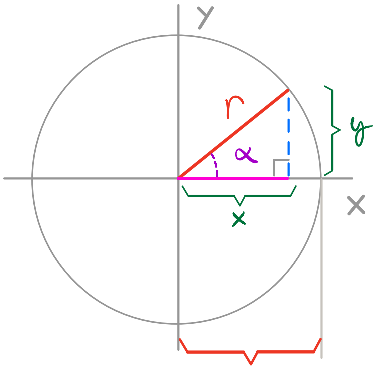
FÓRMULAS DE TRIGONOMETRÍA

Razones (divisiones) trigonométricas de una circunferencia. Definiciones.

Definimos las razones a partir de un triángulo rectángulo inscrito en la circunferencia.



r = radio = hipotenusa

seno $\equiv \dfrac{\text{cateto opuesto}}{\text{hipotenusa}} \Rightarrow \operatorname{sen}\alpha = \dfrac{y}{r}$

coseno $\equiv \dfrac{\text{cateto contiguo}}{\text{hipotenusa}} \Rightarrow \cos\alpha = \dfrac{x}{r}$

tangente $\equiv \dfrac{\text{cateto opuesto}}{\text{cateto contiguo}} \Rightarrow \operatorname{tg}\alpha = \dfrac{y}{x}$

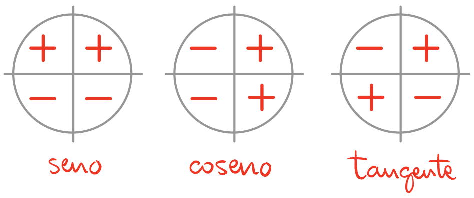
$\operatorname{cosec}\alpha = \dfrac{1}{\operatorname{sen}\alpha} = \dfrac{r}{y}$

$\sec\alpha = \dfrac{1}{\cos\alpha} = \dfrac{r}{x}$

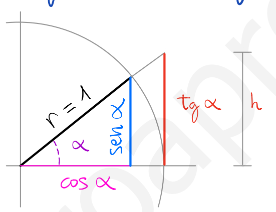
$\operatorname{cotg}\alpha = \dfrac{1}{\operatorname{tg}\alpha} = \dfrac{x}{y}$

Dominio: $\operatorname{sen}\alpha,\ \cos\alpha \in [-1,1]$ $\quad \operatorname{tg}\alpha \in [-\infty,+\infty] = \mathbb{R}$

El coseno es la "base" y el seno la "altura" del triángulo. La tangente es la "pendiente".



Signos por cuadrante



π rad $= 180^\circ$

Factor de conversión radianes – grados

$\operatorname{tg}\alpha = \dfrac{\operatorname{sen}\alpha}{\cos\alpha}$; $\operatorname{cosec}\alpha = \dfrac{1}{\operatorname{sen}\alpha}$; $\sec\alpha = \dfrac{1}{\cos\alpha}$; $\operatorname{cotg}\alpha = \dfrac{1}{\operatorname{tg}\alpha}$

$$\cos^2\alpha + \operatorname{sen}^2\alpha = 1$$

$$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \operatorname{sen}\alpha\cdot\operatorname{sen}\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \operatorname{sen}\alpha\cdot\operatorname{sen}\beta$$

$$\operatorname{sen}(\alpha+\beta) = \operatorname{sen}\alpha\cdot\cos\beta + \cos\alpha\cdot\operatorname{sen}\beta$$

$$\operatorname{sen}(\alpha-\beta) = \operatorname{sen}\alpha\cdot\cos\beta - \cos\alpha\cdot\operatorname{sen}\beta$$

$$\operatorname{sen}2\alpha = 2\operatorname{sen}\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha$$

$$\operatorname{sen}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

TRIÁNGULOS

Teorema del Coseno

$$a^2 = b^2 + c^2 - 2\cdot b\cdot c\cdot\cos\hat{A}$$

$$b^2 = a^2 + c^2 - 2\cdot a\cdot c\cdot\cos\hat{B}$$

$$c^2 = a^2 + b^2 - 2\cdot a\cdot b\cdot\cos\hat{C}$$

Teorema de los senos

$$\frac{a}{\operatorname{sen}\hat{A}} = \frac{b}{\operatorname{sen}\hat{B}} = \frac{c}{\operatorname{sen}\hat{C}}$$

