

1.- (2,5 ptos) Resolver por el método de Gauss:
$$\begin{cases} 2x + 5y - 2z = 10 \\ x + 2y - 2z = 4 \\ 4x + 9y - 6z = 18 \end{cases}$$

2.- a) (1,25 ptos) Simplificar y operar:
$$\frac{2x - 8x^2}{16x^2 - 1} \cdot \frac{16x^2 + 8x + 1}{6x} =$$

b) (1,25 ptos) Desarrollar la expresión: $x - |2 - x| + 3 \cdot |x + 2| =$

3.- a) (1,25 ptos) Resolver:
$$\frac{\log(16 - x^2)}{\log(3x - 4)} = 2$$

b) (1,25 ptos) Resolver: $4^x + 2^{2x-1} = 24$

4.- a) (1,25 ptos) Desarrollar por el binomio de Newton: $(2x^3y - 3xy^3)^5 =$

b) (1,25 ptos) Demostrar la propiedad de los logaritmos:

$$\log_a M - \log_a N = \log_a \frac{M}{N}$$

$$\textcircled{1} \begin{cases} 2x + 5y - 2z = 10 \\ x + 2y - 2z = 4 \\ 4x + 9y - 6z = 18 \end{cases} \xrightarrow{E_2 \leftrightarrow E_1} \begin{cases} x + 2y - 2z = 4 \\ 2x + 5y - 2z = 10 \\ 4x + 9y - 6z = 18 \end{cases} \xrightarrow{\begin{matrix} E_2 - 2E_1 \\ E_3 - 4E_1 \end{matrix}}$$

$$\rightarrow \begin{cases} x + 2y - 2z = 4 \\ / \quad y + 2z = 2 \\ / \quad y + 2z = 2 \end{cases} \rightarrow \begin{cases} x + 2y - 2z = 4 \\ y + 2z = 2 \end{cases} \rightarrow \underline{z = t}$$

$$E_2: y + 2t = 2 \Rightarrow \underline{y = 2 - 2t}$$

$$E_1: x + 2(2 - 2t) - 2t = 4 \Rightarrow x + 4 - 4t - 2t = 4 \Rightarrow \underline{x = 6t}$$

$$\boxed{S: (6t, 2 - 2t, t) \forall t \in \mathbb{R}}$$

$$\textcircled{2} \text{ a) } \frac{2x - 8x^2}{16x^2 - 1} \cdot \frac{16x^2 + 8x + 1}{6x} = \frac{2x(1 - 4x)}{(4x + 1)(4x - 1)} \cdot \frac{(4x + 1)^{\frac{2}{3}}}{\frac{6x}{3}} =$$

$$= \boxed{\frac{-(4x + 1)}{3}}$$

$$\text{b) } x - |2 - x| + 3 \cdot |x + 2| = (*)$$

$$|2 - x| = \begin{cases} 2 - x & \text{si } 2 - x \geq 0 \Rightarrow 2 \geq x \Rightarrow x \leq 2 \\ -2 + x & \text{si } 2 - x < 0 \Rightarrow 2 < x \Rightarrow x > 2 \end{cases}$$



$$|x + 2| = \begin{cases} x + 2 & \text{si } x + 2 \geq 0 \Rightarrow x \geq -2 \\ -x - 2 & \text{si } x + 2 < 0 \Rightarrow x < -2 \end{cases}$$

$$(*) = \begin{cases} x - (2 - x) + 3 \cdot (-x - 2) & \text{si } x < -2 \\ x - (2 - x) + 3 \cdot (x + 2) & \text{si } -2 \leq x \leq 2 \\ x - (-2 + x) + 3 \cdot (x + 2) & \text{si } x > 2 \end{cases} =$$

$$= \begin{cases} -x - 8 & \text{si } x < -2 \\ 5x + 4 & \text{si } -2 \leq x \leq 2 \\ 3x + 8 & \text{si } x > 2 \end{cases}$$

$$\textcircled{3} a) \frac{\log(16-x^2)}{\log(3x-4)} = 2 \Rightarrow \log(16-x^2) = 2 \cdot \log(3x-4) \Rightarrow$$

$$\Rightarrow \log(16-x^2) = \log(3x-4)^2 \Rightarrow 16-x^2 = (3x-4)^2 \Rightarrow 16-x^2 = 9x^2-24x+16$$

$$\Rightarrow 0 = 10x^2 - 24x \Rightarrow x(10x-24) = 0 \Rightarrow \begin{cases} x=0 \\ 10x-24=0 \Rightarrow 10x=24 \Rightarrow \\ \Rightarrow x = \frac{24}{10} = \frac{12}{5} \end{cases}$$

Comprobación:

$$* \text{Si } x=0 \Rightarrow \frac{\log(16-0^2)}{\log(3 \cdot 0 - 4)} = 2 \Rightarrow \boxed{x=0 \text{ NO ES SOLUCIÓN}}$$

$$* \text{Si } x = \frac{12}{5} \Rightarrow \frac{\log(16 - \frac{144}{25})}{\log(3 \cdot \frac{12}{5} - 4)} = 2 \Rightarrow \frac{\log 10,24}{\log 3,2} = 2 \Rightarrow \underline{2=2} \Rightarrow$$

$$\Rightarrow \boxed{x = \frac{12}{5} \text{ es solución}}$$

$$b) 4^x + 2^{2x-1} = 24 \Rightarrow (2^2)^x + \frac{2^{2x}}{2} = 24 \Rightarrow 2 \cdot 2^{2x} + 2^{2x} = 48 \Rightarrow$$

$$\Rightarrow 3 \cdot 2^{2x} = 48 \Rightarrow 2^{2x} = \frac{48}{3} \Rightarrow 2^{2x} = 16 \Rightarrow 2^{2x} = 2^4 \Rightarrow 2x = 4$$

$$\Rightarrow \boxed{x=2}$$

$$\textcircled{4} a) (2x^3y - 3xy^3)^5 =$$

$$= \binom{5}{0}(2x^3y)^5 - \binom{5}{1}(2x^3y)^4 \cdot (3xy^3) + \binom{5}{2}(2x^3y)^3 \cdot (3xy^3)^2 - \binom{5}{3}(2x^3y)^2 \cdot (3xy^3)^3 + \binom{5}{4}(2x^3y) \cdot (3xy^3)^4 - \binom{5}{5}(3xy^3)^5 =$$

$$= 1 \cdot 2^5 \cdot x^{15}y^5 - 5 \cdot 2^4 \cdot x^{12}y^4 \cdot 3xy^3 + 10 \cdot 2^3 \cdot x^9y^3 \cdot 3^2x^2y^6 - 10 \cdot 2^2 \cdot x^6y^2 \cdot 3^3x^3y^9 + 5 \cdot 2x^3y \cdot 3^4x^4y^{12} - 1 \cdot 3^5x^5y^{15} =$$

$$= \boxed{32x^{15}y^5 - 240x^{13}y^7 + 720x^{11}y^9 - 1080x^9y^{11} + 810x^7y^{13} - 243x^5y^{15}}$$

b) VER TEORÍA