

1.- a) (1,25 ptos) Racionalizar: $\frac{4}{\sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}}}$

b) (1.25 ptos) Calcular el valor de:

$$\log(\sqrt[5]{0,001}) + \ln \frac{1}{e} + \log_{\frac{1}{3}} \left(\sqrt{\sqrt{3}} \cdot \sqrt[5]{\sqrt{3}} : \sqrt[3]{27} \right)$$

2.- a) (1,25 ptos) Dada $A = \frac{16 \cdot \sqrt[7]{x^5}}{y^3 \cdot \sqrt[4]{8z^3}}$ calcular $\log_2 A$

b) (1,25 ptos) Sabiendo que $\log_3 M = a$ calcular:

$$\log_3 \frac{M}{27} - \log_3 \frac{81}{\sqrt[4]{M^3}} + \log_3 (243M)$$

3.- a) (1,25 ptos) Desarrollar la expresión: $3 - |x + 1| + |5 - 2x| =$

b) (1,25 ptos) Representa en la recta real $\sqrt{40}$

4.- a) (1,25 ptos) Simplifica: $\frac{(-20)^4 \cdot 21^{-2} \cdot 14}{(-12)^{-3} \cdot 100^2} =$

b) (1,25 ptos) Calcular: $\sqrt{\frac{a}{2b}} \cdot \left(\sqrt[3]{\frac{4b^2}{a^2}} : \sqrt{\frac{2b}{a}} \right)$

1.- a) (1,25 ptos) Racionalizar: $\frac{4}{\sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}}}$

$$\begin{aligned} \frac{4}{\sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}}} &= \frac{4(\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}})}{(\sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}})(\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}})} = \\ &= \frac{4(\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}})}{(\sqrt{4+2\sqrt{3}})^2 - (\sqrt{4-2\sqrt{3}})^2} = \frac{4 \cdot (\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}})}{4 \cdot \sqrt{3}} = \\ &= \frac{\sqrt{3}(\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}})}{\sqrt{3} \cdot \sqrt{3}} = \boxed{\frac{\sqrt{12+6\sqrt{3}} + \sqrt{12-6\sqrt{3}}}{3}} \end{aligned}$$

b) (1.25 ptos) Calcular el valor de:

$$\underbrace{\log(\sqrt[5]{0,001})}_{(1)} + \underbrace{\ln \frac{1}{e}}_{(2)} + \underbrace{\log_{\frac{1}{3}}\left(\sqrt{\sqrt{3}} \cdot \sqrt[5]{\sqrt{3} : \sqrt[3]{27}}}\right)}_{(3)} = (*)$$

$$(1) \log(\sqrt[5]{0,001}) = \log \sqrt[5]{10^{-3}} = \log 10^{-3/5} = -\frac{3}{5}$$

$$(2) \ln \frac{1}{e} = \ln e^{-1} = -1$$

$$\begin{aligned} (3) \log_{\frac{1}{3}}\left(\sqrt{\sqrt{3}} \cdot \sqrt[5]{\sqrt{3} : \sqrt[3]{27}}}\right) &= \log_{\frac{1}{3}}\left(\sqrt[4]{3} \cdot \sqrt[5]{\sqrt{3} : \sqrt[3]{3^3}}}\right) = \\ &= \log_{\frac{1}{3}}\left(3^{1/4} \cdot \sqrt[5]{3^{1/2} : 3}\right) = \log_{\frac{1}{3}}\left(3^{1/4} \cdot \left(3^{-1/2}\right)^{1/5}\right) = \end{aligned}$$

$$= \log_{\frac{1}{3}} \left(3^{\frac{1}{4}} \cdot 3^{-\frac{1}{10}} \right) = \log_{\frac{1}{3}} 3^{\frac{3}{20}} = \log_{\frac{1}{3}} \left(\frac{1}{3} \right)^{-\frac{3}{20}} = -\frac{3}{20}$$

$$\underline{\underline{(*)}} \quad -\frac{3}{5} + (-1) + \left(-\frac{3}{20}\right) = \frac{-12-20-3}{20} = \frac{-35}{20} = \frac{-7}{4}$$

2.- a) (1,25 ptos) Dada $A = \frac{16 \cdot \sqrt[7]{x^5}}{y^3 \cdot \sqrt[4]{8z^3}}$ calcular $\log_2 A$

