1) Simplificar la siguientes expresiones

a) $2\cdot \operatorname{tg}\alpha\cdot\cos^2\dfrac{\alpha}{2}-\operatorname{sen}\alpha$ b) $(\operatorname{sen}\alpha+\cos\alpha)^2$ c) $\operatorname{sen}^4\alpha-\cos^4\alpha$

d) $(1-\sec)\cdot(1+\sec)$ e) $\sqrt{1-\operatorname{sen}\alpha}\cdot\sqrt{1+\operatorname{sen}\alpha}$ f) $\cot(-\alpha)\cdot\cos(-\alpha)-\operatorname{sen}(-$

a) $2\cdot \operatorname{tg}\alpha\cdot\cos^2\dfrac{\alpha}{2}-\operatorname{sen}\alpha$

$$\cos\frac{\alpha}{2}=\sqrt{\frac{1+\cos\alpha}{2}}$$

$$2\cdot\operatorname{tg}\alpha\cdot\cos^2\frac{\alpha}{2}-\operatorname{sen}\alpha=2\cdot\frac{\operatorname{sen}\alpha}{\cos\alpha}\cdot\cos^2\frac{\alpha}{2}-\operatorname{sen}\alpha=2\cdot\frac{\operatorname{sen}\alpha}{\cos\alpha}\cdot\left(\sqrt{\frac{1+\cos\alpha}{2}}\right)^2-\operatorname{sen}\alpha=$$

$$=2\cdot\frac{\operatorname{sen}\alpha}{\cos\alpha}\cdot\frac{1+\cos\alpha}{2}-\operatorname{sen}\alpha=\frac{\operatorname{sen}\alpha\cdot(1+\cos\alpha)}{\cos\alpha}-\operatorname{sen}\alpha=\frac{\operatorname{sen}\alpha\cdot(1+\cos\alpha)-\operatorname{sen}\alpha\cdot\cos\alpha}{\cos\alpha}=$$

$$=\frac{\operatorname{sen}\alpha+\operatorname{sen}\alpha\cdot\cos\alpha-\operatorname{sen}\alpha\cdot\cos\alpha}{\cos\alpha}=\frac{\operatorname{sen}\alpha}{\cos\alpha}=\operatorname{tg}\alpha$$

b) $(\operatorname{sen}\alpha+\cos\alpha)^2$

$$\operatorname{sen}^2\alpha+\cos^2\alpha=1$$

$$(\operatorname{sen}\alpha+\cos\alpha)^2=\operatorname{sen}^2\alpha+2\cdot\operatorname{sen}\alpha\cdot\cos\alpha+\cos^2\alpha=\operatorname{sen}^2\alpha+\cos^2\alpha+2\cdot\operatorname{sen}\alpha\cdot\cos\alpha=1+2\cdot\operatorname{sen}\alpha\cdot\cos\alpha$$

c) $\operatorname{sen}^4\alpha-\cos^4\alpha$

$$\operatorname{sen}^2\alpha+\cos^2\alpha=1\iff\cos^2\alpha=1-\operatorname{sen}^2\alpha$$

$$\operatorname{sen}^4\alpha-\cos^4\alpha=\left(\operatorname{sen}^2\alpha+\cos^2\alpha\right)\cdot\left(\operatorname{sen}^2\alpha-\cos^2\alpha\right)=1\cdot\left(\operatorname{sen}^2\alpha-\cos^2\alpha\right)=$$

$$=\operatorname{sen}^2\alpha-\left(1-\operatorname{sen}^2\alpha\right)=-1+2\operatorname{sen}^2\alpha=2\operatorname{sen}^2\alpha-1$$

d) $(1-\sec)\cdot(1+\sec)$

$$(1-\sec)\cdot(1+\sec)=1-\sec^2\alpha=-\operatorname{tg}^2\alpha$$

e) $\sqrt{1-\text{sen } \alpha} \cdot \sqrt{1+\text{sen } \alpha}$

$$\sqrt{1-\text{sen } \alpha} \cdot \sqrt{1+\text{sen } \alpha} = \sqrt{(1-\text{sen } \alpha)\cdot(1+\text{sen } \alpha)} = \sqrt{1-\text{sen}^2\alpha} = \sqrt{\cos^2\alpha} = \cos\alpha$$

