

1.- a) (1,25 puntos) Desarrollar por el binomio de Newton: $\left(3x^2y^3 - \frac{2x^3}{y^2}\right)^5 =$

b) (1,25 puntos) Hallar el lugar que ocupa el término de grado 84 en el desarrollo del binomio $(3x^3 + 2x^5)^{20}$

2.- a) (1,25 puntos) Resolver la ecuación: $\binom{x}{3} = 2 \binom{x}{1}$

b) (1,25 puntos) Calcular, dejando el resultado final en forma de "entorno":

$$\{x \in \mathbb{R} / |x+3| \leq 2\} \cap E[2, 5]$$

3.- a) (1,25 puntos) Dada $A = \frac{81 \cdot \sqrt[7]{x^5}}{y^3 \cdot \sqrt[4]{27z^3}}$ calcular $\log_3 A$

b) (1,25 puntos) Sabiendo que $\log_2 V = m$ calcular:

$$\log_2 \frac{V}{8} - \log_2 \frac{16}{\sqrt[4]{V^3}} + \log_2 (32V)$$

4.- a) (1,25 puntos) Desarrollar la expresión: $3 - |x + 2| - |2 - 3x| =$

b) a) (1,25 puntos) Simplificar la expresión: $\frac{x^2 - xy}{x^3y - xy^3} : \frac{y}{x^2 + 2xy + y^2} =$

$$\begin{aligned}
 \textcircled{1} a) \left(3x^2y^3 - \frac{2x^3}{y^2} \right)^5 &= \binom{5}{0} (3x^2y^3)^5 - \binom{5}{1} (3x^2y^3)^4 \cdot \frac{2x^3}{y^2} + \\
 &+ \binom{5}{2} (3x^2y^3)^3 \cdot \left(\frac{2x^3}{y^2} \right)^2 - \binom{5}{3} (3x^2y^3)^2 \cdot \left(\frac{2x^3}{y^2} \right)^3 + \\
 &+ \binom{5}{4} 3x^2y^3 \cdot \left(\frac{2x^3}{y^2} \right)^4 - \binom{5}{5} \left(\frac{2x^3}{y^2} \right)^5 = \\
 &= 1 \cdot 3^5 \cdot x^{10} \cdot y^{15} - 5 \cdot 3^4 \cdot x^8 y^{12} \cdot \frac{2x^3}{y^2} + 10 \cdot 3^3 x^6 y^9 \cdot \frac{2^2 x^6}{y^4} \\
 &- 10 \cdot 3^2 \cdot x^4 y^6 \cdot \frac{2^3 x^9}{y^6} + 5 \cdot 3 x^2 y^3 \cdot \frac{2^4 x^{12}}{y^8} - 1 \cdot \frac{2^5 x^{15}}{y^{10}} = \\
 &= \left\{ 243 x^{10} y^{15} - 810 x^{11} y^{10} + 1080 x^{12} y^5 - 720 x^{13} + \right. \\
 &\quad \left. + 240 \frac{x^{14}}{y^5} - 32 \frac{x^{15}}{y^{10}} \right\}
 \end{aligned}$$

$$\begin{aligned}
 b) T_k \text{ de grado } 84 \text{ en } (3x^3 + 2x^5)^{20} \\
 T_k &= \binom{20}{k-1} (3x^3)^{20-(k-1)} \cdot (2x^5)^{k-1} = \\
 &= \binom{20}{k-1} 3^{21-k} (x^3)^{21-k} \cdot 2^{k-1} \cdot (x^5)^{k-1} \Rightarrow \\
 &\Rightarrow X^{63-3k} \cdot X^{5k-5} = X^{84} \Rightarrow \\
 &\Rightarrow X^{58+2k} = X^{84} \Rightarrow 58+2k=84 \Rightarrow \\
 &\Rightarrow 2k=84-58 \Rightarrow 2k=26 \Rightarrow \boxed{k=13}
 \end{aligned}$$

$$\textcircled{2} \text{ a) } \binom{x}{3} = 2 \binom{x}{1} \Rightarrow \frac{x!}{3! (x-3)!} = 2 \cdot \frac{x!}{1! (x-1)!} \Rightarrow$$

$$\Rightarrow \frac{x \cdot (x-1) \cdot (x-2) \cdot \cancel{(x-3)!}}{3 \cdot 2 \cdot 1 \cdot \cancel{(x-3)!}} = \frac{2 \cdot x \cdot \cancel{(x-1)!}}{1 \cdot \cancel{(x-1)!}}$$

Multiplíco en cruz igualando:

$$x \cdot (x-1) \cdot (x-2) = 12x$$

$$x \cdot (x-1) \cdot (x-2) - 12x = 0$$

$$x \cdot ((x-1) \cdot (x-2) - 12) = 0$$

$$x \cdot (x^2 - 3x + 2 - 12) = 0$$

$$x \cdot (x^2 - 3x - 10) = 0$$

$$\begin{cases} \underline{x=0} \\ x^2 - 3x - 10 = 0 \end{cases} \left| \begin{array}{l} \textcircled{+} 3 \\ \textcircled{-} -10 \end{array} \right. \Rightarrow \begin{cases} \underline{x=5} \\ \underline{x=-2} \end{cases}$$

