

Potencias y raíces.

I. Ejercicios de potencias y raíces:

1. $\frac{25^2 \cdot 2^3 \cdot 6^2}{3^3 \cdot 8^2 \cdot 5^4} =$
2. $\frac{15^2 \cdot 2^3 \cdot 35^2}{3^3 \cdot 5^2 \cdot 2^4 \cdot 7} =$
3. $\frac{12^2 \cdot 2^3 \cdot 30^2}{3^3 \cdot 8^2 \cdot 5^4} =$
4. $\frac{4^2 \cdot 2^3 \cdot 15^2}{3^3 \cdot 5^2 \cdot 2^4} =$
5. $\frac{4^2 \cdot 12^3 \cdot 15^2}{9^3 \cdot 8^2 \cdot 3^3} =$
6. $\frac{8^4 \cdot 15^3 \cdot 18^2 \cdot 12^{-3}}{20^3 \cdot 27^2 \cdot 3^{-3}} =$
7. $\frac{27^{-1} \cdot 81 \cdot 3^4 \cdot \left(\frac{2}{3}\right)^{-1} \cdot 2^3}{36 \cdot \left(\frac{1}{3}\right)^{-2} \cdot \frac{4 \cdot 27}{3 \cdot 16} \cdot 2^0} =$
8. $\frac{(-27)^3 \cdot 32^{-5} \cdot (-8)^5 \cdot (25^2)^{-6}}{(-72)^4 \cdot (-50^3)^4} =$
9. $\frac{a^3 \cdot b^2 \cdot c \cdot b^0}{a^{-1} \cdot b^{-2} \cdot c^{-2} \cdot a} =$
10. $\left(\frac{(a \cdot a^3)^{-2}}{a^{-4}}\right)^{-3} =$
11. $\frac{\left(\frac{2}{5}\right)^3 \cdot \left(\frac{2}{3}\right)^2 \cdot \left(\frac{5}{6}\right)^3}{\left(\frac{4}{3}\right)^5 \cdot \left(\frac{1}{8}\right)^2} =$
12. $\frac{10^{-3} \cdot 2^5 \cdot 14^2 \cdot 5}{6^5 \cdot 12^3} =$
13. $\frac{\left(\left(\frac{2}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^{-5}\right)^{-3}}{\left(\left(\frac{2}{3}\right)^{-5} : \left(\frac{2}{3}\right)^{-8}\right)^{-2}} =$
14. $\frac{6 \cdot 12^3 \cdot 18^2 \cdot 3^2 \cdot 108^2}{27^2 \cdot 3^2 \cdot 16 \cdot 48 \cdot 36} =$
15. $\frac{15^2 \cdot 5^{-2} \cdot 5^3 \cdot 45^2}{(5^3)^2 \cdot 27 \cdot 3^{-2}} =$

II. Realiza las siguientes operaciones:

1. $\sqrt{\frac{27}{4}}$
2. $\sqrt[5]{\frac{5x^{10}}{y^8}}$
3. $\sqrt[3]{\frac{8x^4y^{32}}{n^6}}$
4. $\sqrt[4]{\frac{x^581}{y^6}}$
5. $\sqrt{8a^3b^2}$
6. $\sqrt[3]{24a^5b^3}$
7. $\sqrt{\frac{16a^3}{b^2c^3}}$
8. $\sqrt[4]{\frac{81a^5b^2}{c^7d^8}}$
9. $\sqrt{\frac{a^4}{b^6}}$
10. $\sqrt[3]{\frac{a^3b^9}{c^{13}}}$
11. $\sqrt[3]{\frac{a^7bc^4}{d^5}}$
12. $\sqrt{a^2\sqrt{b^3c^5}}$
13. $\sqrt[3]{\frac{\sqrt{x^6a^2}}{\sqrt[3]{b^9c^3}}}$
14. $\sqrt[3]{\frac{a^5b^6c}{c^5b}}$
15. $\sqrt[4]{\frac{81a^5b^2}{c^7d^8}}$

III. Introduce en las raíces todos los términos:

1. $ab\sqrt{cd}$
2. $ab\sqrt[3]{ab^2}$
3. $\frac{ab^2}{c} \sqrt[3]{\frac{2c^2}{b}}$
4. $\frac{abc}{d} \sqrt{\frac{cd}{a}}$
5. $2ab \sqrt[5]{\frac{b}{4a}}$
6. $\sqrt[3]{bc^2} \sqrt{\frac{a}{c}}$
7. $\sqrt{ab} \sqrt{\frac{c}{ab}}$
8. $ca^2 \sqrt[4]{\frac{ab^2}{c^3}}$
9. $-2ab \sqrt[3]{\frac{b}{4a}}$
10. $\frac{\sqrt{x^2y^3}}{x}$
11. $\frac{3}{2} \sqrt[3]{\frac{4}{9}}$
12. $\frac{3}{2} \sqrt[3]{\frac{4}{3}}$