f) $\cot(-\alpha)\cdot\cos(-\alpha)-\text{sen}(-\alpha)$

Por ángulos opuestos: $\begin{cases} \cos(-\alpha)=\cos\alpha \\ \text{sen}(-\alpha)=-\text{sen } \alpha \\ \cot(-\alpha)=-\cot\alpha \end{cases}$

$$\cot(-\alpha)\cdot\cos(-\alpha)-\text{sen}(-\alpha) = -\cot\alpha\cdot\cos\alpha+\text{sen } \alpha = -\frac{\cos\alpha}{\text{sen } \alpha}\cdot\cos\alpha+\text{sen } \alpha =$$

$$= \frac{\cos\alpha\cdot\cos\alpha+\text{sen}^2\alpha}{\text{sen } \alpha} = \frac{\cos^2\alpha+\text{sen}^2\alpha}{\text{sen } \alpha} = \frac{1}{\text{sen } \alpha} = \text{cosec } \alpha = -\text{cosec}(-\alpha)$$

2) Simplificar la siguientes expresiones

a) $\cos^3\alpha+\text{sen}^2\alpha\cdot\cos\alpha$

b) $(1+\text{sen}\alpha)\cdot(1-\text{sen}\alpha)$

c) $\dfrac{\text{sen}\alpha}{\cos\alpha}+\dfrac{\cos\alpha}{1+\text{sen}\alpha}$

d) $\dfrac{\text{sen}^2\alpha\cdot(1+\cos\alpha)}{1-\cos\alpha}$

e) $\dfrac{\cos\alpha}{\text{tg } \alpha\cdot(1-\text{sen}\alpha)}$

f) $\dfrac{\sec^2\alpha}{\text{cosec}^2\alpha-\sec^2\alpha}+\dfrac{\cot^2\alpha}{\cot^2\alpha-1}$

a) $\cos^3\alpha+\text{sen}^2\alpha\cdot\cos\alpha$

$$\cos^3\alpha+\text{sen}^2\alpha\cdot\cos\alpha = \cos^2\alpha\cdot\cos\alpha+\text{sen}^2\alpha\cdot\cos\alpha = \cos\alpha\cdot\left(\cos^2\alpha+\text{sen}^2\alpha\right) = \cos\alpha\cdot 1 = \cos\alpha$$

b) $(1+\text{sen}\alpha)\cdot(1-\text{sen}\alpha)$

$$(1+\text{sen}\alpha)\cdot(1-\text{sen}\alpha) = 1-\text{sen } \alpha+\text{sen } \alpha-\text{sen}^2\alpha = 1-\text{sen}^2\alpha = \cos^2\alpha = \frac{1}{\sec^2\alpha}$$

c) $\dfrac{\operatorname{sen}\alpha}{\cos\alpha}+\dfrac{\cos\alpha}{1+\operatorname{sen}\alpha}$

$$\frac{\operatorname{sen}\alpha}{\cos\alpha}+\frac{\cos\alpha}{1+\operatorname{sen}\alpha}=\frac{\operatorname{sen}\alpha\cdot\left(1+\operatorname{sen}\alpha\right)\cdot\cos\alpha\cdot\operatorname{sen}\alpha}{\cos\alpha\cdot\left(1+\operatorname{sen}\alpha\right)}=\frac{\operatorname{sen}\alpha+\operatorname{sen}^2\alpha+\cos^2\alpha}{\cos\alpha\cdot\left(1+\operatorname{sen}\alpha\right)}=$$

$$=\frac{\operatorname{sen}\alpha+1}{\cos\alpha\cdot\left(1+\operatorname{sen}\alpha\right)}=\frac{1}{\cos\alpha}=\sec\alpha$$

d) $\dfrac{\operatorname{sen}^2\alpha\cdot\left(1+\cos\alpha\right)}{1-\cos\alpha}$

$$\frac{\operatorname{sen}^2\alpha\cdot\left(1+\cos\alpha\right)}{1-\cos\alpha}=\frac{\left(1-\cos^2\alpha\right)\cdot\left(1+\cos\alpha\right)}{1-\cos\alpha}=\frac{\left(1-\cos\alpha\right)\cdot\left(1+\cos\alpha\right)\cdot\left(1+\cos\alpha\right)}{1-\cos\alpha}=\left(1+\cos\alpha\right)^2$$

e) $\dfrac{\cos\alpha}{\operatorname{tg}\alpha\cdot\left(1-\operatorname{sen}\alpha\right)}$

