1.- (3 puntos)

a) Desarrollar la expresión de tg 3a dejándola en función de tg a

b) Sea a un ángulo del II cuadrante y sen a = 3/5, calcular las razones de $\frac{a}{2}$ (seno, coseno y tangente)

2.- (2 puntos)

a) Resolver la ecuación: $2\,\text{sen}^2 x + \cos 2x = 4\cos^2 x$

b) Calcular en función de h la sec 203°, siendo cotg 67° = h

3.- (3 puntos)

a) Resuelve el triángulo y calcula su área si se conocen A = 80°, a = 10 m y b = 5 m

b) Calcular $\sqrt[5]{\dfrac{-1-\sqrt{3}i}{-3+\sqrt{3}i}}$

4.- (2 puntos)

a) Dado el vector u = (-10, 12), hallar sus coordenadas respecto de la base B = {v, w} donde v = (3, -4) y w = (1, -1)

b) Dados los complejos $z_1 = 2_{60°}$, $z_2 = -1 + i$ y $z_3 = 2(\cos 210° + i\,\text{sen}\,210°)$ calcular

$$\frac{z_1 \cdot \overline{z_2}}{z_3}$$ en forma binómica

① a) $tg\,3a = tg\,(2a+a) = \dfrac{tg\,2a + tg\,a}{1 + tg\,2a \cdot tg\,a} = \dfrac{\dfrac{2tg\,a}{1+tg^2a} + tg\,a}{1 + \dfrac{2tg\,a}{1+tg^2a}\cdot tg\,a} =$

$$= \dfrac{\dfrac{2tg\,a + tg\,a + tg^3a}{\cancel{1+tg^2a}}}{\dfrac{1+tg^2a + 2tg^2a}{\cancel{1+tg^2a}}} = \boxed{\dfrac{3tg\,a + tg^3a}{1+3tg^2a}}$$

b) $\alpha \in II$, $sen\,\alpha = \dfrac{3}{5} \Rightarrow sen^2\alpha + cos^2\alpha = 1 \Rightarrow \dfrac{9}{25} + cos^2\alpha = 1 \Rightarrow$

$cos^2\alpha = 1 - \dfrac{9}{25} \Rightarrow cos^2\alpha = \dfrac{16}{25} \Rightarrow \left.\begin{array}{l} cos\,\alpha = \pm\sqrt{\dfrac{16}{25}} \\ \alpha \in II \end{array}\right\} cos\,\alpha = -\dfrac{4}{5}$.

$\dfrac{\alpha}{2} \in I$:

$$sen\,\frac{\alpha}{2} = +\sqrt{\frac{1-cos\alpha}{2}} = \sqrt{\frac{1+\frac{4}{5}}{2}} = \sqrt{\frac{\frac{9}{5}}{2}} = \sqrt{\frac{9}{10}} = \boxed{\frac{3}{\sqrt{10}}}$$

$$cos\,\frac{\alpha}{2} = +\sqrt{\frac{1+cos\alpha}{2}} = \sqrt{\frac{1-\frac{4}{5}}{2}} = \sqrt{\frac{\frac{1}{5}}{2}} = \sqrt{\frac{1}{10}} = \boxed{\frac{1}{\sqrt{10}}}$$

$$tg\,\frac{\alpha}{2} = \frac{\frac{3}{\cancel{\sqrt{10}}}}{\frac{1}{\cancel{\sqrt{10}}}} = \boxed{3}$$

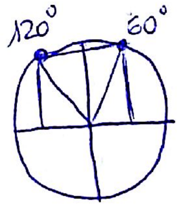
② a) $2\,sen^2x + cos\,2x = 4\,cos^2x$

$2\,sen^2x + cos^2x - sen^2x = 4\,cos^2x$

$sen^2x - 3\,cos^2x = 0 \Rightarrow sen^2x - 3(1-sen^2x) = 0 \Rightarrow$

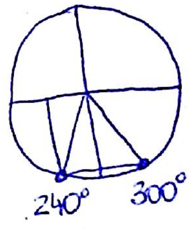
$\Rightarrow sen^2x - 3 + 3\,sen^2x = 0 \Rightarrow 4\,sen^2x - 3 = 0 \Rightarrow 4\,sen^2x = 3 \Rightarrow$

