1) $\lim_{x\to+\infty}\left(\frac{\sqrt{x^2+1}}{\sqrt{x^2-x}}\right)^x = 1^\infty = \lim_{x\to+\infty}\left[\sqrt{\frac{x^2+1}{x^2-x}}\right]^x = \lim_{x\to+\infty}\left(\frac{x^2+1}{x^2-x}\right)^{x/2} = L$

$\ln L = \ln \lim_{x\to+\infty}\left(\frac{x^2+1}{x^2-x}\right)^{x/2} = \lim_{x\to+\infty}\left[\ln\left(\frac{x^2+1}{x^2-x}\right)^{x/2}\right] =$

$= \lim_{x\to+\infty}\left[\frac{x}{2}\ln\left(\frac{x^2+1}{x^2-x}\right)\right] = \infty\cdot 0 \text{ Ind} = \lim_{x\to+\infty}\frac{\ln\left(\frac{x^2+1}{x^2-x}\right)}{\frac{2}{x}} = \frac{0}{0}\text{ Ind} =$

$= \lim_{x\to+\infty}\frac{\ln(x^2+1)-\ln(x^2-x)}{2/x} = \frac{0}{0}\text{ Ind}(\text{L'Hop}) =$

$= \lim_{x\to+\infty}\frac{\frac{2x}{x^2+1}-\frac{2x-1}{x^2-x}}{\frac{-2}{x^2}} = \lim_{x\to+\infty}\frac{-x^2[\cancel{2x^3}-2x^2-\cancel{2x^3}-2x+x^2+1]}{2(x^2+1)(x^2-x)} =$

$= \lim_{x\to+\infty}\frac{+x^4}{2x^4} = \frac{1}{2} \rightarrow \ln L = \frac{1}{2} \rightarrow L = e^{1/2} = \sqrt{e}$

otra forma de hacerlo:

• $\lim_{x\to+\infty}\left(\frac{x^2+1}{x^2-x}\right)^{x/2} = \lim_{x\to+\infty}\left(1+\frac{x^2+1}{x^2-x}-1\right)^{x/2} = \lim_{x\to+\infty}\left(1+\frac{x+1}{x^2-x}\right)^{x/2} =$

$= \lim_{x\to+\infty}\left[\left(1+\frac{1}{\frac{x^2-x}{x+1}}\right)^{\frac{x^2-x}{x+1}}\right]^{\frac{x}{2}\cdot\frac{x+1}{x^2-x}} = e^{\lim_{x\to+\infty}\frac{x(x+1)}{2(x^2-x)}} = e^{1/2} = \underline{\underline{\sqrt{e}}}$

2) $\lim_{x\to+\infty}\left(\sqrt{x^2-3x+2}-\sqrt{x^2-x}\right) = \infty-\infty \text{ Ind} =$

$= \lim_{x\to+\infty}\frac{\left(\sqrt{x^2-3x+2}-\sqrt{x^2-x}\right)\left(\sqrt{x^2-3x+2}+\sqrt{x^2-x}\right)}{\sqrt{x^2-3x+2}+\sqrt{x^2-x}} =$

$= \lim_{x\to+\infty}\frac{\left(\sqrt{x^2-3x+2}\right)^2-\left(\sqrt{x^2-x}\right)^2}{\sqrt{x^2-3x+2}+\sqrt{x^2-x}} = \lim_{x\to+\infty}\frac{-2x+2}{\sqrt{x^2-3x+2}+\sqrt{x^2-x}} =$

$= \lim_{x\to+\infty}\frac{-2x}{\sqrt{x^2}+\sqrt{x^2}} = \lim_{x\to+\infty}\frac{-2x}{2x} = \underline{\underline{-1}}$

3) $\lim_{x\to 2\pi}\frac{\cos^2 x+2\cos x-3}{6\cos x-6} = \frac{0}{0}\text{ Ind }(\text{L'Hop}) =$

$= \lim_{x\to 2\pi}\frac{-2\,\text{sen}\,x\cos x-2\,\text{sen}\,x}{-6\,\text{sen}\,x} = \lim_{x\to 2\pi}\frac{-2\cancel{\text{sen}\,x}(\cos x+1)}{-6\cancel{\text{sen}\,x}} = \frac{2\cdot 2}{6} = \frac{2}{3}$

* Se pueden aplicar infinitésimos equivalentes, aunque al sacar factor común es innecesario.