$$\begin{aligned} \log_2 A &= \log_2 \frac{16 \cdot \sqrt[7]{x^5}}{y^3 \cdot \sqrt[4]{8z^3}} = \log_2 (16 \cdot x^{\frac{5}{7}}) - \log_2 (y^3 \cdot (2^3 \cdot z^3)^{\frac{1}{4}}) \\ &= \log_2 2^4 + \log_2 x^{\frac{5}{7}} - \left(\log_2 y^3 + \log_2 (2^3 \cdot z^3)^{\frac{1}{4}} \right) = \\ &= 4 + \frac{5}{7} \log_2 x - 3 \log_2 y - \frac{1}{4} \log_2 (2^3 \cdot z^3) = \\ &= 4 + \frac{5}{7} \log_2 x - 3 \log_2 y - \frac{1}{4} (\log_2 2^3 + \log_2 z^3) = \\ &= 4 + \frac{5}{7} \log_2 x - 3 \log_2 y - \frac{1}{4} (3 + 3 \log_2 z) = \\ &= 4 + \frac{5}{7} \log_2 x - 3 \log_2 y - \frac{3}{4} - \frac{3}{4} \log_2 z = \\ &= \boxed{\frac{13}{4} + \frac{5}{7} \log_2 x - 3 \log_2 y - \frac{3}{4} \log_2 z} \end{aligned}$$

b) (1,25 ptos) Sabiendo que $\log_3 M = a$ calcular:

$$\log_3 \frac{M}{27} - \log_3 \frac{81}{\sqrt[4]{M^3}} + \log_3 (243M) = (*)$$

(1) (2) (3)

$$(1) \log_3 \frac{M}{27} = \log_3 M - \log_3 3^3 = a - 3$$

$$(2) \log_3 \frac{81}{\sqrt[4]{M^3}} = \log_3 3^4 - \log_3 M^{3/4} = 4 - \frac{3}{4} \log_3 M =$$
$$= 4 - \frac{3}{4} a$$

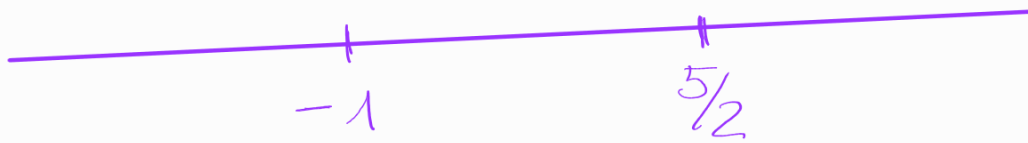
$$(3) \log_3 (243M) = \log_3 3^5 + \log_3 M = 5 + a$$

$$\underline{(*)} \quad a - 3 - \left(4 - \frac{3}{4} a\right) + 5 + a =$$
$$= a - 3 - 4 + \frac{3}{4} a + 5 + a =$$
$$= \boxed{-2 + \frac{11}{4} a}$$

3.- a) (1,25 ptos) Desarrollar la expresión: $3 - |x + 1| + |5 - 2x| = (*)$

$$|x+1| = \begin{cases} \underline{x+1} & \text{si } x+1 \geq 0 \rightarrow \underline{x \geq -1} \\ \underline{-x-1} & \text{si } x+1 < 0 \rightarrow \underline{x < -1} \end{cases}$$

$$|5-2x| = \begin{cases} \underline{5-2x} & \text{si } 5-2x \geq 0 \Rightarrow 5 \geq 2x \Rightarrow \underline{\frac{5}{2} \geq x \Rightarrow x \leq \frac{5}{2}} \\ \underline{-5+2x} & \text{si } 5-2x < 0 \Rightarrow 5 < 2x \Rightarrow \underline{\frac{5}{2} < x \Rightarrow x > \frac{5}{2}} \end{cases}$$



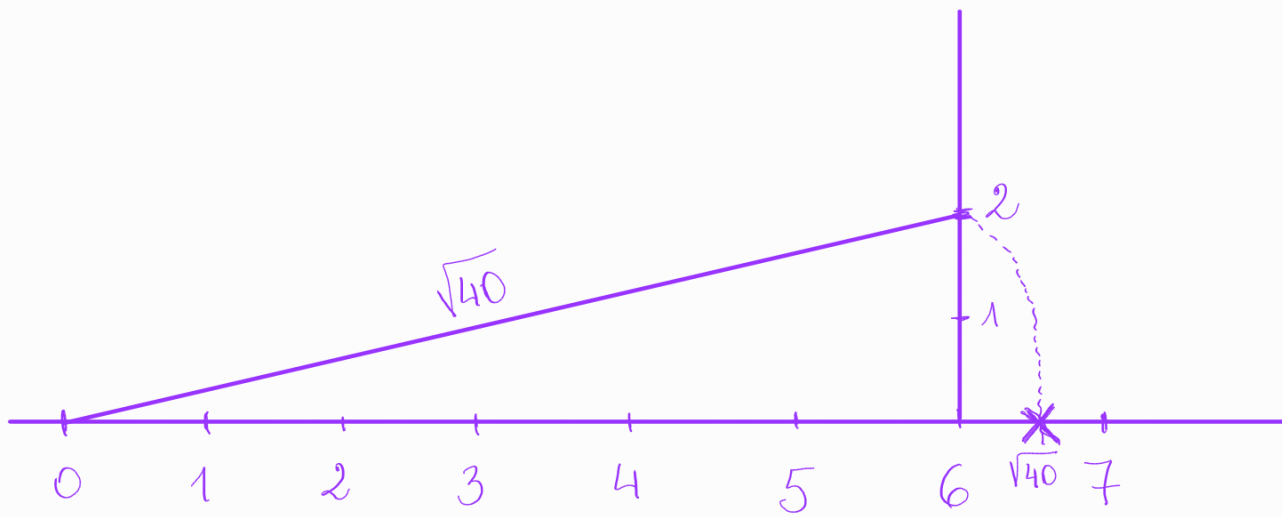
$$(*) = \begin{cases} \text{si } x < -1 : 3 - (-x-1) + (5-2x) \\ \text{si } -1 \leq x \leq \frac{5}{2} : 3 - (x+1) + (5-2x) \\ \text{si } x > \frac{5}{2} : 3 - (x+1) + (-5+2x) \end{cases} =$$