$\Rightarrow sen^2x = \dfrac{3}{4} \Rightarrow sen\,x = \pm\sqrt{\dfrac{3}{4}} = \pm\dfrac{\sqrt{3}}{2}$

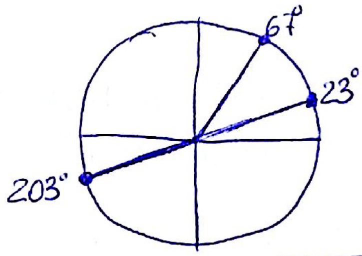


$sen\,x = \dfrac{\sqrt{3}}{2} \Rightarrow \boxed{x = \begin{cases} 60° + 2\pi K \\ 120° + 2\pi K \end{cases}}$

$sen\,x = -\dfrac{\sqrt{3}}{2} \Rightarrow \boxed{x = \begin{cases} 240° + 2\pi K \\ 300° + 2\pi K \end{cases}}$



② b) $\sec 203^\circ$, si $\cotg 67^\circ = h$



$h = \cotg 67^\circ = \tg 23^\circ$ por ser complementarios los ángulos.

$$\sec^2 203^\circ = 1 + \tg^2 203^\circ \Longrightarrow \sec^2 203^\circ = 1 + h^2 \underset{203^\circ \in \text{III}}{\Rightarrow} \boxed{\sec 203^\circ = -\sqrt{1+h^2}}$$

Como $\tg 23^\circ = \tg 203^\circ = h$

③ a) $A = 80^\circ$, $a = 10\,m$ y $b = 5\,m$

$$\frac{a}{\operatorname{sen} A} = \frac{b}{\operatorname{sen} B} \Rightarrow \operatorname{sen} B = \frac{b \cdot \operatorname{sen} A}{a} = \frac{5 \cdot \operatorname{sen} 80^\circ}{10} = \frac{\operatorname{sen} 80^\circ}{2} = 0{,}4924$$

$$B = \operatorname{arc\,sen} 0{,}4924 = \begin{cases} \boxed{29{,}49^\circ} \\ \cancel{+150{,}5^\circ} \text{ NO VALE } (B = 80^\circ) \end{cases}$$

$$\boxed{C = 180^\circ - 80^\circ - 29{,}49^\circ = \boxed{70{,}51^\circ}}$$

$$\frac{a}{\operatorname{sen} A} = \frac{c}{\operatorname{sen} C} \Rightarrow \frac{10}{\operatorname{sen} 80^\circ} = \frac{c}{\operatorname{sen} 70{,}51^\circ} \Rightarrow \boxed{c = \frac{10 \cdot \operatorname{sen} 70{,}51^\circ}{\operatorname{sen} 80^\circ} = 9{,}57\,m}$$

$$A = \frac{1}{2} a \cdot b \cdot \operatorname{sen} C = \frac{1}{2} \cdot 10 \cdot 5 \cdot \operatorname{sen} 70{,}51^\circ = 25 \cdot \operatorname{sen} 70{,}51^\circ = \boxed{23{,}57\,m^2}$$

b) $\sqrt[5]{\dfrac{-1-\sqrt{3}i}{-3+\sqrt{3}i}}$

$$-1-\sqrt{3}i \Rightarrow |-1-\sqrt{3}i| = \sqrt{(-1)^2 + (-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

$$\alpha = \operatorname{arc\,tg} \frac{-\sqrt{3}}{-1} = \operatorname{arc\,tg} \sqrt{3} = \begin{cases} \cancel{60^\circ} \text{ Afijo } (-1, -\sqrt{3}) \in \text{III} \\ 240^\circ \end{cases}$$