Otra forma de hacerlo:

$$\lim_{x\to 2\pi} \frac{\cos^2 x + 2\cos x - 3}{6\cos x - 6} = L \qquad \cos^2 x + 2\cos x - 3 = 0 \Rightarrow \cos x = \frac{-2\pm\sqrt{4+12}}{2} \Rightarrow$$

$$\Rightarrow \cos x = \frac{-2\pm 4}{2} \begin{cases} \cos x = 1 \\ \cos x = -3 \end{cases}$$

$$L = \lim_{x\to 2\pi} \frac{\cancel{(\cos x - 1)}(\cos x + 3)}{6\cancel{(\cos x - 1)}} = \frac{1+3}{6} = \frac{4}{6} = 2/3$$

4) $$\lim_{x\to +\infty} \left(\frac{x^2 - x + 2}{x^2 + 2x + 5}\right)^{\sqrt{2x+1}} = 1^{\infty} \text{ Ind}$$

Por L'Hopital puede resultar muy largo.

$$\lim_{x\to +\infty} \left(1 + \frac{x^2 - x + 2}{x^2 + 2x + 5} - 1\right)^{\sqrt{2x+1}} = \lim_{x\to +\infty} \left(1 + \frac{-3x-3}{x^2+2x+5}\right)^{\sqrt{2x+1}} =$$

$$= \lim_{x\to +\infty} \left[\left(1 + \frac{1}{\frac{x^2+2x+5}{-3x-3}}\right)^{\frac{x^2+2x+5}{-3x-3}}\right]^{\sqrt{2x+1}\cdot\frac{(-3x-3)}{x^2+2x+5}} = e^{\lim_{x\to +\infty} \frac{(-3x-3)\sqrt{2x+1}}{x^2+2x+5}} =*$$

$= e^0 = 1$ * grado del denominador > grado del numerador.

5) $$\lim_{x\to 0} \frac{1 - 2x - e^x + \operatorname{sen}(3x)}{x^2} = \frac{0}{0} \text{ Ind (L'Hopital)} =$$

$$= \lim_{x\to 0} \frac{-2 - e^x + 3\cos(3x)}{2x} = \frac{0}{0} \text{ Ind (L'Hop)} = \lim_{x\to 0} \frac{-e^x - 9\operatorname{sen}(3x)}{2} = -\frac{1}{2}$$

6) $$\lim_{x\to +\infty} \frac{(5x^2+2)(x-6)}{(x^2-1)(2x-1)} = \lim_{x\to +\infty} \frac{5x^3 + \cdots}{2x^3 + \cdots} = \frac{5}{2}$$

7) $$\lim_{x\to 3} \left(\frac{x}{x-3} - \frac{4x^2}{x^2-9}\right) = \infty - \infty \text{ Ind} = \lim_{x\to 3} \frac{x(x+3) - 4x^2}{(x-3)(x+3)} =$$

$$= \lim_{x\to 3} \frac{-3x^2 + 3x}{(x-3)(x+3)} = \nexists \begin{cases} \lim_{x\to 3^-} f(x) = +\infty \\ \lim_{x\to 3^+} f(x) = -\infty \end{cases}$$

8) $$\lim_{x\to +\infty} \left(\frac{5x^2 + x}{5x^2 - 3}\right)^{\frac{2x^2-1}{x}} = 1^{\infty} \text{ Ind} = \lim_{x\to +\infty} \left(1 + \frac{5x^2+x}{5x^2-3} - 1\right)^{\frac{2x^2-1}{x}} =$$

$$= \lim_{x\to +\infty} \left(1 + \frac{x+3}{5x^2-3}\right)^{\frac{2x^2-1}{x}} = \lim_{x\to +\infty} \left[\left(1 + \frac{1}{\frac{5x^2-3}{x+3}}\right)^{\frac{5x^2-3}{x+3}}\right]^{\frac{2x^2-1}{x}\cdot\frac{x+3}{5x^2-3}} =$$

$$= e^{\lim_{x\to +\infty} \frac{(2x^2-1)(x+3)}{x(5x^2-3)}} = e^{2/5} = \sqrt[5]{e^2}$$

9) $\lim\limits_{x\to+\infty} \dfrac{x^2+3x}{\sqrt{2x^2+1}} = \lim\limits_{x\to+\infty} \dfrac{x^2}{\sqrt{2}\,x} = +\infty$