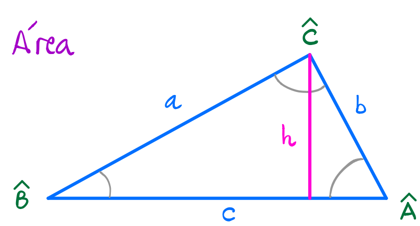
Área

$$A = \frac{1}{2}\cdot c\cdot b\cdot\operatorname{sen}\hat{A}$$

$$A = \frac{1}{2}\cdot c\cdot a\cdot\operatorname{sen}\hat{B}$$

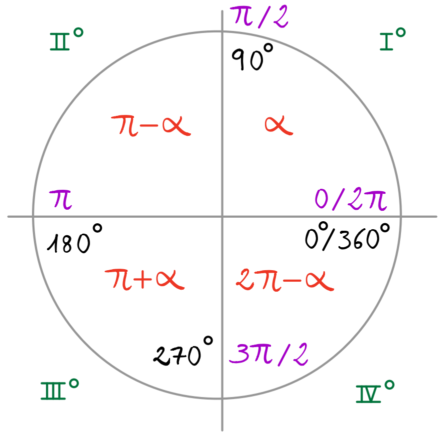
$$A = \frac{1}{2}\cdot a\cdot b\cdot\operatorname{sen}\hat{C}$$

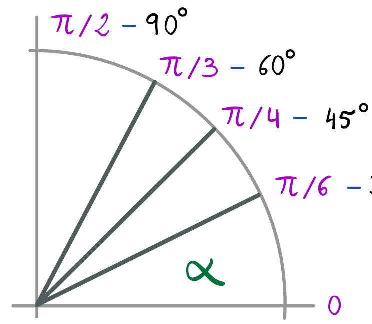
$$\hat{A}+\hat{B}+\hat{C} = 180^\circ$$



Circunferencia goniométrica (r=1)
Cuadrantes y reducción al 1er cuadrante

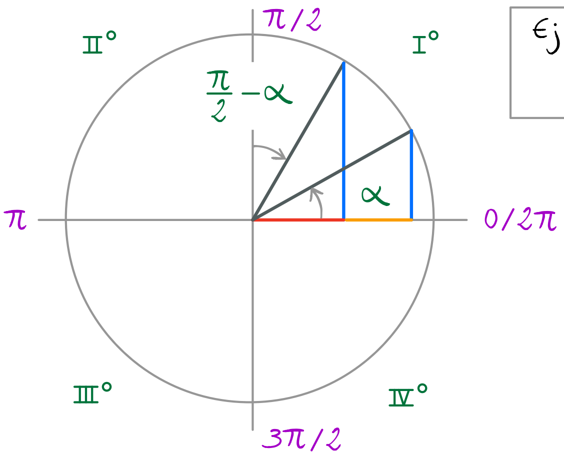
	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sen α	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos α	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tg α	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞





I° Ángulos complementarios $\boxed{\alpha,\ \frac{\pi}{2}-\alpha}$

Ej: $90^\circ - 30^\circ = 60^\circ$
$\frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

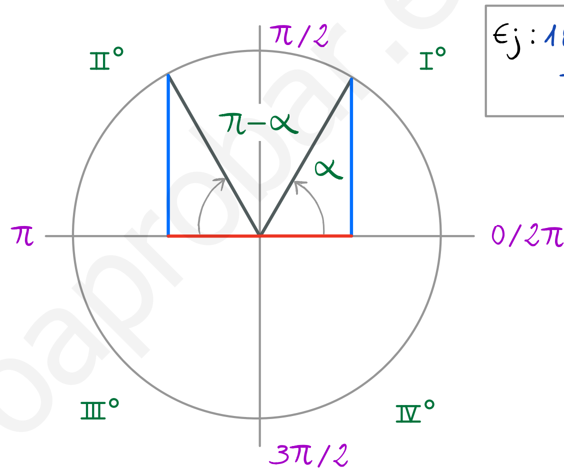


$$\text{sen}\,\alpha = \cos\left(\frac{\pi}{2}-\alpha\right) \ ; \ \cos\alpha = \text{sen}\left(\frac{\pi}{2}-\alpha\right)$$

