

**2nd TERM GENERAL EXAM**

Name:

Remember: in each question, write the steps you have taken to reach the solution. (1 point each question)

1. Solve the simultaneous equation:
$$\left. \begin{array}{l} y - x = 8 \\ x^2 + y^2 = 34 \end{array} \right\}$$
2. A motorist buys 24 litres of petrol and 5 litres of oil for \$33.75, while another motorist buys 18 litres of petrol and 10 litres of oil for \$30. Find the cost of one litre of petrol and one litre of oil.
3. Solve the equation: $2x - \frac{15}{4} = \frac{6x+3}{4} - \frac{9-x}{2}$
4. A ball is dropped from a height of 10 m. Each time it strikes the ground it bounces vertically to a height that is $\frac{3}{4}$ of the preceding height. Find the total distance the ball will travel if it is assumed to bounce infinitely.
5. Work out and simplify: $3(2x - 1)^2 - 3x(4x - 3) - (1 + 2x)(1 - 2x) =$
6. Classify this system depending on the number of solutions. In order to solve it use the most convenient method:
$$\left. \begin{array}{l} 3x - 2(y - 1) = 5 \\ 2(x + 1) - y = 3 - (x - y) \end{array} \right\}$$
7. Solve the equation: $\frac{(x-2)(x+2)}{8} - \frac{x-6}{2} = \frac{10+x}{4}$
8. Find the sum of the 80 first multiples of 3.
9. How many pounds of coffee worth \$1.50 per pound should be mixed with 10 pounds of coffee worth \$2.00 per pound to obtain a mixture worth \$1.70 per pound?
10. The base of a rectangle is 5 cm more than its height. If the height is decreased by 2 cm, the area of the new rectangle is 60 cm². What were the dimensions of the original rectangle?

10. SOLUTION

$$1. \begin{cases} y - x = 8 \\ x^2 + y^2 = 34 \end{cases} \left\{ \begin{array}{l} y = 8 + x \\ x^2 + y^2 = 34 \end{array} \right. \Rightarrow x^2 + (8 + x)^2 = 34 \Rightarrow x^2 + 64 + 16x + x^2 = 34$$

$$2x^2 + 16x + 30 = 0 \Rightarrow x^2 + 8x + 15 = 0 \rightarrow x = \frac{-8 \pm \sqrt{64 - 60}}{2} = \frac{-8 \pm 2}{2} = \begin{cases} -3 \\ -5 \end{cases}$$

If $x = -3 \rightarrow y = 8 - 3 = 5$, If $x = -5 \rightarrow y = 8 - 5 = 3$

Solution: $x = -3, y = 5$ and $x = -5, y = 3$

2. A motorist buys 24 litres of petrol and 5 litres of oil for \$33.75, while another motorist buys 18 litres of petrol and 10 litres of oil for \$30. Find the cost of one litre of petrol and one litre of oil.

x - cost of petrol per litre, y - cost of oil per litre

$$\begin{cases} 24x + 5y = 33.75 \\ 18x + 10y = 30 \end{cases} \rightarrow y = \frac{30 - 18x}{10} \rightarrow 24x + 5 \cdot \frac{30 - 18x}{10} = 33.75$$

$$24x + 15 - 9x = 33.75 \Rightarrow 15x = 33.75 - 15 \Rightarrow 15x = 18.75 \Rightarrow x = 1.25$$

$$y = \frac{30 - 18 \cdot 1.25}{10} = 0.75$$

Answer: Petrol costs \$1.75 per litre and oil costs \$0.75 per litre

$$3. \text{ Solve the equation: } 2x - \frac{15}{4} = \frac{6x+3}{4} - \frac{9-x}{2} \rightarrow \frac{8x-15}{4} = \frac{6x+3}{4} - \frac{18-2x}{4}$$

$$8x - 15 = 6x + 3 - 18 + 2x \rightarrow 8x - 6x - 2x = 3 - 18 + 15 \rightarrow 0x = 0$$