10) $\lim\limits_{x\to-\infty} \boxed{\dfrac{x^2+3x}{\sqrt{2x^2+1}}}\,^{*} = \lim\limits_{x\to+\infty} \dfrac{(-x)^2+3(-x)}{\sqrt{2(-x)^2+1}} = \lim\limits_{x\to+\infty} \dfrac{x^2-3x}{\sqrt{2x^2+1}} = +\infty$

* Antes de operar el límite comprobamos el dominio de:

$f(x) = \dfrac{x^2+3x}{\sqrt{2x^2+1}}$ $\quad 2x^2+1>0 \to 2x^2+1$ es siempre positivo. $\text{Dom} f = \mathbb{R}$.

11) $\lim\limits_{x\to+\infty} \dfrac{x^2+3x}{\sqrt{2x^3+1}} = +\infty$

12) $\lim\limits_{x\to-\infty} \dfrac{x^2+3x}{\sqrt{2x^3+1}}\,^{*}$ $\nexists$ límite.

* Calculamos el dominio de $f(x) = \dfrac{x^2+3x}{\sqrt{2x^3+1}}$.

$2x^3+1>0 \to 2x^3+1=0 \to x = \sqrt[3]{\dfrac{-1}{2}} = \dfrac{-1}{\sqrt[3]{2}} \cdot \dfrac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}} = \dfrac{-\sqrt[3]{2^2}}{2}$ → Si estudiamos el signo

$\text{Dom} f = \left(\dfrac{-\sqrt[3]{2^2}}{2}, +\infty\right).$

13) $\lim\limits_{x\to 0} [x\cdot \text{cotg}(2x)] = 0\cdot\infty \text{ Ind} = \lim\limits_{x\to 0} \dfrac{x}{\text{tg}(2x)} = \dfrac{0}{0} \text{ Ind (L'Hop)} =$

$= \lim\limits_{x\to 0} \dfrac{1}{2(1+\text{tg}^2(2x))} = \dfrac{1}{2(1+0)} = 1/2$

Otra forma:

$\lim\limits_{x\to 0} [x\,\text{cotg}(2x)] = 0\cdot\infty \text{ Ind} = \lim\limits_{x\to 0} \dfrac{x}{\text{tg}(2x)} = \dfrac{0}{0} \text{ Ind} = \lim\limits_{x\to 0} \dfrac{x}{\frac{\text{sen}(2x)}{\cos(2x)}} =$

$= \lim\limits_{x\to 0} \dfrac{x\cos(2x)}{\text{sen}(2x)}\,^{*} = \dfrac{0}{0} \text{ Ind} = \lim\limits_{x\to 0} \dfrac{\cancel{x}\cos(2x)}{2\cancel{x}} = \lim\limits_{x\to 0} \dfrac{\cos(2x)}{2} = 1/2$

* Aplicando infinitésimos equivalentes: $x\to 0$: $\text{sen}\,x \simeq x$.

14) $\lim\limits_{x\to \pi/2} \left[\left(\dfrac{\pi}{2}-x\right)\text{tg}\,x\right] = 0\cdot\infty \text{ Ind} = \lim\limits_{x\to\pi/2} \dfrac{\frac{\pi}{2}-x}{\text{cotg}\,x}\,^{*} = \dfrac{0}{0} \text{ Ind (L'Hop)} =$

$= \lim\limits_{x\to\pi/2} \dfrac{-1}{-1/\text{sen}^2 x} = \lim\limits_{x\to\pi/2} \text{sen}^2 x = 1$

* $y = \text{cotg}\,x = \dfrac{1}{\text{tg}\,x} \to y' = \dfrac{-1/\cos^2 x}{\text{tg}^2 x} = \dfrac{-1/\cos^2 x}{\frac{\text{sen}^2 x}{\cos^2 x}} = \dfrac{-1}{\text{sen}^2 x}$

15) $\lim\limits_{x\to\sqrt{2}} \dfrac{e^{2x^2+x-1} - e^{x^2+x+1}}{x^2-2} = \dfrac{0}{0} \text{ Ind (L'Hôp)} =$

$= \lim\limits_{x\to\sqrt{2}} \dfrac{(4x+1)e^{2x^2+x-1} - (2x+1)e^{x^2+x+1}}{2x} = \dfrac{(4\sqrt{2}+1)e^{3+\sqrt{2}} - (2\sqrt{2}+1)e^{3+\sqrt{2}}}{2\sqrt{2}} =$