$$\frac{\cos\alpha}{\operatorname{tg}\alpha\cdot\left(1-\operatorname{sen}\alpha\right)}=\frac{\cos\alpha}{\frac{\operatorname{sen}\alpha}{\cos\alpha}\cdot\left(1-\operatorname{sen}\alpha\right)}=\frac{\cos^2\alpha}{\operatorname{sen}\alpha\cdot\left(1-\operatorname{sen}\alpha\right)}=\frac{1-\operatorname{sen}^2\alpha}{\operatorname{sen}\alpha\cdot\left(1-\operatorname{sen}\alpha\right)}=\frac{\left(1+\operatorname{sen}\alpha\right)\cdot\left(1-\operatorname{sen}\alpha\right)}{\operatorname{sen}\alpha\cdot\left(1-\operatorname{sen}\alpha\right)}=\frac{1+\operatorname{sen}\alpha}{\operatorname{sen}\alpha}$$

f) $\dfrac{\sec^2\alpha}{\operatorname{cosec}^2\alpha-\sec^2\alpha}+\dfrac{\cot^2\alpha}{\cot^2\alpha-1}$

$$\frac{\sec^2\alpha}{\operatorname{cosec}^2\alpha-\sec^2\alpha}+\frac{\cos^2\alpha}{\cot^2\alpha-1}=\frac{\frac{1}{\cos^2\alpha}}{\frac{1}{\operatorname{sen}^2\alpha}-\frac{1}{\cos^2\alpha}}+\frac{\frac{\cos^2\alpha}{\operatorname{sen}^2\alpha}}{\frac{\cos^2\alpha}{\operatorname{sen}^2\alpha}-1}=\frac{\frac{1}{\cos^2\alpha}}{\frac{\cos^2\alpha-\operatorname{sen}^2\alpha}{\operatorname{sen}^2\alpha\cdot\cos^2\alpha}}+\frac{\frac{\cos^2\alpha}{\operatorname{sen}^2\alpha}}{\frac{\cos^2\alpha-\operatorname{sen}^2\alpha}{\operatorname{sen}^2\alpha}}=$$

- $$\frac{\frac{1}{\cos^2\alpha}}{\frac{\cos^2\alpha-\operatorname{sen}^2\alpha}{\operatorname{sen}^2\alpha\cdot\cos^2\alpha}}=\frac{1}{\cos^2\alpha}:\frac{\cos^2\alpha-\operatorname{sen}^2\alpha}{\operatorname{sen}^2\alpha\cdot\cos^2\alpha}=\frac{\operatorname{sen}^2\alpha\cdot\cos^2\alpha}{\cos^2\alpha\left(\cos^2\alpha-\operatorname{sen}^2\alpha\right)}=\frac{\operatorname{sen}^2\alpha}{\cos^2\alpha-\operatorname{sen}^2\alpha}$$

- $$\frac{\frac{\cos^2\alpha}{\operatorname{sen}^2\alpha}}{\frac{\cos^2\alpha-\operatorname{sen}^2\alpha}{\operatorname{sen}^2\alpha}}=\frac{\cos^2\alpha}{\operatorname{sen}^2\alpha}:\frac{\cos^2\alpha-\operatorname{sen}^2\alpha}{\operatorname{sen}^2\alpha}=\frac{\cos^2\alpha\cdot\operatorname{sen}^2\alpha}{\operatorname{sen}^2\alpha\left(\cos^2\alpha-\operatorname{sen}^2\alpha\right)}=\frac{\cos^2\alpha}{\cos^2\alpha-\operatorname{sen}^2\alpha}$$

$$=\frac{\operatorname{sen}^2\alpha}{\cos^2\alpha\text{-}\operatorname{sen}^2\alpha}+\frac{\cos^2\alpha}{\cos^2\alpha\text{-}\operatorname{sen}^2\alpha}=\frac{\cos^2\alpha+\operatorname{sen}^2\alpha}{\cos^2\alpha-\operatorname{sen}^2\alpha}=\frac{1}{\cos^2\alpha-\operatorname{sen}^2\alpha}$$