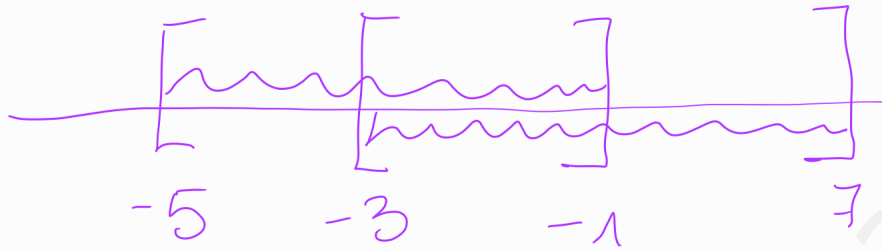
Por tener $\binom{x}{3} \Rightarrow \underline{x \geq 3} \Rightarrow \begin{cases} x=0 \\ x=-2 \end{cases} \underline{\text{NO VALEN}}$

Solución: $\boxed{x=5}$

$$\text{b) } \{x \in \mathbb{R} / |x+3| \leq 2\} \cap E[2,5]$$

$$|x+3| \leq 2 \Leftrightarrow E[-3, 2] = [-3-2, -3+2] = [-5, -1]$$

$$E[2, 5] = [2-5, 2+5] = [-3, 7]$$



$$[-5, -1] \cap [-3, 7] = [-3, -1]$$

$$\text{Centro} = \frac{-3 + (-1)}{2} = \frac{-4}{2} = -2$$

$$\text{Radio} = \frac{(-1) - (-3)}{2} = \frac{-1+3}{2} = \frac{2}{2} = 1$$

$$\text{Solución } \boxed{E[-2, 1]}$$

$$(3) a) A = \frac{81 \cdot \sqrt[7]{x^5}}{y^3 \cdot \sqrt[4]{27z^3}} \quad \text{¿} \log_3 A \text{?}$$

$$\log_3 A = \log_3 \left(\frac{81 \cdot x^{5/7}}{y^3 \cdot 3^{3/4} \cdot z^{3/4}} \right) =$$

$$\begin{aligned}
 &= \log_3 (81 \cdot X^{5/7}) - \log_3 (y^3 \cdot 3^{3/4} \cdot z^{3/4}) = \\
 &= \log_3 81 + \log_3 X^{5/7} - [\log_3 y^3 + \log_3 3^{3/4} + \log_3 z^{3/4}] = \\
 &= \log_3 3^4 + \frac{5}{7} \log_3 X - 3 \log_3 y - \frac{3}{4} \log_3 3 - \frac{3}{4} \log_3 z = \\
 &= 4 \log_3 3 + \frac{5}{7} \log_3 X - 3 \log_3 y - \frac{3}{4} \cdot 1 - \frac{3}{4} \log_3 z = \\
 &= (4 \cdot 1) + \frac{5}{7} \log_3 X - 3 \log_3 y - \frac{3}{4} - \frac{3}{4} \log_3 z = \\
 &= \boxed{\frac{13}{4} + \frac{5}{7} \log_3 X - 3 \log_3 y - \frac{3}{4} \log_3 z}
 \end{aligned}$$

b) Si $\log_2 V = m$

$$\begin{aligned}
 &\log_2 \frac{V}{8} - \log_2 \frac{16}{\sqrt[4]{V^3}} + \log_2 (32V) = \\
 &= \log_2 V - \log_2 8 - (\log_2 16 - \log_2 V^{3/4}) + \log_2 32 + \log_2 V \\
 &\stackrel{(*)}{=} m - 3 - (4 - \frac{3}{4} \log_2 V) + 5 + m = \\
 &= m - 3 - 4 + \frac{3}{4} \cdot m + 5 + m = 2m + \frac{3}{4}m - 2 =
 \end{aligned}$$

$$= \boxed{\frac{11}{4}m - 2}$$

$$* \begin{cases} \log_2 8 = \log_2 2^3 = 3 \log_2 2 = 3 \cdot 1 = 3 \\ \log_2 16 = \log_2 2^4 = 4 \log_2 2 = 4 \cdot 1 = 4 \\ \log_2 32 = \log_2 2^5 = 5 \log_2 2 = 5 \cdot 1 = 5 \end{cases}$$

$$(4) a) 3 - |x+2| - |2-3x| = (*)$$

$$|x+2| = \begin{cases} x+2 & \text{si } x+2 \geq 0 \Rightarrow x \geq -2 \\ -x-2 & \text{si } x+2 < 0 \Rightarrow x < -2 \end{cases}$$

$$|2-3x| = \begin{cases} 2-3x & \text{si } 2-3x \geq 0 \Rightarrow 2 \geq 3x \Rightarrow x \leq \frac{2}{3} \\ -2+3x & \text{si } 2-3x < 0 \Rightarrow 2 < 3x \Rightarrow x > \frac{2}{3} \end{cases}$$



$$* \begin{cases} 3 - (-x-2) - (2-3x) & \text{si } x < -2 \\ 3 - (x+2) - (2-3x) & \text{si } -2 \leq x \leq \frac{2}{3} \\ 3 - (x+2) - (-2+3x) & \text{si } x > \frac{2}{3} \end{cases} =$$

$$= \begin{cases} 3+4x & \text{si } x < -2 \\ -1+2x & \text{si } -2 \leq x \leq \frac{2}{3} \\ 3-4x & \text{si } x > \frac{2}{3} \end{cases}$$

$$b) \frac{x^2 - xy}{x^3y - xy^3} \circ \frac{y}{x^2 + 2xy + y^2} = \frac{x(x-y)}{xy(x^2 - y^2)} \circ \frac{y}{(x+y)^2}$$

$$= \frac{\cancel{x} \cdot \cancel{(x-y)} \cdot (x+y)}{\cancel{x} \cdot y \cdot \frac{\cancel{(x+y)} \cancel{(x-y)}}{(x^2 - y^2)} \cdot y} = \boxed{\frac{x+y}{y^2}}$$

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