

1.- Resolver: $\sqrt{3x-6} + \sqrt{2x+6} = \sqrt{9x+4}$

2.- Resolver: $\log 8 + (x^2 - 5x + 7) \cdot \log 3 = \log 24$

3.- Resolver: $\begin{cases} 2^x + 5^y = 9 \\ 2^{x+2} + 5^{y+1} = 41 \end{cases}$

4.- a) Si $\log A = \frac{1}{2} \cdot \log a + 3 \log b - \frac{1}{3} \cdot \log c - 2$ calcular el valor de A

b) Calcular: $-\log_2 (\log_2 \sqrt{\sqrt{2}}) =$

5.- Resolver: $\begin{cases} x - y = 4 \\ x^2 + y^2 = 58 \end{cases}$

① $\sqrt{3x-6} + \sqrt{2x+6} = \sqrt{9x+4} \Rightarrow (\sqrt{3x-6} + \sqrt{2x+6})^2 = (\sqrt{9x+4})^2 \Rightarrow$
 $\Rightarrow 3x-6 + 2 \cdot \sqrt{3x-6} \cdot \sqrt{2x+6} + 2x+6 = 9x+4 \Rightarrow 5x + 2 \cdot \sqrt{(3x-6)(2x+6)} = 9x+4$
 $\Rightarrow 2 \sqrt{6x^2+18x-12x-36} = 9x+4-5x \Rightarrow 2 \sqrt{6x^2+6x-36} = 4x+4 \Rightarrow$
 $\Rightarrow (2 \cdot \sqrt{6x^2+6x-36})^2 = (4x+4)^2 \Rightarrow 4 \cdot (6x^2+6x-36) = 16x^2+32x+16 \Rightarrow$
 $\Rightarrow 24x^2+24x-144 = 16x^2+32x+16 \Rightarrow 24x^2-16x^2+24x-32x-144-16=0 \Rightarrow$
 $\Rightarrow 8x^2-8x-160=0 \Rightarrow x^2-x-20=0 \Rightarrow \begin{cases} x_1=5 \\ x_2=-4 \end{cases} \text{ (Cardano)}$
 $\oplus 1$
 $\ominus 20$

Comprobación:

Si $x=5 \Rightarrow \sqrt{9} + \sqrt{16} = \sqrt{49} \Rightarrow 3+4=7 \Rightarrow \boxed{7=7}$ Sí es solución

Si $x=-4 \Rightarrow \sqrt{-18} + \sqrt{-2} = \sqrt{-32} \Rightarrow \cancel{\text{soluc. real}} \Rightarrow \text{NO es solución}$

② $\log 8 + (x^2 - 5x + 7) \cdot \log 3 = \log 24 \Rightarrow \log 8 + \log 3^{x^2-5x+7} = \log 24 \Rightarrow$
 $\Rightarrow \log 8 \cdot 3^{x^2-5x+7} = \log 24 \Rightarrow 8 \cdot 3^{x^2-5x+7} = 24 \Rightarrow 3^{x^2-5x+7} = \frac{24}{8} \Rightarrow$
 $\Rightarrow 3^{x^2-5x+7} = 3^1 \Rightarrow x^2-5x+7=1 \Rightarrow x^2-5x+7-1=0 \Rightarrow$
 $\Rightarrow x^2-5x+6=0 \Rightarrow \begin{cases} x_1=3 \\ x_2=2 \end{cases}$
 $\oplus 5$
 $\ominus 6$
 Comprobación:
 Si $x=3 \Rightarrow \frac{\log 8 + 1 \cdot \log 3}{\log(8 \cdot 3)} = \log 24 \Rightarrow \text{CIERTO}$
 Si $x=2 \Rightarrow \frac{\log 8 + 1 \cdot \log 3}{\log(8 \cdot 3)} = \log 24 \Rightarrow \text{CIERTO}$

③ $\begin{cases} 2^x + 5^y = 9 \\ 2^{x+2} + 5^{y+1} = 41 \end{cases} \Rightarrow \begin{cases} 2^x + 5^y = 9 \\ 2^3 \cdot 2^x + 5 \cdot 5^y = 41 \end{cases} \Rightarrow \begin{cases} 2^x = L \\ 5^y = M \end{cases} \Rightarrow \begin{cases} L + M = 9 \\ 4L + 5M = 41 \end{cases}$
 $\Rightarrow \begin{cases} 4L + 4M = 36 \\ 4L + 5M = 41 \end{cases} \Rightarrow \begin{matrix} 4L+4M=36 \\ -M=-5 \end{matrix} \Rightarrow \begin{matrix} M=5 \\ 5^y=5^1 \Rightarrow y=1 \end{matrix}$
 $\Rightarrow \begin{matrix} L+5=9 \Rightarrow L=9-5 \Rightarrow \\ L=4 \\ 2^x=2^2 \Rightarrow x=2 \end{matrix}$

$\boxed{S(2,1)}$

④ a) Si $\log A = \frac{1}{2} \cdot \log a + 3 \log b - \frac{1}{3} \cdot \log c - 2$ ¿A?

$$\begin{aligned} \log A &= \log a^{1/2} + \log b^3 - \log c^{1/3} - \log 100 = \\ &= \log a^{1/2} + \log b^3 - (\log c^{1/3} + \log 100) = \\ &= \log (a^{1/2} \cdot b^3) - \log (c^{1/3} \cdot 100) = \log \frac{a^{1/2} \cdot b^3}{c^{1/3} \cdot 100} \end{aligned}$$

$$A = \frac{\sqrt{a} \cdot b^3}{100 \cdot \sqrt[3]{c}}$$

b) $-\log_2(\log_2 \sqrt{\sqrt{2}}) = -\log_2(\log_2 \sqrt[8]{2}) = -\log_2(\underbrace{\log_2 2^{1/8}}_{\frac{1}{8} \text{ por definición de logaritmo}}) =$
 $= -\log_2 \frac{1}{8} = -\log_2 \frac{1}{2^3} = -\underbrace{\log_2 2^{-3}}_{-3 \text{ por definición de logaritmo}} = -(-3) = \boxed{3}$

⑤ $\begin{cases} x - y = 4 \Rightarrow x = 4 + y \\ x^2 + y^2 = 58 \Rightarrow (4 + y)^2 + y^2 = 58 \Rightarrow 16 + 8y + y^2 + y^2 = 58 \end{cases}$
 $\Rightarrow 2y^2 + 8y + 16 - 58 = 0 \Rightarrow 2y^2 + 8y - 42 = 0$ (Divido entre 2)
 $\Rightarrow y^2 + 4y - 21 = 0$ $\left\{ \begin{array}{l} y_1 = -7 \Rightarrow x_1 = 4 + (-7) = -3 \\ y_2 = 3 \Rightarrow x_2 = 4 + 3 = 7 \end{array} \right.$

$$\begin{array}{l} \oplus -4 \\ \ominus 21 \end{array}$$

$$S_1(-3, -7)$$

$$S_2(7, 3)$$