

1.- (2 puntos)

a) Resolver la ecuación: $\cos 8x + \cos 6x = 2 \cdot \cos 210^\circ \cdot \cos x$

b) Demostrar: $\frac{\sin(2\alpha)}{1 + \cos(2\alpha)} = \tan \alpha$

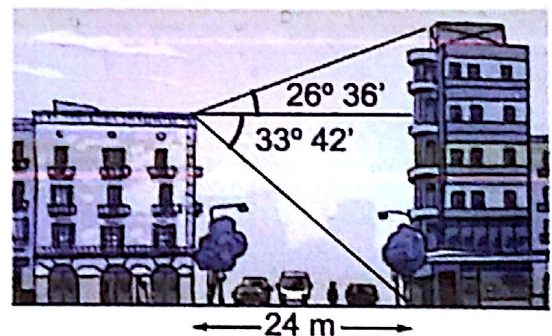
2.- (2 puntos)

Sabiendo que $\sin \alpha = -0,2$ y $\cos \beta = -0,5$ siendo $\frac{3\pi}{2} < \alpha < 2\pi$ y $\pi < \beta < \frac{3\pi}{2}$ calcular: a) $\cos(\alpha + \beta)$ b) $\tan\left(\frac{\beta}{2}\right)$

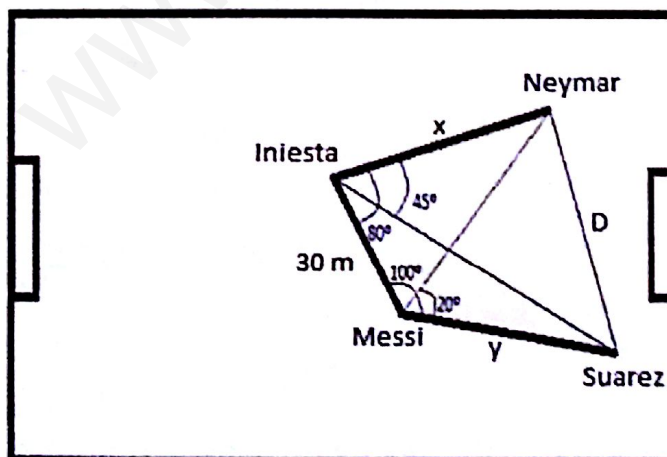
3.- (3 puntos)

a) Calcular la altura de los dos edificios del dibujo:

b) Resolver el sistema en el intervalo $[0^\circ, 360^\circ]$

$$\begin{cases} \tan x + \tan y = 2 \\ x - y = \pi \end{cases}$$


4.- (3 puntos)

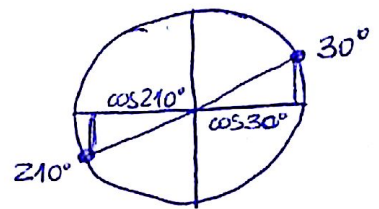


Ataca el Barcelona, ya le gana 0-3 al Real Madrid, y el periodista Manolo Lama quiere saber, con los datos que veis en el dibujo, las distancias siguientes:

- Distancia entre Iniesta y Neymar (x)
- Distancia entre Messi y Suarez (y)
- Distancia entre los delanteros Neymar y Suarez (D)

$$\textcircled{1} \text{ a) } \cos 8x + \cos 6x = 2 \cos 210^\circ \cdot \cos x$$

$$2 \cos \frac{8x+6x}{2} \cdot \cos \frac{8x-6x}{2} = 2(-\cos 30^\circ) \cdot \cos x$$



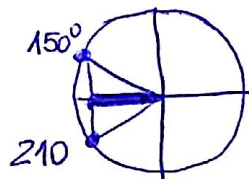
$$2 \cos 7x \cdot \cos x = 2 \cdot \left(-\frac{\sqrt{3}}{2}\right) \cdot \cos x$$

$$2 \cos 7x \cdot \cos x + \sqrt{3} \cos x = 0$$

$$\cos x \cdot (2 \cos 7x + \sqrt{3}) = 0 \Rightarrow$$

$$\left\{ \begin{array}{l} 1) \cos x = 0 \Rightarrow \begin{cases} x = 90^\circ + 2\pi K \\ x = 270^\circ + 2\pi K \end{cases} \Rightarrow \\ \Rightarrow \boxed{x = 90^\circ + \pi K} \quad K \in \mathbb{Z} \\ 2) 2 \cos 7x + \sqrt{3} = 0 \Rightarrow \end{array} \right.$$