It is an identity, infinite solutions

4. A ball is dropped from a height of 10 m. Each time it strikes the ground it bounces vertically to a height that is $\frac{3}{4}$ of the preceding height. Find the total distance the ball will travel if it is assumed to bounce infinitely.

$$1^{\text{st}} \text{ height } 10 \text{ m, } 2^{\text{nd}} \text{ height } \frac{3}{4} \text{ of } 10 = \frac{30}{4} = \frac{15}{2} \text{ m, } 3^{\text{rd}} \text{ height } \frac{3}{4} \text{ of } \frac{15}{2} = \frac{45}{8} \text{ m}$$

$$\text{It is a GP: } a_1 = 10, r = \frac{3}{4} \rightarrow S = \frac{a_1}{1-r} = \frac{10}{1-\frac{3}{4}} = \frac{10}{\frac{1}{4}} = 40 \text{ m}$$

$$5. 3(2x-1)^2 - 3x(4x-3) - (1+2x)(1-2x) = 3(4x^2 - 4x + 1) - 12x^2 + 9x - (1 - 4x^2) = 12x^2 - 12x + 3 - 12x^2 + 9x - 1 + 4x^2 = 4x^2 - 3x + 2$$

6. Classify this system depending on the number of solutions. In order to solve it

use the most convenient method:
$$\left. \begin{array}{l} 3x - 2(y - 1) = 5 \\ 2(x + 1) - y = 3 - (x - y) \end{array} \right\}$$

$$\left. \begin{array}{l} 3x - 2y + 2 = 5 \\ 2x + 2 - y = 3 - x + y \end{array} \right\} \rightarrow \left. \begin{array}{l} 3x - 2y = 3 \\ 3x - 2y = 1 \end{array} \right\} \rightarrow \text{Subtract} \rightarrow 0x = 2 \text{ No solution}$$

Inconsistent system

7. Solve the equation: $\frac{(x-2)(x+2)}{8} - \frac{x-6}{2} = \frac{10+x}{4} \rightarrow \frac{x^2-4}{8} - \frac{x-6}{2} = \frac{10+x}{4}$

$$\frac{x^2-4}{8} - \frac{4x-24}{8} = \frac{20+2x}{8} \rightarrow x^2 - 4 - 4x + 24 = 20 + 2x \rightarrow x^2 - 6x = 0$$

$$x^2 - 6x = 0 \rightarrow x(x-6) = 0 \rightarrow \begin{cases} x = 0 \\ x = 6 \end{cases}$$

8. Find the sum of the 80 first multiples of 3.

They are: 3, 6, 9, 12, it is a AP with common difference $d = 3$ and the first term is also 3.

$$a_{80} = a_1 + 79d = 3 + 79 \cdot 3 = 240 \rightarrow S_{80} = \frac{(3+240)80}{2} = 9720$$

9. How many pounds of coffee worth \$1.50 per pound should be mixed with 10 pounds of coffee worth \$2.00 per pound to obtain a mixture worth \$1.70 per pound?

	Coffee 1	Coffee 2	Mixture
Cost per pound	\$1.50	\$2.00	\$1.70
Pounds	x	10	x + 10

$$1.50x + 2.00 \cdot 10 = 1.70(x + 10) \rightarrow 1.50x + 20 = 1.70x + 17$$

$$0.20x = 3 \rightarrow x = \frac{3}{0.20} = 15$$

Answer: we have to put 15 pounds of the coffee 1

10. The base of a rectangle is 5 cm more than its height. If the height is decreased by 2 cm, the area of the new rectangle is 60 cm².

What were the dimensions of the original rectangle?

Height- x, Base- x + 5

$$\text{New area: } (x-2)(x+5) = 60 \rightarrow x^2 - 2x + 5x - 10 = 60 \rightarrow x^2 + 3x - 70 = 0$$

$$x = \frac{-3 \pm \sqrt{9+280}}{2} = \frac{-3 \pm 17}{2} = \begin{cases} 7 \\ -10 \text{ Negative, impossible!} \end{cases}$$

Dimensions: height 7 cm, base 12 cm