rightarrow y = 10 \text{ hours}$$

$$x = 230 - 5y = 230 - 50 = 180$$

ANSWER: They will meet after 10 hours (the hunter) and 50 km from the place the hunter left

3. In a parking lot, there are 458 vehicles; cars and motorcycles. Find the number of each type of vehicle knowing that in total there are 1764 wheels.

$$\left. \begin{array}{l} \left. \begin{array}{l} x = \text{cars} \\ y = \text{motorcycles} \end{array} \right\} \rightarrow \begin{array}{l} x + y = 458 \\ 4x + 2y = 1764 \end{array} \right\} \rightarrow \begin{array}{l} -2x - 2y = -916 \\ 4x + 2y = 1764 \end{array} \right\} \rightarrow 2x = 848$$

$$x = 424 \rightarrow y = 458 - 424 = 34$$

ANSWER: there are 424 cars and 34 motorcycles in the parking

4. Solve the simultaneous equation:

$$\left. \begin{array}{l} 2x + y = 1 \\ 2x^2 + y^2 = 3 \end{array} \right\} \rightarrow y = 1 - 2x \Rightarrow 2x^2 + (1 - 2x)^2 = 3 \rightarrow 2x^2 + 1 - 4x + 4x^2 = 3$$

$$6x^2 - 4x - 2 = 0 \rightarrow 3x^2 - 2x - 1 = 0 \rightarrow x = \frac{2 \pm \sqrt{4 + 12}}{6} = \left\langle \begin{array}{l} 1 \\ -\frac{1}{3} \end{array} \right\rangle$$

$$y = 1 - 2x = \begin{cases} 1 - 2 \cdot 1 = -1 \\ 1 - 2 \cdot \left(-\frac{1}{3}\right) = 1 + \frac{2}{3} = \frac{5}{3} \end{cases}$$

$$\text{SOLUTION: } x_1 = 1, y_1 = -1 \text{ and } x_1 = -\frac{1}{3}, y_1 = \frac{5}{3}$$

5. Classify these systems depending on the number of solutions. In order to solve them use the most convenient method in each one (and graphing):

a) $\left. \begin{array}{l} x - 2y = 7(1 - y) \\ 2(x + y) - 3(y - 2) = x - 6y \end{array} \right\} \rightarrow \left. \begin{array}{l} x - 2y = 7 - 7y \\ 2x + 2y - 3y + 6 = x - 6y \end{array} \right\} \text{by addition:}$

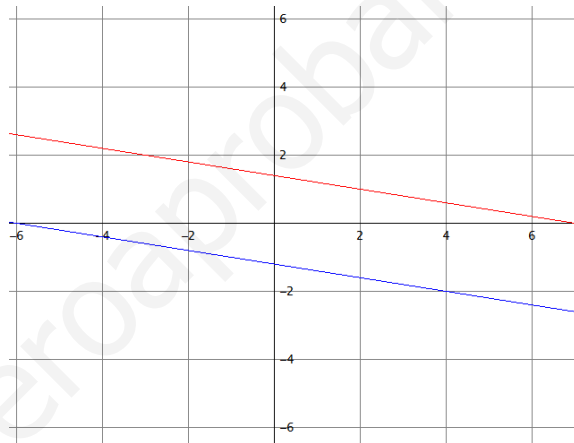
$$\left. \begin{array}{l} x + 5y = 7 \\ x + 5y = -6 \end{array} \right\} \rightarrow 0x = 1$$

No solution

By graphing:

$y = \frac{7-x}{5}$	x	2	7
	y	1	0

$y = \frac{-6-x}{5}$	x	-6	-1
	y	0	-1



PARALLEL LINES
INCONSISTENT SYSTEM

b) $\left. \begin{array}{l} 2x + \frac{3y}{4} = 5 \\ 5x - \frac{y}{2} = 3 \end{array} \right\} \rightarrow \left. \begin{array}{l} 8x + 3y = 20 \\ 10x - y = 6 \end{array} \right\} \rightarrow y = 10x - 6 \text{ by substitution}$

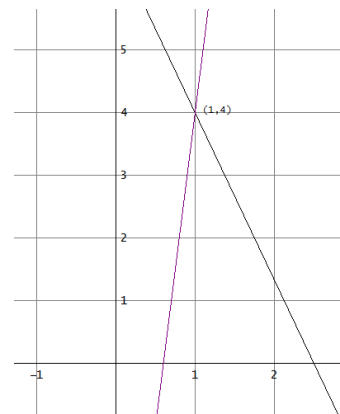
$$8x + 3(10x - 6) = 20 \rightarrow 8x + 30x - 18 = 20 \rightarrow 38x = 38 \rightarrow x = 1$$

$$y = 10x - 6 = 10 - 6 = 4 \text{ SOLUTION: } x = 1, y = 4$$

By graphing:

$y = \frac{20-8x}{3}$	x	1	-2
	y	4	12

$y = 10x - 6$	x	1	0
	y	4	-6



CONSISTENT INDEPENDENT SYSTEM