$$\Rightarrow 2 \cos 7x = -\sqrt{3} \Rightarrow \cos 7x = -\frac{\sqrt{3}}{2} \Rightarrow \left\{ \begin{array}{l} 7x = 150^\circ + 2\pi K \Rightarrow x = \frac{150^\circ}{7} + \frac{2\pi}{7} K \\ 7x = 210^\circ + 2\pi K \Rightarrow x = \frac{210^\circ}{7} + \frac{2\pi}{7} K \end{array} \right. \Rightarrow \boxed{x = \frac{150^\circ}{7} + \frac{2\pi}{7} K}$$



$$\text{b) } \frac{\sin 2\alpha}{1 + \cos 2\alpha} = \tan \alpha$$

$$\boxed{\frac{\sin 2\alpha}{1 + \cos 2\alpha} = \frac{2 \sin \alpha \cos \alpha}{1 + \cos^2 \alpha - \sin^2 \alpha} = \frac{2 \sin \alpha \cos \alpha}{1 - \sin^2 \alpha + \cos^2 \alpha} = \frac{2 \sin \alpha \cos \alpha}{\cos^2 \alpha + \cos^2 \alpha}}$$

$$= \frac{\cancel{2} \sin \alpha \cos \alpha}{\cancel{2} \cos^2 \alpha} = \frac{\sin \alpha}{\cos \alpha} = \boxed{\tan \alpha}$$

$$\textcircled{2} \sin \alpha = -0,2, \cos \beta = -0,5, \alpha \in \text{IV} \text{ y } \beta \in \text{III}$$

$$\text{a) } \boxed{\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta = 0,98 \cdot (-0,5) - (-0,2) \cdot (-0,87) = -0,664}$$

$$\cos^2 \alpha + \sin^2 \alpha = 1 \Rightarrow \cos^2 \alpha + (-0,2)^2 = 1 \Rightarrow \cos^2 \alpha + 0,04 = 1 \Rightarrow \cos^2 \alpha = 1 - 0,04 \Rightarrow$$

$$\Rightarrow \cos^2 \alpha = 0,96 \Rightarrow \cos \alpha = \pm \sqrt{0,96} = \pm 0,98$$

$\alpha \in \text{IV}$

$$\sin^2 \beta + \cos^2 \beta = 1 \Rightarrow \sin^2 \beta + (-0,5)^2 = 1 \Rightarrow \sin^2 \beta + 0,25 = 1 \Rightarrow \sin^2 \beta = 1 - 0,25 \Rightarrow$$

$$\Rightarrow \sin^2 \beta = 0,75 \Rightarrow \sin \beta = \pm \sqrt{0,75} = -0,87$$

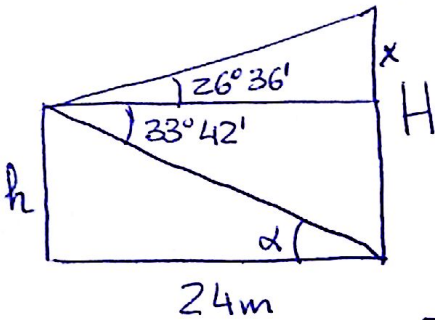
$\beta \in \text{III}$

$$b) \boxed{\operatorname{tg}\left(\frac{\beta}{2}\right) = \pm \sqrt{\frac{1-\cos\beta}{1+\cos\beta}}} = -\sqrt{\frac{1-\cos\beta}{1+\cos\beta}} = -\sqrt{\frac{1-(-0,5)}{1+(-0,5)}} =$$

$$\beta \in \text{III} \Rightarrow \beta/2 \in \text{II} // = -\sqrt{\frac{1,5}{0,5}} = \boxed{-\sqrt{3}}$$

3

a)



$\alpha = 33^\circ 42'$ por ser ángulos alternos internos.