3) Simplificar la siguientes expresiones

a) $\dfrac{1-\cos 2\alpha}{\cos^2\alpha}$ b) $\dfrac{\operatorname{sen}\alpha\left(\operatorname{sen}^2\alpha-1\right)}{\cos\alpha\left(1-\cos^2\alpha\right)}\cdot \operatorname{tg}\alpha$ c) $\dfrac{\operatorname{tg}\alpha+\operatorname{tg}\alpha\cdot\cot^2\alpha}{1+\operatorname{tg}^2\alpha}$

d) $\dfrac{\sec(-\alpha)}{\operatorname{tg}(-\alpha)}$ e) $\dfrac{\operatorname{cosec}\alpha}{1+\cot^2\alpha}$ f) $\dfrac{\operatorname{sen}^4\alpha+\operatorname{sen}^2\alpha\cdot\cos^2\alpha}{\cos^4\alpha-\cos^2\alpha\cdot\operatorname{sen}^2\alpha}\cdot\cot\alpha$

a) $\dfrac{1-\cos 2\alpha}{\cos^2\alpha}$

$\cos 2\alpha=\cos^2\alpha-\operatorname{sen}^2\alpha$

$$\frac{1-\cos 2\alpha}{\cos^2\alpha}=\frac{1-\left(\cos^2\alpha-\operatorname{sen}^2\alpha\right)}{\cos^2\alpha}=\frac{1-\cos^2\alpha+\operatorname{sen}^2\alpha}{\cos^2\alpha}=\frac{\left(1-\cos^2\alpha\right)+\operatorname{sen}^2\alpha}{\cos^2\alpha}=$$

$$=\frac{\operatorname{sen}^2\alpha+\operatorname{sen}^2\alpha}{\cos^2\alpha}=\frac{2\operatorname{sen}^2\alpha}{\cos^2\alpha}=2\operatorname{tg}^2\alpha$$

b) $\dfrac{\operatorname{sen}\alpha\left(\operatorname{sen}^2\alpha-1\right)}{\cos\alpha\left(1-\cos^2\alpha\right)}\cdot\operatorname{tg}\alpha$

$$\operatorname{sen}^2\alpha+\cos^2\alpha=1\begin{cases}\cos^2\alpha=\operatorname{sen}^2\alpha-1\\ \operatorname{sen}^2\alpha-1=-\cos^2\alpha\\ \operatorname{sen}^2\alpha=1-\cos^2\alpha\end{cases}$$

$$\frac{\operatorname{sen}\alpha\left(\operatorname{sen}^2\alpha-1\right)}{\cos\alpha\left(1-\cos^2\alpha\right)}\cdot\operatorname{tg}\alpha=\frac{-\operatorname{sen}\alpha\cdot\cos^2\alpha}{\cos\alpha\cdot\operatorname{sen}^2\alpha}\cdot\operatorname{tg}\alpha=\frac{-\operatorname{sen}\alpha\cdot\cos^2\alpha}{\cos\alpha\cdot\operatorname{sen}^2\alpha}\cdot\frac{\operatorname{sen}\alpha}{\cos\alpha}=\frac{-\operatorname{sen}^2\alpha\cdot\cos^2\alpha}{\cos^2\alpha\cdot\operatorname{sen}^2\alpha}=-1$$

c) $\dfrac{\operatorname{tg}\alpha+\operatorname{tg}\alpha\cdot\cot^2\alpha}{1+\operatorname{tg}^2\alpha}$

$\operatorname{cosec}^2\alpha=1+\cot^2\alpha$

$1+\operatorname{tg}^2\alpha=\sec^2\alpha$

$$\frac{\operatorname{tg}\alpha+\operatorname{tg}\alpha\cdot\cot^2\alpha}{1+\operatorname{tg}^2\alpha}=\frac{\operatorname{tg}\alpha\cdot\left(1+\cot^2\alpha\right)}{1+\operatorname{tg}^2\alpha}=\frac{\operatorname{tg}\alpha\cdot\operatorname{cosec}^2\alpha}{\sec^2\alpha}=\frac{\dfrac{\operatorname{sen}\alpha}{\cos\alpha}\cdot\dfrac{1}{\operatorname{sen}^2\alpha}}{\dfrac{1}{\cos^2\alpha}}=\frac{\dfrac{\operatorname{sen}\alpha}{\cos\alpha\cdot\operatorname{sen}^2\alpha}}{\dfrac{1}{\cos^2\alpha}}=\frac{\dfrac{1}{\cos\alpha\cdot\operatorname{sen}\alpha}}{\dfrac{1}{\cos^2\alpha}}=$$