$$= \begin{cases} \text{si } x < -1 : 3 + x + 1 + 5 - 2x \\ \text{si } -1 \leq x \leq \frac{5}{2} : 3 - x - 1 + 5 - 2x \\ \text{si } x > \frac{5}{2} : 3 - x - 1 - 5 + 2x \end{cases} =$$

$$= \begin{cases} -x + 9 & \text{si } x < -1 \\ -3x + 7 & \text{si } -1 \leq x \leq \frac{5}{2} \\ x - 3 & \text{si } x > \frac{5}{2} \end{cases}$$

b) (1,25 ptos) Representa en la recta real $\sqrt{40}$

$$(\sqrt{40})^2 = 6^2 + 2^2 \quad (\text{T}^\circ \text{ Pitágoras}) \quad 6 \leq \sqrt{40} \leq 7$$



4.- a) (1,25 ptos) Simplifica: $\frac{(-20)^4 \cdot 21^{-2} \cdot 14}{(-12)^{-3} \cdot 100^2} = (*)$

$$(-20)^4 = 20^4 \text{ (exponente par)}$$

$$(-12)^3 = -12^3 \text{ (exponente impar)}$$

$$(*) = \frac{20^4 \cdot 14 \cdot (-12)^3}{21^2 \cdot 100^2} = \frac{(2^2 \cdot 5)^4 \cdot 2 \cdot 7 \cdot (- (2^2 \cdot 3)^3)}{(3 \cdot 7)^2 \cdot (2^2 \cdot 5^2)^2} =$$

$$= \frac{- 2^8 \cdot \cancel{5^4} \cdot 2 \cdot 7 \cdot 2^6 \cdot 3^3}{3^2 \cdot 7^2 \cdot 2^4 \cdot \cancel{5^4}} = - \frac{2^{15} \cdot 7 \cdot 3^3}{3^2 \cdot 7^2 \cdot 2^4} =$$

$$= \boxed{- 2^{11} \cdot 7^{-1} \cdot 3} = \boxed{- \frac{2^{11} \cdot 3}{7}}$$

b) (1,25 ptos) Calcular: $\sqrt{\frac{a}{2b}} \cdot \left(\sqrt[3]{\frac{4b^2}{a^2}} : \sqrt{\frac{2b}{a}} \right) =$

$$= \sqrt{\frac{a}{2b}} \cdot \left(\sqrt[6]{\frac{4^2 \cdot b^4}{a^4}} \div \sqrt[6]{\frac{2^3 b^3}{a^3}} \right) = \sqrt{\frac{a}{2b}} \cdot \sqrt[6]{\frac{4^2 \cdot b^4 \cdot a^3}{a^4 \cdot 2^3 \cdot b^3}} =$$

$$= \sqrt[6]{\frac{a^3}{2^3 \cdot b^3}} \cdot \sqrt[6]{\frac{2^4 \cdot b^4 \cdot a^3}{a^4 \cdot 2^3 \cdot b^3}} = \sqrt[6]{\frac{a^3 \cdot 2^4 \cdot b^4 \cdot a^3}{2^3 \cdot b^3 \cdot a^4 \cdot 2^3 \cdot b^3}} =$$

$$= \sqrt[6]{\frac{2^4 \cdot a^6 \cdot b^4}{2^6 \cdot a^4 \cdot b^6}} = \sqrt[6]{\frac{a^2}{2^2 \cdot b^2}} = \sqrt[6]{\left(\frac{a}{2b}\right)^2} = \sqrt[3]{\frac{a}{2b}} =$$

$$= \frac{\sqrt[3]{a}}{\sqrt[3]{2b}} = \frac{\sqrt[3]{a} \sqrt[3]{2^2 b^2}}{\sqrt[3]{2b} \cdot \sqrt[3]{2^2 b^2}} = \frac{\sqrt[3]{4ab^2}}{\sqrt[3]{2^3 b^3}} = \boxed{\frac{\sqrt[3]{4ab^2}}{2b}}$$