

$$20) \lim_{x \rightarrow 0} \frac{x \cos x - \sin x}{x^3} = \frac{0}{0} \text{ Ind (L'Hop)} = \lim_{x \rightarrow 0} \frac{\cancel{\cos x} - x \cancel{\sin x} - \cancel{\cos x}}{3x^2} =$$

$$= \frac{0}{0} \text{ Ind (L'Hop)} = \lim_{x \rightarrow 0} \frac{-(\sin x + x \cos x)}{6x} = \lim_{x \rightarrow 0} \frac{-\sin x - x \cos x}{6x} =$$

$$= \frac{0}{0} \text{ Ind (L'Hop)} = \lim_{x \rightarrow 0} \frac{-\cos x - (\cos x - x \sin x)}{6} = \frac{-1 - (1 - 0)}{6} = \frac{-2}{6} = -\frac{1}{3}$$

$$21) \lim_{x \rightarrow +\infty} \left(x \ln \frac{x+1}{x} \right) = \lim_{x \rightarrow +\infty} \left[\ln \left(\frac{x+1}{x} \right)^x \right] = \ln \lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x} \right)^x =$$

$$= \ln e = 1 \quad \text{Puede resolverse por L'Hôpital.}$$

$$22) \lim_{x \rightarrow 1} \frac{x^3 - x^2 - x + 1}{2x^3 - 5x^2 + 4x - 1} = \frac{0}{0} \text{ Ind} = L$$

$$\bullet x^3 - x^2 - x + 1 = 0$$

$$\begin{array}{c|cccc} 1 & 1 & -1 & -1 & 1 \\ \hline & & & & \end{array} \quad x^2 - 1 = 0 \Rightarrow x = \pm 1.$$

$$\begin{array}{c|cccc} 1 & & 1 & 0 & -1 \\ \hline & 1 & 0 & -1 & 0 \end{array} \quad x^3 - x^2 - x + 1 = (x-1)^2(x+1)$$

$$\bullet 2x^3 - 5x^2 + 4x - 1 = 0.$$

$$\begin{array}{c|cccc} 2 & -5 & 4 & -1 \\ \hline & & & & \end{array} \quad 2x^2 - 3x + 1 = 0 \Rightarrow x = \frac{3 \pm \sqrt{9-8}}{4} \left. \begin{array}{l} x=1 \\ x=\frac{2}{4} = \frac{1}{2} \end{array} \right\}$$

$$\begin{array}{c|cccc} 1 & & 2 & -3 & 1 \\ \hline & 2 & -3 & 1 & 0 \end{array} \quad 2x^3 - 5x^2 + 4x - 1 = (x-1)^2(2x-1)$$

$$L = \lim_{x \rightarrow 1} \frac{(x-1)^2(x+1)}{(x-1)^2(2x-1)} = \frac{2}{1} = 2$$

$$23) \lim_{x \rightarrow 1} x^{\frac{x}{\sin(\pi x)}} = 1^\infty \text{ Ind} = L$$

$$\ln L = \ln \lim_{x \rightarrow 1} x^{\frac{x}{\sin(\pi x)}} = \lim_{x \rightarrow 1} \ln x^{\frac{x}{\sin(\pi x)}} = \lim_{x \rightarrow 1} \left[\frac{x}{\sin(\pi x)} \cdot \ln x \right] =$$

$$= \infty \cdot 0 \text{ Ind} = \lim_{x \rightarrow 1} \frac{x \ln x}{\sin(\pi x)} = \frac{0}{0} \text{ Ind (L'Hop)} = \lim_{x \rightarrow 1} \frac{\ln x + x \cdot \frac{1}{x}}{\pi \cos(\pi x)} =$$

$$= \frac{0+1}{-\pi} = -\frac{1}{\pi}$$

$$\ln L = -\frac{1}{\pi} \Rightarrow L = e^{-1/\pi} = \frac{1}{e^{1/\pi}} = \frac{1}{\sqrt[\pi]{e}}$$

$$24) \lim_{x \rightarrow +\infty} \left[(4x+1) [\ln(x-3) - \ln(x+2)] \right] = \infty - \infty \text{ Ind} =$$