IV. Realiza las siguientes operaciones:

1. $\sqrt{125} - 2\sqrt{5} + \sqrt{20} =$
2. $\sqrt{75} - \sqrt{8} + 3\sqrt{12} - 2\sqrt{32} =$
3. $\sqrt{4a^2b} - \sqrt{9ab^2} + 3\sqrt{a} - 2a\sqrt{b} =$
4. $2\sqrt{a} + \sqrt{8b} - 3\sqrt{2b} + 3\sqrt{a^3} =$
5. $\sqrt{\frac{16}{3}} - 2\sqrt{\frac{4}{3}} + 3\sqrt{\frac{1}{27}} - 2\sqrt{\frac{25}{3}} =$
6. $\sqrt{\frac{8}{9}} - 3\sqrt{\frac{2}{9}} - 2\sqrt{\frac{2}{16}} + \sqrt{32} =$
7. $\sqrt{125} - \sqrt{\frac{5}{4}} + 2\sqrt{\frac{45}{9}} - 2\sqrt{5} =$
8. $\sqrt{50} + \sqrt{8} + \sqrt{2} - 3\sqrt{2} =$
9. $\sqrt{18} + \sqrt{20} - 2\sqrt{8} + \sqrt{45} =$
10. $3\sqrt{a} - \sqrt{4b} + 3\sqrt{4a} + 3\sqrt{b} =$

Bloque 1. Potencias,

$$\bullet a^n \cdot a^m = a^{n+m}$$

$$2^3 \cdot 2^5 = 2^{3+5}$$

$$2^3 = 2 \cdot 2 \cdot 2$$

$$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

$$\left\{ \begin{array}{l} 2^3 \cdot 2^5 = \overbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}^{2^8} = 2^8 \\ \underbrace{2 \cdot 2 \cdot 2}_{2^3} \cdot \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{2^5} \end{array} \right.$$

$$\bullet \frac{a^n}{a^m} = a^{n-m}$$

$$\left\{ \begin{array}{l} \frac{2^5}{2^3} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} = 2^{5-3} = 2^2 \\ \frac{2^3}{2^5} = 2^{3-5} = 2^{-2} = \frac{1}{2^2} \end{array} \right.$$

$$\bullet (a^n)^m = a^{n \cdot m}$$

$$(2^3)^5 = 2^{3 \cdot 5} = 2^{15}$$

$$\bullet a^{-m} = \frac{1}{a^m} \quad 2^{-2} = \frac{1}{2^2}$$

$$\frac{1}{a^{-m}} = a^m \quad \frac{1}{2^{-2}} = 2^2$$

$$\bullet a^n \cdot b^n = (a \cdot b)^n; \quad \frac{a^n}{b^n} = \left(\frac{a}{b} \right)^n$$

$$\bullet a^0 = 1; \quad a^1 = a; \quad 1^n = 1$$

$$\bullet (-a)^n: \quad \begin{array}{l} \bullet n \text{ impar: } (-a)^n = -a^n, \quad (-3)^3 = -3^3 \\ \bullet n \text{ es par: } (-a)^n = a^n, \quad (-3)^4 = 3^4 \end{array}$$

$$1: \frac{25^2 \cdot 2^3 \cdot 6^2}{3^3 \cdot 8^2 \cdot 5^4} = \frac{(5^2)^2 \cdot 2^3 \cdot (3 \cdot 2)^2}{3^3 \cdot (2^3)^2 \cdot 5^4} = \frac{5^4 \cdot 2^3 \cdot 3^2 \cdot 2^2}{3^3 \cdot 2^6 \cdot 5^4} =$$

$$= \frac{2^5 \cdot 3^2}{3^3 \cdot 2^6} = 2^{-1} \cdot 3^{-1} = \frac{1}{2 \cdot 3} = \frac{1}{6}$$

$$2: \frac{15^2 \cdot 2^3 \cdot 35^2}{3^3 \cdot 5^2 \cdot 2^4 \cdot 7} = \frac{(5 \cdot 3)^2 \cdot 2^3 \cdot (7 \cdot 5)^2}{3^3 \cdot 5^2 \cdot 2^4 \cdot 7} = \frac{5^2 \cdot 3^2 \cdot 2^3 \cdot 7^2 \cdot 5^2}{3^3 \cdot 5^2 \cdot 2^4 \cdot 7} =$$

$$(a \cdot b)^n = a^n \cdot b^n$$

$$\frac{a^n}{a^m} = a^{n-m}$$

$$= 5^2 \cdot 3^{-1} \cdot 2^{-1} \cdot 7 = \frac{5^2 \cdot 7}{3 \cdot 2} = \frac{5^2 \cdot 7}{6}$$