$$\text{tg}\,\alpha = \frac{1}{\text{tg}\left(\frac{\pi}{2}-\alpha\right)}$$

II° Ángulos suplementarios $\boxed{\alpha,\ \pi-\alpha}$ $\boxed{\text{I}^\circ\ \ \pi-\alpha}$

Ej: $180^\circ - 30^\circ = 150^\circ$
$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

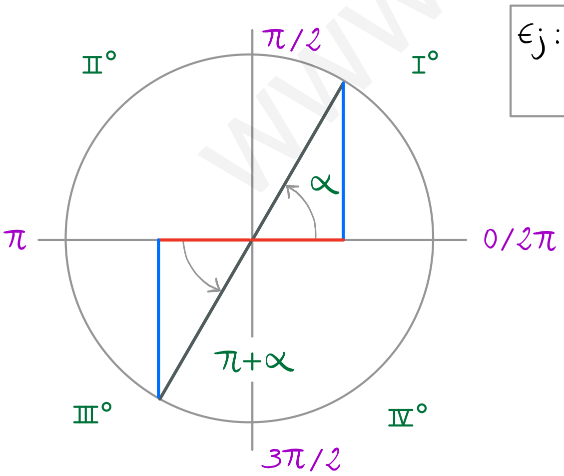


$$\text{sen}\,\alpha = \text{sen}(\pi-\alpha) \ ; \ \cos\alpha = -\cos(\pi-\alpha)$$

$$\text{tg}\,\alpha = -\text{tg}(\pi-\alpha)$$

III° Ángulos del 3er cuadrante $\boxed{\alpha,\ \pi+\alpha}$ $\boxed{\text{I}^\circ\ \ \alpha-\pi}$

Ej: $180^\circ + 30^\circ = 210^\circ$
$\pi + \frac{\pi}{6} = \frac{7\pi}{6}$

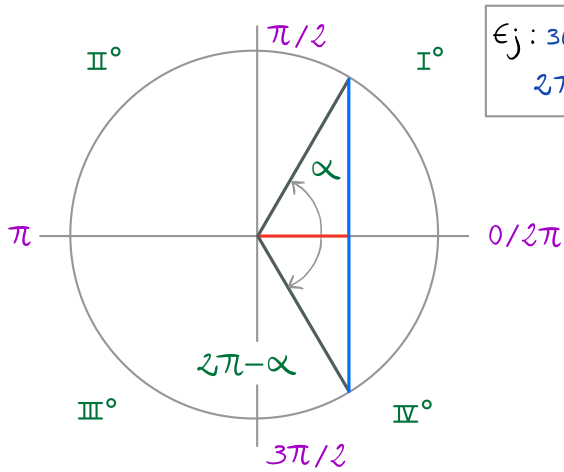


$$\text{sen}\,\alpha = -\text{sen}(\pi+\alpha) \ ; \ \cos\alpha = -\cos(\pi+\alpha)$$

$$\text{tg}\,\alpha = \text{tg}(\pi+\alpha)$$

IV° Ángulos opuestos $\boxed{\alpha,\ 2\pi-\alpha}$ $\boxed{\text{I}^\circ\ 2\pi-\alpha}$

Ej: $360^\circ - 30^\circ = 330^\circ$
$2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$



$$\text{sen}\,\alpha = -\text{sen}(2\pi-\alpha) \ ; \ \cos\alpha = \cos(2\pi-\alpha)$$

$$\text{tg}\,\alpha = -\text{tg}(2\pi-\alpha)$$