$$\operatorname{tg} 33^\circ 42' = \frac{h}{24} \Rightarrow h = 24 \cdot \operatorname{tg} 33^\circ 42' \Rightarrow$$

$$\boxed{h = 16\text{m}}$$

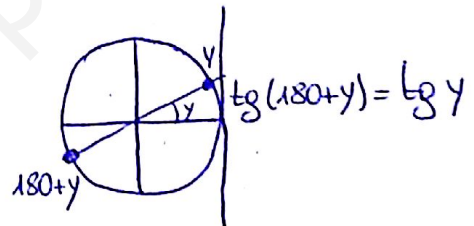
$$H = h + x$$

$$\operatorname{tg} 26^\circ 36' = \frac{x}{24} \Rightarrow \underline{x = 24 \cdot \operatorname{tg} 26^\circ 36' = 12,02\text{m}}$$

$$\boxed{H = 16 + 12,02 = 28,02\text{m}}$$

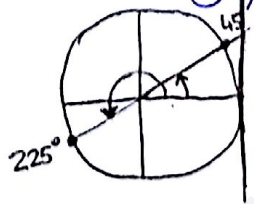
$$b) \left\{ \begin{array}{l} \operatorname{tg} x + \operatorname{tg} y = 2 \\ x, y \in (0, 360) \end{array} \right.$$

$$x - y = \pi \Rightarrow x = 180^\circ + y$$



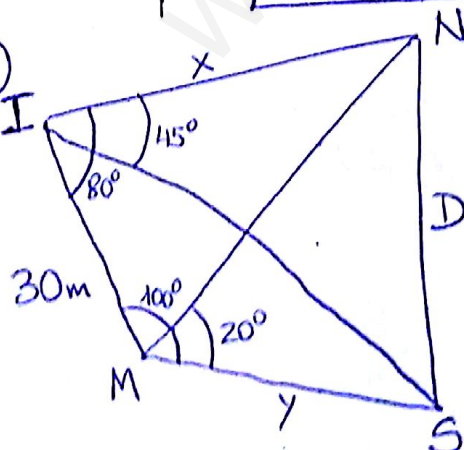
$$\operatorname{tg}(180+y) + \operatorname{tg} y = 2 \Rightarrow \operatorname{tg} y + \operatorname{tg} y = 2 \Rightarrow 2 \operatorname{tg} y = 2 \Rightarrow$$

$$\Rightarrow \operatorname{tg} y = 1 \Rightarrow \left\{ \begin{array}{l} y_1 = 45^\circ \Rightarrow x_1 = 180^\circ + 45^\circ = 225^\circ \\ y_2 = 225^\circ \Rightarrow y_2 = 180^\circ + 225^\circ = 405^\circ \notin (0, 360^\circ) \end{array} \right.$$



$$\boxed{S_1(225^\circ, 45^\circ)}$$

4



$$a) \text{ En } \triangle IMN \left\{ \begin{array}{l} I = 80^\circ \\ M = 100^\circ - 20^\circ = 80^\circ \\ \text{(ISÓCELES)} \end{array} \right. \Rightarrow N = 180^\circ - 80^\circ - 80^\circ = 20^\circ$$

$$\frac{30}{\operatorname{sen} 20^\circ} = \frac{x}{\operatorname{sen} 80^\circ} \Rightarrow \boxed{x = \frac{30 \cdot \operatorname{sen} 80^\circ}{\operatorname{sen} 20^\circ} = 86,38\text{m}}$$

$$b) \text{ En } \triangle IMS \left\{ \begin{array}{l} M = 100^\circ \\ I = 80^\circ - 45^\circ = 35^\circ \end{array} \right. \Rightarrow S = 180^\circ - 100^\circ - 35^\circ = 45^\circ$$

$$\frac{30}{\operatorname{sen} 45^\circ} = \frac{y}{\operatorname{sen} 35^\circ} \Rightarrow \boxed{y = \frac{30 \cdot \operatorname{sen} 35^\circ}{\operatorname{sen} 45^\circ} = 24,33\text{m}}$$

$$c) \overline{MN} = \overline{IN} = 86,38\text{m} \text{ (isóceles } \triangle IMN) \\ \text{En } \triangle MSN \text{ conozco } \overline{MN}, \overline{MS} (y) \text{ y el ángulo opuesto a } D \Rightarrow \text{Aplico T}^e \text{ COSENO} \\ D^2 = \overline{MN}^2 + \overline{MS}^2 - 2 \overline{MN} \overline{MS} \cdot \cos 20^\circ = 86,38^2 + 24,33^2 - 2 \cdot 86,38 \cdot 24,33 \cdot \cos 20^\circ = 4103,69 \\ \boxed{D = \sqrt{4103,69} = 64,06\text{m}}$$