$$= \lim_{x \rightarrow +\infty} \left[\frac{\ln(x-3) - \ln(x+2)}{\frac{1}{4x+1}} \right] = \frac{\infty}{0} \text{ Ind (L'Hop)} =$$

$$= \lim_{x \rightarrow +\infty} \frac{\frac{1}{x-3} - \frac{1}{x+2}}{-4} = \lim_{x \rightarrow +\infty} \frac{\frac{x+2-x-3}{(x-3)(x+2)}}{-4} = \lim_{x \rightarrow +\infty} \frac{5(4x+1)^2}{-4(x-3)(x+2)} =$$

$$= \lim_{x \rightarrow +\infty} \frac{5 \cdot 16x^2 + \dots}{-4x^2 \dots} = \frac{5 \cdot 16}{4} = 5 \cdot 4 = \underline{\underline{20}}$$

$$25) \lim_{x \rightarrow 0^+} (\sqrt{x} \cdot \log x) = 0 \cdot (-\infty) \text{ Ind} = \lim_{x \rightarrow 0^+} \frac{\log x}{\frac{1}{\sqrt{x}}} = \frac{\infty}{\infty} \text{ Ind (L'Hôp)}$$

$$= \lim_{x \rightarrow 0^+} \frac{\frac{1}{x \cdot \log 10}}{\frac{-1}{2\sqrt{x}}} = \lim_{x \rightarrow 0^+} \frac{-2\sqrt{x}}{x \log 10} = \frac{0}{\log 10} = \underline{\underline{0}}$$

$$26) \lim_{x \rightarrow +\infty} \left(\frac{4x-2}{2x^3+6x^2+1} + 1 \right)^{\frac{x^2-2x}{4x-2}} = 1^\infty =$$

$$= \lim_{x \rightarrow +\infty} \left[1 + \frac{1}{\frac{2x^3+6x^2+1}{4x-2}} \right]^{\frac{x^2-2x}{4x-2}} = e^{\lim_{x \rightarrow +\infty} \frac{4x^3 \dots}{2x^3 \dots}} = e^2$$

27) Calcular a para que f sea continua en $\mathbb{R}$.

$$f(x) = \begin{cases} |x-4| & \text{si } x < a \\ x^2 - 6x + 8 & \text{si } x \geq a \end{cases}$$

$$g(x) = |x-4| \Rightarrow x-4=0 \Rightarrow x=4$$

Signo de $x-4$:

$x-4$	$-$	$+$
x	$-\infty$	$+\infty$

$$g(x) = |x-4| = \begin{cases} -x+4 & \text{si } x < 4 \\ x-4 & \text{si } x > 4 \end{cases}$$

Para que f sea continua $|x-4|$ tiene que cortar a $x^2 - 6x + 8$:

• Corte entre $g_1(x) = -x+4$ y $S(x) = x^2 - 6x + 8$.

$$-x+4 = x^2 - 6x + 8 \Rightarrow x^2 - 5x + 4 = 0 \Rightarrow x = \frac{5 \pm \sqrt{25-16}}{2} \begin{cases} x=4 \\ x=2 \end{cases}$$

• Corte entre $g_2(x) = x-4$ y $S(x) = x^2 - 6x + 8$:

$$x-4 = x^2 - 6x + 8 \Rightarrow x^2 - 7x + 12 = 0 \Rightarrow x = \frac{7 \pm \sqrt{49-48}}{2} = \begin{cases} x=4 \\ x=3 \end{cases}$$

no es válida para $x > 4$.

Por lo tanto $a = \underline{\underline{2}}$ ó $\underline{\underline{4}}$.

$$28) \lim_{x \rightarrow +\infty} (x+e^x)^{\frac{2}{x}} = \infty^0 \text{ Ind} = L$$

$$\ln L = \ln \lim_{x \rightarrow +\infty} (x+e^x)^{\frac{2}{x}} = \lim_{x \rightarrow +\infty} \left[\ln(x+e^x)^{\frac{2}{x}} \right] = \lim_{x \rightarrow +\infty} \left[\frac{2}{x} \ln(x+e^x) \right]$$

$$= 0 \cdot \infty \text{ Ind} = \lim_{x \rightarrow +\infty} \frac{2 \ln(x+e^x)}{x} = \frac{\infty}{\infty} \text{ Ind (L'Hôp)} = \lim_{x \rightarrow +\infty} \frac{2 \frac{1+e^x}{x+e^x}}{1} =$$

$$= \lim_{x \rightarrow +\infty} \frac{2(1+e^x)}{x+e^x} = \lim_{x \rightarrow +\infty} \frac{\frac{2}{e^x} + \frac{2e^x}{e^x}}{\frac{x}{e^x} + \frac{e^x}{e^x}} = \frac{0+2}{0+1} = 2$$