$A \in \text{III}$

$\Rightarrow 2_{240^\circ}$

$$-3+\sqrt{3}i \Rightarrow |-3+\sqrt{3}i| = \sqrt{(-3)^2 + (\sqrt{3})^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}$$

$$\beta = \operatorname{arc\,tg} \frac{\sqrt{3}}{-3} = \begin{cases} -30^\circ \rightarrow \cancel{330^\circ} \text{ Afijo } (-3, \sqrt{3}) \in \text{II} \\ 150^\circ \end{cases}$$

$B \in \text{II}$

$\Rightarrow 2\sqrt{3}_{150^\circ}$

$$\sqrt[5]{\frac{2_{240^\circ}}{2\sqrt{3}_{150^\circ}}} = \sqrt[5]{\left(\frac{\cancel{2}}{\cancel{2}\sqrt{3}}\right)_{90^\circ}} = \sqrt[5]{\left(\frac{1}{\sqrt{3}}\right)_{90^\circ}} = \left(\sqrt[5]{\frac{1}{\sqrt{3}}}\right)_{\underbrace{\frac{90^\circ + 2\pi k}{5}}_{18^\circ + 72k \ \forall k = 0,1,2,3,4}} = \begin{cases} \text{Si } k=0 \Rightarrow \left(\sqrt[5]{1/\sqrt{3}}\right)_{18^\circ} \\ \text{Si } k=1 \Rightarrow \left(\sqrt[5]{1/\sqrt{3}}\right)_{90^\circ} \\ \text{Si } k=2 \Rightarrow \left(\sqrt[5]{1/\sqrt{3}}\right)_{162^\circ} \\ \text{Si } k=3 \Rightarrow \left(\sqrt[5]{1/\sqrt{3}}\right)_{234^\circ} \\ \text{Si } k=4 \Rightarrow \left(\sqrt[5]{1/\sqrt{3}}\right)_{306^\circ} \end{cases}$$

(4)

a) $\vec{u}=(-10,12)$ $\quad B=\{(3,-4),(1,-1)\}$

$$(-10,12)=\alpha(3,-4)+\beta(1,-1)=(3\alpha+\beta,-4\alpha-\beta)$$

$$\begin{cases}3\alpha+\beta=-10\\-4\alpha-\beta=12\end{cases} \longrightarrow 3\cdot(-2)+\beta=-10 \Rightarrow -6+\beta=-10 \Rightarrow$$

$$\Rightarrow \beta=-10+6 \Rightarrow \beta=-4$$

$\oplus \quad -\alpha=2 \Rightarrow \alpha=-2$

$\boxed{\vec{u}=(-2,-4)}$ respecto de B

b) $z_1=2_{60°}$ $\quad z_2=-1+i \Rightarrow \overline{z_2}=-1-i$, $z_3=2(\cos 210°+i\,\text{sen}\,210°)=2_{210°}$

$$\overline{z_2}=\sqrt{2}_{225°}\begin{cases}|\overline{z_2}|=\sqrt{(-1)^2+(-1)^2}=\sqrt{1+1}=\sqrt{2}\\ \alpha=\text{arctg}\,\frac{-1}{-1}=\text{arctg}\,1=\begin{cases}\cancel{45°} & A\in \text{III}\\ 225°\end{cases}\\ A(-1,-1)\in \text{III}\end{cases}$$

$$\frac{z_1\cdot\overline{z_2}}{z_3}=\frac{2_{60°}\cdot\sqrt{2}_{225°}}{2_{210°}}=\left(\frac{\cancel{2}\sqrt{2}}{\cancel{2}}\right)_{60°+225°-210°}=\sqrt{2}_{75°}=$$

$$=\sqrt{2}(\cos 75°+i\,\text{sen}\,75°)=\boxed{\sqrt{2}\cos 75°+\sqrt{2}\,\text{sen}\,75°\cdot i}$$