EXAM 2_4 (PROGRESSIONS)

Name:.....

Remember: in each question, write the steps you have taken to reach the solution.

1) For each of the following sequences, determine whether it is arithmetic or geometric. Find the tenth and twentieth terms, and the sum of the first 20 terms. (4 points)

a. 8, 3, -2, …

b. 27, 9, 3, …

c. 1, 11, 21, …

d. 4, -8, 16,…

2) The third term of a positive geometric progression is 36 and the fifth term is 16. Find the first term and the sum of all its terms. (1.5 points)

3) In an arithmetic progression the first term is 3 and the common difference is 2. The sum of n first term is 120, find n. (1.5 points)

4) A theatre has 60 seats in the first row, 68 seats in the second row, 76 seats in the third row, and so on in the same increasing pattern. If the theatre has 20 rows of seats, how many seats are in the theatre? (1.5 points)

5) The sixth and the tenth term of a geometric progression are 70 and 17920 respectively. Find the first term and the common ratio. (1.5 points)

SOLUTION

1) For each of the following sequences, determine whether it is arithmetic or geometric. Find the tenth and twentieth terms, and the sum of the first 20 terms.

a. 8, 3, -2, … AP $d=-5,\quad a_1=8\quad a_n=8+(n-1)(-5)$

$a_{10}=8+9(-5)=-37$, $a_{20}=8+19(-5)=-87$

$$S_{20}=\frac{(a_1+a_{20})20}{2}=(8-87)\cdot 10=-790$$

b. 27, 9, 3, … GP $r=\frac{1}{3}$, $a_1=27\rightarrow a_n=27\cdot\left(\frac{1}{3}\right)^{n-1}$

$$a_{10}=27\cdot\left(\frac{1}{3}\right)^{9}=729;\ a_{20}=27\cdot\left(\frac{1}{3}\right)^{19}=\frac{1}{3^{16}}$$

$$S_{20}=\frac{\frac{1}{3^{16}}\cdot\frac{1}{3}-27}{\frac{1}{3}-1}=40.4999....$$

c. 1, 11, 21, … AP $d=10,\quad a_1=1\quad a_n=1+(n-1)10$

$a_{10}=1+90=91$, $a_{20}=1+190=191$

$$S_{20}=\frac{(a_1+a_{20})20}{2}=(1+191)\cdot 10=1920$$

d. 4, -8, 16,… GP $r=-2$, $a_1=4\rightarrow a_n=4\cdot(-2)^{n-1}$

$a_{10}=4\cdot(-2)^9=-2048;\ a_{20}=4\cdot(-2)^{19}=-2097152$

$$S_{20}=\frac{-2097152\cdot(-2)-4}{-2-1}=-1398100$$

2) The third term of a positive geometric progression is 36 and the fifth term is 16. Find the first term and the sum of all its terms.

GP $a_3=36$, $a_5=16\rightarrow a_5=a_3\cdot r^2\rightarrow 16=36r^2\rightarrow r^2=\frac{16}{36}\rightarrow r=\frac{4}{6}=\frac{2}{3}$

$$a_3=a_1\cdot r^2\rightarrow 36=a_1\left(\frac{2}{3}\right)^2\rightarrow 36=\frac{4a_1}{9}\rightarrow\frac{36\cdot 9}{4}=a_1\rightarrow a_1=81;$$

$$S=\frac{a_1}{1-r}=\frac{81}{1-\frac{2}{3}}=\frac{81}{\frac{1}{3}}=243$$

3) In an arithmetic progression the first term is 3 and the common difference is 2. The sum of n first term is 120, find n.

$AP,\ d=2\quad a_1=3\quad S_n=\dfrac{(a_1+a_n)n}{2}\rightarrow 120=\dfrac{(3+a_n)n}{2}$

$a_n=3+(n-1)2=3+2n-2=2n+1$

$120=\dfrac{(3+2n+1)n}{2}\rightarrow 240=3n+2n^2+n\rightarrow 2n^2+4n-240=0\rightarrow n^2+2n-120=0$

$n=\dfrac{-2\pm\sqrt{4+480}}{2}=\dfrac{-2\pm 22}{2}=\begin{cases}10\\-12\ NO!\end{cases}$

Answer n is 10

4) A theatre has 60 seats in the first row, 68 seats in the second row, 76 seats in the third row, and so on in the same increasing pattern. If the theatre has 20 rows of seats, how many seats are in the theatre?

60, 68, 76,..... It is a AP, difference $d=8\quad a_1=60$

We have to calculate the sum of 20 terms

$S_{20}=\dfrac{(a_1+a_{20})20}{2}\quad$ with $a_{20}=60+19\cdot 8=212$

$S_{20}=\dfrac{(60+212)20}{2}=2720$

There are 2720 seats in the theatre

5) The sixth and the tenth term of a geometric progression are 70 and 17920 respectively. Find the first term and the common ratio.

GP

$a_6=70,\ a_{10}=17920\rightarrow$

$a_{10}=a_6\cdot r^4\rightarrow 17920=70r^4\rightarrow r^4=\dfrac{17920}{70}\rightarrow r^4=256\rightarrow r=4$

$a_6=a_1\cdot r^5\rightarrow 70=a_1\cdot 4^5\rightarrow a_1=\dfrac{70}{1024}=\dfrac{35}{512}$