Dividiendo cada término por e^x

$$\ln L = 2 \Rightarrow \underline{\underline{L = e^2}}$$

29) Determina el valor de k para que f sea continua en $x=0$.

$$f(x) = \begin{cases} (1+x)^{\frac{\ln k}{x}} & \text{si } x \neq 0 \\ 5 & \text{si } x = 0 \end{cases}$$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} (1+x)^{\frac{\ln k}{x}} = 1^\infty \text{ Ind} = L$$

$$\ln L = \ln \lim_{x \rightarrow 0} (1+x)^{\frac{\ln k}{x}} = \lim_{x \rightarrow 0} \ln (1+x)^{\frac{\ln k}{x}} = \lim_{x \rightarrow 0} \left[\frac{\ln k}{x} \cdot \ln(1+x) \right] =$$

$$= \infty \cdot 0 \text{ Ind} = \lim_{x \rightarrow 0} \frac{\ln k \cdot \ln(1+x)}{x} = \frac{0}{0} \text{ Ind (L'Hôp)} =$$

$$= \lim_{x \rightarrow 0} \frac{\ln k \cdot \frac{1}{1+x}}{1} = \lim_{x \rightarrow 0} \frac{\ln k}{1+x} = \ln k.$$

$$\ln L = \ln k \Rightarrow \underline{\underline{L = k}}$$

Para que f sea continua: $f(0) = \lim_{x \rightarrow 0} f(x)$.

$$\left. \begin{aligned} f(0) &= 5 \\ \lim_{x \rightarrow 0} f(x) &= k \end{aligned} \right\} \underline{\underline{k = 5}}$$

$$30) \lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right) = \infty - \infty \text{ Ind} = L$$

$$\begin{array}{l} \bullet 1-x^3=0 \\ \begin{array}{c|ccc} -1 & 0 & 0 & 1 \\ 1 & -1 & -1 & -1 \\ \hline & -1 & -1 & -1 \\ & & & 0 \end{array} \end{array} \quad \begin{aligned} 1-x^3 &= (x-1)(-x^2-x-1) = \\ &= (1-x)(x^2+x+1) \end{aligned}$$

$$L = \lim_{x \rightarrow 1} \frac{x^2+x+1-3}{(1-x)(x^2+x+1)} = \frac{0}{0} \text{ Ind} = L$$

$$\bullet x^2+x-2=0 \Rightarrow x = \frac{-1 \pm \sqrt{1+8}}{2} = \begin{cases} x=1 \\ x=-2 \end{cases}$$

$$L = \lim_{x \rightarrow 1} \frac{(x-1)(x+2)}{-(x-1)(x^2+x+1)} = \underline{\underline{\frac{-3}{4}}}$$

$$31) \lim_{x \rightarrow 0} \frac{1-\cos x^2}{8x^4} = \frac{0}{0} \text{ Ind (L'Hôp)} = \lim_{x \rightarrow 0} \frac{+2x \sin x^2}{16 \cdot 32x^3} = \frac{0}{0} \text{ Ind (L'H)} =$$

$$= \lim_{x \rightarrow 0} \frac{\sin x^2 + x \cdot 2x \cos x^2}{48x^2} = \frac{0}{0} \text{ Ind (L'Hôp)} = \lim_{x \rightarrow 0} \frac{2x \cos x^2 + 4x \cos x^2 - 4x^3 \sin x^2}{96x}$$

$$= \lim_{x \rightarrow 0} \frac{6x \cos x^2 - 4x^3 \sin x^2}{48 \cdot 96x} = \frac{3-0}{48} = \frac{3}{48} = \underline{\underline{\frac{1}{16}}}$$