$$=\frac{1}{\cos\alpha\cdot\operatorname{sen}\alpha}:\frac{1}{\cos^2\alpha}=\frac{\cos^2\alpha}{\cos\alpha\cdot\operatorname{sen}\alpha}=\frac{\cos\alpha}{\operatorname{sen}\alpha}=\cot\alpha$$

d) $\dfrac{\sec(-\alpha)}{\operatorname{tg}(-\alpha)}$

Por ángulos opuestos: $\sec(-\alpha) = \sec\alpha$

$$\frac{\sec(-\alpha)}{\operatorname{tg}(-\alpha)} = \frac{\sec\alpha}{-\operatorname{tg}\alpha} = \frac{\dfrac{1}{\cos\alpha}}{-\dfrac{\operatorname{sen}\alpha}{\cos\alpha}} = -\left(\frac{1}{\cos\alpha} : \frac{\operatorname{sen}\alpha}{\cos\alpha}\right) = \frac{-\cos\alpha}{\operatorname{sen}\alpha\cos\alpha} = -\frac{1}{\operatorname{sen}\alpha} = -\operatorname{cosec}\alpha$$

e) $\dfrac{\operatorname{cosec}\alpha}{1+\cot^2\alpha}$

$\operatorname{sen}^2\alpha + \cos^2\alpha = 1$

$$\frac{\operatorname{cosec}\alpha}{1+\cot^2\alpha} = \frac{\dfrac{1}{\operatorname{sen}\alpha}}{1+\dfrac{\cos^2\alpha}{\operatorname{sen}^2\alpha}} = \frac{\dfrac{1}{\operatorname{sen}\alpha}}{\dfrac{\operatorname{sen}^2\alpha+\cos^2\alpha}{\operatorname{sen}^2\alpha}} = \frac{\dfrac{1}{\operatorname{sen}\alpha}}{\dfrac{1}{\operatorname{sen}^2\alpha}} = \frac{1}{\operatorname{sen}\alpha} : \frac{1}{\operatorname{sen}^2\alpha} = \frac{\operatorname{sen}^2\alpha}{\operatorname{sen}\alpha} = \operatorname{sen}\alpha$$

f) $\dfrac{\operatorname{sen}^4\alpha + \operatorname{sen}^2\alpha\cdot\cos^2\alpha}{\cos^4\alpha - \cos^2\alpha\cdot\operatorname{sen}^2\alpha}\cdot\cot\alpha$

$\cot\alpha = \dfrac{\cos\alpha}{\operatorname{sen}\alpha}$

$$\frac{\operatorname{sen}^4\alpha - \operatorname{sen}^2\alpha\cdot\cos^2\alpha}{\cos^4\alpha - \cos^2\alpha\cdot\operatorname{sen}^2\alpha}\cdot\cot\alpha = \frac{\operatorname{sen}^2\alpha\left(\operatorname{sen}^2\alpha-\cos^2\alpha\right)}{\cos^2\alpha\left(\cos^2\alpha-\operatorname{sen}^2\alpha\right)}\cdot\frac{\cos\alpha}{\operatorname{sen}\alpha} = \frac{-\operatorname{sen}^2\alpha\left(-\operatorname{sen}^2\alpha+\cos^2\alpha\right)\cdot\cos\alpha}{\cos^2\alpha\left(\cos^2\alpha-\operatorname{sen}^2\alpha\right)\cdot\operatorname{sen}\alpha} =$$

$$= \frac{-\operatorname{sen}^2\alpha\left(\cos^2\alpha-\operatorname{sen}^2\alpha\right)\cdot\cos\alpha}{\cos^2\alpha\left(\cos^2\alpha-\operatorname{sen}^2\alpha\right)\cdot\operatorname{sen}\alpha} = \frac{-\operatorname{sen}^2\alpha\cdot\cos\alpha}{\cos^2\alpha\cdot\operatorname{sen}\alpha} = -\frac{\operatorname{sen}\alpha}{\cos\alpha} = -\operatorname{tg}\alpha$$

4) Comprueba las siguientes identidades.