$a^{-n} = \frac{1}{a^n}$

$$3: \frac{12^2 \cdot 2^3 \cdot 30^2}{3^3 \cdot 8^2 \cdot 5^4} = \frac{(2^2 \cdot 3)^2 \cdot 2^3 \cdot (3 \cdot 2 \cdot 5)^2}{3^3 \cdot (2^3)^2 \cdot 5^4} = \frac{2^4 \cdot 3^2 \cdot 2^3 \cdot 3^2 \cdot 2^2 \cdot 5^2}{3^3 \cdot 2^6 \cdot 5^4}$$

$12 = 2^2 \cdot 3$
 $30 = 3 \cdot 2 \cdot 5$
 $8 = 2^3$

$(ab)^n = a^n b^n$
 $(a^n)^m = a^{n \cdot m}$

$a^n \cdot a^m = a^{n+m}$

$$= \frac{2^9 \cdot 3^4 \cdot 5^2}{3^3 \cdot 2^6 \cdot 5^4} = 2^3 \cdot 3 \cdot 5^{-2} = \frac{2^3 \cdot 3}{5^2}$$

$\frac{a^n}{a^m} = a^{n-m}$ $a^{-n} = \frac{1}{a^n}$

$$4: \frac{4^2 \cdot 2^3 \cdot 15^2}{3^3 \cdot 5^2 \cdot 2^4} = \frac{(2^2)^2 \cdot 2^3 \cdot (5 \cdot 3)^2}{3^3 \cdot 5^2 \cdot 2^4} = \frac{2^4 \cdot 2^3 \cdot 5^2 \cdot 3^2}{3^3 \cdot 5^2 \cdot 2^4} = 2^3 \cdot 3^{-1} = \frac{2^3}{3}$$

$$5: \frac{4^2 \cdot 12^3 \cdot 15^2}{9^3 \cdot 8^2 \cdot 3^3} = \frac{(2^2)^2 \cdot (2^2 \cdot 3)^3 \cdot (5 \cdot 3)^2}{(3^2)^3 \cdot (2^3)^2 \cdot 3^3} = \frac{2^4 \cdot 2^6 \cdot 3^3 \cdot 5^2 \cdot 3^2}{3^6 \cdot 2^6 \cdot 3^3} =$$

$$= \frac{2^{10} \cdot 3^5 \cdot 5^2}{3^9 \cdot 2^6} = 2^4 \cdot 3^4 \cdot 5^2 = \frac{2^4 \cdot 5^2}{3^4}$$

$$6: \frac{8^4 \cdot 15^3 \cdot 18^2 \cdot 12^{-3}}{20^3 \cdot 27^2 \cdot 3^{-3}} = \frac{(2^3)^4 \cdot (5 \cdot 3)^3 \cdot (3^2 \cdot 2)^2 \cdot (2^2 \cdot 3)^{-3}}{(2^2 \cdot 5)^3 \cdot (3^3)^2 \cdot 3^{-3}} =$$

$$= \frac{2^{12} \cdot 5^3 \cdot 3^3 \cdot 3^4 \cdot 2^2 \cdot 2^{-6} \cdot 3^{-3}}{2^6 \cdot 5^3 \cdot 3^6 \cdot 3^{-3}} = \frac{2^{14} \cdot 5^3 \cdot 3^7 \cdot 3^3}{2^6 \cdot 5^3 \cdot 3^6 \cdot 2^6 \cdot 3^3} = \frac{2^{14} \cdot 5^3 \cdot 3^{10}}{2^{12} \cdot 5^3 \cdot 3^9} =$$

$$= 2^2 \cdot 3$$

$$7: \frac{27^{-1} \cdot 81 \cdot 3^4 \cdot \left(\frac{2^3}{3}\right)^{-1} \cdot 2^3}{36 \cdot \left(\frac{1}{3}\right)^{-2} \cdot \frac{4}{3} \cdot \frac{27}{16} \cdot 2^0} = \frac{(3^3)^{-1} \cdot 3^4 \cdot 3^4 \cdot \left(\frac{2^3}{3}\right)^{-1} \cdot 2^3}{2^2 \cdot 3^2 \cdot \left(\frac{1}{3}\right)^{-2} \cdot \frac{2^2}{3} \cdot \frac{3^3}{2^4}}$$

