

EXAM 1_1 (Numbers/Percentages)

1) a) Which of the following numbers are rational numbers? (2 points)

$0.838338333\dots$; $0.8\bar{3}$; 0.83 ; 0.8383 ; $0.\bar{83}$

b) Arrange them in ascending order.

c) Express each rational number as a fraction.

d) Write three numbers between $-0.8\bar{3}$ and -0.83

2) Work out and simplify, writing the steps you have taken to reach the solution: (1.5 points)

a) $\frac{4}{3} \div \frac{2}{9} + 3^{-2} - \frac{9}{2} \div \frac{3}{7} =$

b) $\left(\frac{1}{2} - 1\right)^2 \cdot \frac{6}{5} - \frac{3}{2} \cdot \left(\frac{2}{5} - \frac{1}{2}\right) =$

3) Write the following expressions as a single positive power, writing the steps you have taken to reach the solution: (1.5 points)

a) $\left(\frac{1}{2}\right)^{-3} \cdot \frac{4^{-2} \cdot 12^3 \cdot 8^2}{6^3 \cdot 16^3} =$

b) $\left(\frac{3}{2}\right)^{-2} \cdot \left(\frac{9}{8}\right)^2 \cdot 12 =$

4) There were twelve bananas on the counter. One-sixth of the bananas were eaten yesterday. Three-fourths of the bananas were eaten today. What fraction of the bananas have been eaten in all? (1 point)

5) $\frac{3}{5}$ of the rabbits on the farm are grey, $\frac{2}{3}$ of the rest are white, and the remaining rabbits, 20, are black. How many rabbits are on the farm? (1.25 points)

6) Martha bought a watch at 80% of the regular price. She paid €33.6 for the watch. What was the regular price? (1 point)

7) Tommy paid €21 for a book he purchased from a book club. This was a saving of €7 off the regular price. What percent discount did the book club give Tommy? (1.25 points)

SOLUTION

1) a) Which of the following numbers are rational numbers?

$0.838338333\dots$; $0.8\overline{3}$; 0.83 ; 0.8383 ; $0.8\overline{\overline{3}}$

Rational numbers: $0.8\overline{3}$; 0.83 ; 0.8383 ; $0.8\overline{\overline{3}}$

b) Arrange them in ascending order.

$0.83 < 0.8\overline{3} < 0.8383 < 0.838338333\dots < 0.8\overline{\overline{3}}$

c) Express each rational number as a fraction.

$$0.8\overline{3} \rightarrow N = 0.83333\dots; 10N = 8.3333\dots; 100N = 83.3333\dots \Rightarrow 90N = 75 \rightarrow N = \frac{75}{90} = \frac{5}{6}$$

$$0.83 = \frac{83}{100}; \quad 0.8383 = \frac{8383}{10000}$$

$$0.8\overline{\overline{3}} \rightarrow N = 0.838383\dots; 100N = 83.838383\dots \Rightarrow 99N = 83 \Rightarrow N = \frac{83}{99}$$

d) Write three numbers between $-0.8\overline{3}$ and -0.83

$-0.8\overline{3} < -0.833 < -0.832 < -0.831 < -0.83$

2) Work out and simplify, writing the steps you have taken to reach the solution:

$$\begin{aligned} \text{a) } \frac{4}{3} \div \frac{2}{9} + 3^{-2} - \frac{9}{2} \div \frac{3}{7} &= \frac{4 \cdot 9}{3 \cdot 2} + \frac{1}{3^2} - \frac{9 \cdot 7}{2 \cdot 3} = \frac{2^2 \cdot 3^2}{2 \cdot 3} + \frac{1}{3^2} - \frac{3^2 \cdot 7}{2 \cdot 3} = 6 + \frac{1}{9} - \frac{21}{2} = \\ &= \frac{108}{18} + \frac{2}{18} - \frac{189}{18} = -\frac{79}{18} \end{aligned}$$

$$\begin{aligned} \text{b) } \left(\frac{1}{2} - 1\right)^2 \cdot \frac{6}{5} - \frac{3}{2} \cdot \left(\frac{2}{5} - \frac{1}{2}\right) &= \left(-\frac{1}{2}\right)^2 \cdot \frac{6}{5} - \frac{3}{2} \cdot \frac{4-5}{10} = \frac{1}{4} \cdot \frac{6}{5} - \frac{3}{2} \cdot \left(-\frac{1}{10}\right) = \\ &= \frac{6}{20} + \frac{3}{20} = \frac{9}{20} \end{aligned}$$

3) Write the following expressions as a single positive power, writing the steps you have taken to reach the solution:

a)

$$\left(\frac{1}{2}\right)^{-3} \cdot \frac{4^{-2} \cdot 12^3 \cdot 8^2}{6^3 \cdot 16^3} = 2^3 \cdot \frac{(2^2)^{-2} \cdot (2^2 \cdot 3)^3 \cdot (2^3)^2}{(2 \cdot 3)^3 \cdot (2^4)^3} = \frac{2^3 \cdot 2^{-4} \cdot 2^6 \cdot 3^3 \cdot 2^6}{2^3 \cdot 3^3 \cdot 2^{12}} = \frac{2^{11}}{2^{15}} = \left(\frac{1}{2}\right)^4$$

$$\text{b) } \left(\frac{3}{2}\right)^{-2} \cdot \left(\frac{9}{8}\right)^2 \cdot 12 = \left(\frac{2}{3}\right)^2 \cdot \left(\frac{3^2}{2^3}\right)^2 \cdot 2^2 \cdot 3 = \frac{2^2 \cdot 3^4 \cdot 2^2 \cdot 3}{3^2 \cdot 2^6} = \frac{2^4 \cdot 3^5}{3^2 \cdot 2^6} = \frac{3^3}{2^2}$$

4) There were twelve bananas on the counter. One-sixth of the bananas were eaten yesterday. Three-fourths of the bananas were eaten today. What fraction of the bananas have been eaten in all?

$$\text{The total is } \frac{1}{6} + \frac{3}{4} = \frac{2}{12} + \frac{9}{12} = \frac{11}{12}$$

Answer: $\frac{11}{12}$ of the bananas have been eaten

5) $\frac{3}{5}$ of the rabbits on the farm are grey, $\frac{2}{3}$ of the rest are white, and the remaining rabbits, 20, are black. How many rabbits are on the farm?

					3/5 are grey (grey)
					2/3 of the rest are white (yellow)
			10	10	Remaining 20: 10 each little square

Answer: The total is $15 \times 10 = 150$ rabbits

6) Martha bought a watch at 80% of the regular price. She paid €33.6 for the watch. What was the regular price?

$$\frac{33.6}{80} = \frac{x}{100} \Rightarrow x = \frac{33.6 \cdot 100}{80} = 42$$

Answer: The regular Price was €42

7) Tommy paid €21 for a book he purchased from a book club. This was a saving of €7 off the regular price. What percent discount did the book club give Tommy?

If Tommy paid €21, the regular price was €28, so

$$\frac{7}{x} = \frac{28}{100} \Rightarrow x = \frac{7 \cdot 100}{28} = 25$$

Answer: the percent of discount was 25%