a) $\cot^2\alpha + 1 = \dfrac{1}{\operatorname{sen}^2\alpha}$ b) $\operatorname{tg}\alpha \cdot \cot\alpha = \operatorname{sen}^2\alpha + \cos^2\alpha$ c) $\operatorname{sen}\alpha \cdot \sec\alpha = \operatorname{tg}\alpha$

d) $(\operatorname{sen}\alpha + \cos\alpha)^2 = 1 + \dfrac{2\operatorname{sen}\alpha}{\cos\alpha}$ e) $\operatorname{tg}\alpha + \cot\alpha = \sec\alpha \cdot \operatorname{cosec}\alpha$ f) $\dfrac{\operatorname{sen}\alpha \cdot \cos\alpha}{\cos^2\alpha - \operatorname{sen}^2\alpha} = \dfrac{\operatorname{tg}\alpha}{1 - \operatorname{tg}^2\alpha}$

a) $\cot^2\alpha + 1 = \dfrac{1}{\operatorname{sen}^2\alpha}$

$$\cot^2\alpha + 1 = \left(\frac{\cos\alpha}{\operatorname{sen}\alpha}\right)^2 + 1 = \frac{\cos^2\alpha}{\operatorname{sen}^2\alpha} + 1 = \frac{\cos^2\alpha + \operatorname{sen}^2\alpha}{\operatorname{sen}^2\alpha} = \frac{1}{\operatorname{sen}^2\alpha}$$

b) $\operatorname{tg}\alpha \cdot \cot\alpha = \operatorname{sen}^2\alpha + \cos^2\alpha$

$$\operatorname{tg}\alpha \cdot \cot\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} \cdot \frac{\cos\alpha}{\operatorname{sen}\alpha} = 1 = \operatorname{sen}^2\alpha + \cos^2\alpha$$

c) $\operatorname{sen}\alpha \cdot \sec\alpha = \operatorname{tg}\alpha$

$$\operatorname{sen}\alpha \cdot \sec\alpha = \operatorname{sen}\alpha \cdot \frac{1}{\cos\alpha} = \frac{\operatorname{sen}\alpha}{\cos\alpha} = \operatorname{tg}\alpha$$

d) $(\operatorname{sen}\alpha + \cos\alpha)^2 = 1 + \dfrac{2\operatorname{sen}\alpha}{\cos\alpha}$

$$(\operatorname{sen}\alpha + \cos\alpha)^2 = \operatorname{sen}^2\alpha + 2 \cdot \operatorname{sen}\alpha \cdot \cos\alpha + \cos^2\alpha = \operatorname{sen}^2\alpha + \cos^2\alpha + 2 \cdot \operatorname{sen}\alpha \cdot \cos\alpha = 1 + 2 \cdot \operatorname{sen}\alpha \cdot \cos\alpha =$$

$$= 1 + \frac{2 \cdot \operatorname{sen}\alpha}{\sec\alpha}$$

e) $\operatorname{tg}\alpha + \cot\alpha = \sec\alpha \cdot \operatorname{cosec}\alpha$

$$\operatorname{tg}\alpha + \cot\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} + \frac{\cos\alpha}{\operatorname{sen}\alpha} = \frac{\operatorname{sen}\alpha \cdot \operatorname{sen}\alpha + \cos\alpha \cdot \cos\alpha}{\cos\alpha \cdot \operatorname{sen}\alpha} = \frac{\operatorname{sen}^2\alpha + \cos^2\alpha}{\cos\alpha \cdot \operatorname{sen}\alpha} =$$

$$= \frac{1}{\cos\alpha \cdot \operatorname{sen}\alpha} = \frac{1}{\cos\alpha} \cdot \frac{1}{\operatorname{sen}\alpha} = \sec\alpha \cdot \operatorname{cosec}\alpha$$

f) $\dfrac{\operatorname{sen}\alpha \cdot \cos\alpha}{\cos^2\alpha - \operatorname{sen}^2\alpha} = \dfrac{\operatorname{tg}\alpha}{1 - \operatorname{tg}^2\alpha}$

$$\frac{\operatorname{sen}\alpha \cdot \cos\alpha}{\cos^2\alpha - \operatorname{sen}^2\alpha} = \frac{\dfrac{\operatorname{sen}\alpha \cdot \cos\alpha}{\cos^2\alpha}}{\dfrac{\cos^2\alpha - \operatorname{sen}^2\alpha}{\cos^2\alpha}} = \frac{\dfrac{\operatorname{sen}\alpha}{\cos\alpha}}{\dfrac{\cos^2\alpha}{\cos^2\alpha} - \dfrac{\operatorname{sen}^2\alpha}{\cos^2\alpha}} = \frac{\operatorname{tg}\alpha}{1 - \operatorname{tg}^2\alpha}$$