$= \dfrac{e^{3+\sqrt{2}}(4\sqrt{2}+1-2\sqrt{2}-1)}{2\sqrt{2}} = \dfrac{\cancel{2\sqrt{2}}\,e^{3+\sqrt{2}}}{\cancel{2\sqrt{2}}} = e^{3+\sqrt{2}}$

16) $$\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\ln(1+x)}\right)=\infty-\infty \text{ Ind}=\lim_{x\to 0}\frac{\ln(1+x)-x}{x\ln(1+x)}=$$

$$=\frac{0}{0}\text{ Ind }(\text{L'Hôp})=\lim_{x\to 0}\frac{\frac{1}{1+x}-1}{\ln(1+x)+\frac{x}{1+x}}=\lim_{x\to 0}\frac{\frac{-x}{\cancel{1+x}}}{\frac{(1+x)\ln(1+x)+x}{\cancel{1+x}}}=$$

$$=\lim_{x\to 0}\frac{-x}{(1+x)\ln(1+x)+x}=\frac{0}{0}\text{ Ind }(\text{L'Hôp})=\lim_{x\to 0}\frac{-1}{\ln(1+x)+\frac{\cancel{1+x}}{\cancel{1+x}}+1}=$$

$$=\frac{-1}{0+1+1}=-1/2$$

17) $$\lim_{x\to 0}\frac{x^3\cos x}{\operatorname{sen}x+\sqrt{x}}=\frac{0}{0}\text{ Ind }(\text{L'Hôp})=$$

$$=\lim_{x\to 0}\frac{3x^2\cos x-x^3\operatorname{sen}x}{\cos x+\frac{1}{2\sqrt{x}}}=\lim_{x\to 0}\frac{x^2(3\cos x-x\operatorname{sen}x)}{\frac{2\sqrt{x}\cos x+1}{2\sqrt{x}}}=$$

$$=\lim_{x\to 0}\frac{2\sqrt{x}\,x^2(3\cos x-x\operatorname{sen}x)}{2\sqrt{x}\cos x+1}=\frac{0}{0+1}=0$$

18) $$\lim_{x\to +\infty}\left(\sqrt{x^2+2x}-\sqrt{x^2-x+3}\right)=\infty-\infty\text{ Ind}=$$

$$=\lim_{x\to+\infty}\frac{\left(\sqrt{x^2+2x}-\sqrt{x^2-x+3}\right)\left(\sqrt{x^2+2x}+\sqrt{x^2-x+3}\right)}{\sqrt{x^2+2x}+\sqrt{x^2-x+3}}=$$

$$=\lim_{x\to+\infty}\frac{\left(\sqrt{\cancel{x^2}+2x}\right)^2-\left(\sqrt{\cancel{x^2}-x+3}\right)^2}{\sqrt{x^2+2x}+\sqrt{x^2-x+3}}=\lim_{x\to+\infty}\frac{3x-3}{\sqrt{x^2+2x}+\sqrt{x^2-x+3}}=$$

$$=\lim_{x\to+\infty}\frac{3x}{\sqrt{x^2}+\sqrt{x^2}}=\lim_{x\to+\infty}\frac{3x}{2x}=3/2$$

19) $$\lim_{x\to+\infty}\left(\frac{2x+1}{2x+4}\right)^{\frac{x^2}{x+1}}=1^{\infty}\text{ Ind}=$$

$$=\lim_{x\to+\infty}\left(1+\frac{2x+1}{2x+4}-1\right)^{\frac{x^2}{x+1}}=\lim_{x\to+\infty}\left(1+\frac{-3}{2x+4}\right)^{\frac{x^2}{x+1}}=$$

$$=\lim_{x\to+\infty}\left[\left(1+\frac{1}{\frac{2x+4}{-3}}\right)^{\frac{2x+4}{-3}}\right]^{\frac{x^2}{x+1}\cdot\frac{-3}{2x+4}}=e^{\lim_{x\to+\infty}\frac{-3x^2}{(x+1)(2x+4)}}=$$

$$=e^{-3/2}=\frac{1}{e^{3/2}}=\frac{1}{\sqrt{e^3}}=\frac{1}{e\sqrt{e}}\cdot\frac{\sqrt{e}}{\sqrt{e}}=\frac{\sqrt{e}}{e^2}$$