$$32) \text{ Calcula } k \text{ sabiendo que } \lim_{x \rightarrow 0} [\cos(kx)]^{\frac{1}{x^2}} = \frac{1}{e^2} = e^{-2}$$

$$\lim_{x \rightarrow 0} [\cos kx]^{1/x^2} = 1^\infty \text{ Ind} = L$$

$$\ln L = \ln \lim_{x \rightarrow 0} (\cos kx)^{1/x^2} = \lim_{x \rightarrow 0} \ln (\cos kx)^{1/x^2} = \lim_{x \rightarrow 0} \frac{1}{x^2} \ln (\cos kx) =$$

$$= \infty \cdot 0 \text{ Ind} = \lim_{x \rightarrow 0} \frac{\ln (\cos kx)}{x^2} = \frac{0}{0} \text{ Ind (L'Hôp)} = \lim_{x \rightarrow 0} \frac{-k \cdot \sin kx}{2x} =$$

$$= \lim_{x \rightarrow 0} \frac{-k \sin kx}{2x \cdot \cos kx} = \frac{0}{0} \text{ Ind (L'Hôp)} = \lim_{x \rightarrow 0} \frac{-k^2 \cos kx}{2 \cos kx - 2x \cdot k \sin kx} =$$

$$= \frac{-k^2}{2-0} = \frac{-k^2}{2} \quad \ln L = \frac{-k^2}{2} \rightarrow L = e^{-k^2/2}$$

• Como la solución del límite: $J' = \frac{1}{e^2} = e^{-2}$

$$\text{Como } L = J' \rightarrow e^{-k^2/2} = e^{-2} \rightarrow \frac{-k^2}{2} = -2 \rightarrow k^2 = 4 \rightarrow k = \pm 2$$

$$33) \lim_{x \rightarrow 2} \frac{x^2 - \sqrt{8x}}{\sqrt{2x} - 2} = \frac{0}{0} \text{ Ind} = \lim_{x \rightarrow 2} \frac{(x^2 - \sqrt{8x})(x^2 + \sqrt{8x})(\sqrt{2x} + 2)}{(\sqrt{2x} - 2)(\sqrt{2x} + 2)(x^2 + \sqrt{8x})} =$$

$$= \lim_{x \rightarrow 2} \frac{(x^2)^2 - (\sqrt{8x})^2}{(\sqrt{2x})^2 - 2^2} = \lim_{x \rightarrow 2} \frac{x^4 - 8x}{2x - 4} = L$$

$$\cdot x^4 - 8x = x(x^3 - 8) = 0$$

$$x^2 + 2x + 4 = 0 \rightarrow x = \frac{-2 \pm \sqrt{4-16}}{2} \quad \text{No Sol.}$$

$$x^4 - 8x = x(x-2)(x^2 + 2x + 4)$$

$$L = \lim_{x \rightarrow 2} \frac{x(x-2)(x^2 + 2x + 4)}{2(x-2)} = \frac{2 \cdot 12}{2} = 12$$

34) ¿Para qué valores de k es f continua en $x=0$?

$$f(x) = \begin{cases} \left(\frac{x+1}{2x+1}\right)^{1/x} & \text{Si } x < 0 \\ 6x+k & \text{Si } x \geq 0 \end{cases}$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (6x+k) = k$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \left(\frac{x+1}{2x+1}\right)^{1/x} = 1^\infty \text{ Ind} = \lim_{x \rightarrow 0^-} \left(1 + \frac{x+1}{2x+1} - 1\right)^{1/x} =$$

$$= \lim_{x \rightarrow 0^+} \left[\left(1 + \frac{-x}{2x+1}\right)^{\frac{2x+1}{-x}} \right]^{\frac{1}{x} \cdot \frac{-x}{2x+1}} = e^{\lim_{x \rightarrow 0^+} \frac{-1}{2x+1}} = e^{-1} = \frac{1}{e}$$

$$\text{Como } \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x) \rightarrow k = \frac{1}{e}$$

Para $k = \frac{1}{e}$: $\exists \lim_{x \rightarrow 0} f(x) = \frac{1}{e} = f(0)$. por lo tanto f es continua.