$27 = 3^3$
 $81 = 3^4$
 $36 = 2^2 \cdot 3^2$
 $16 = 2^4$
 $2^0 = 1$

$(a^n)^m = a^{n \cdot m}$
 $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$$\begin{aligned}
 &= \frac{3^{-3} \cdot 3^4 \cdot 3^4 \cdot \frac{2^{-3}}{3^{-1}} \cdot 2^3}{2^2 \cdot 3^2 \cdot \frac{1^{-2}}{3^{-2}} \cdot \frac{2^2}{3} \cdot \frac{3^3}{2^4}} = \frac{\frac{3^{-3} \cdot 3^4 \cdot 3^4 \cdot 2^{-3} \cdot 2^3}{3^{-1}}}{\frac{2^2 \cdot 3^2 \cdot 1^{-2} \cdot 2^2 \cdot 3^3}{3^{-2} \cdot 3 \cdot 2^4}} = \frac{3^4 \cdot 3^4 \cdot 2^3 \cdot 3}{3^3 \cdot 2^3} \leftarrow \\
 &= \frac{\frac{3^9 \cdot 2^3}{3^3 \cdot 2^3}}{\frac{2^4 \cdot 3^3}{3 \cdot 2^4}} = \frac{3^6}{3^6} = 1
 \end{aligned}$$

$a^{-n} = \frac{1}{a^n}$; $\frac{1}{a^{-n}} = a^n$
 $a^n \cdot a^m = a^{n+m}$
 $\frac{a^n}{a^m} = a^{n-m}$

$$8: \frac{(-27)^3 \cdot 32^{-5} \cdot (-8)^5 \cdot (25)^{-6}}{(-72)^4 \cdot (-50)^4} = \frac{27^3 \cdot 32^{-5} \cdot 8^5 \cdot (25)^{-6}}{72^4 \cdot (50)^4} =$$

$$\begin{aligned}
 27 &= 3^3 \\
 32 &= 2^5 \\
 72 &= 2^3 \cdot 3^2 \\
 50 &= 2 \cdot 5^2
 \end{aligned}$$

$$= \frac{(3^3)^3 \cdot (2^5)^{-5} \cdot (2^3)^5 \cdot ((5^2)^{-6})}{(2^3 \cdot 3^2)^4 \cdot ((2 \cdot 5^2)^4)} = \frac{3^9 \cdot 2^{-25} \cdot 2^{15} \cdot 5^{-24}}{2^{12} \cdot 3^8 \cdot 2^{12} \cdot 5^{24}} = \frac{3^9 \cdot 2^{-10} \cdot 5^{-24}}{2^{24} \cdot 3^8 \cdot 5^{24}}$$

$(a^m)^n = a^{m \cdot n}$
 $a^{-n} = \frac{1}{a^n}$

$$= \frac{3^9 \cdot 2^{-10}}{2^{24} \cdot 3^8 \cdot 5^{24} \cdot 2^{25} \cdot 5^{24}} = \frac{3^9 \cdot 2^{-34}}{2^{49} \cdot 3^8 \cdot 5^{48}} = \frac{3 \cdot 2^{-34}}{5^{48}} = \frac{3}{2^{34} \cdot 5^{48}}$$

$a^n \cdot a^m = a^{n+m}$
 $\frac{a^n}{a^m} = a^{n-m}$
 $a^{-n} = \frac{1}{a^n}$

$$= \frac{3}{2^{34} \cdot 5^{48}}$$

$$9: \frac{a^3 \cdot b^2 \cdot c \cdot b^4}{a^{-1} \cdot b^{-2} \cdot c^{-2} \cdot a} = \frac{a^3 \cdot b^2 \cdot c \cdot a \cdot b^2 \cdot c^2}{a^{-1} \cdot b^{-2} \cdot c^{-2} \cdot a} = a^3 \cdot b^4 \cdot c^3$$

$$10: \left(\frac{(a \cdot a^3)^{-2}}{a^{-4}} \right)^{-3} = \frac{(a^4)^6}{a^{12}} = \frac{a^{24}}{a^{12}} = a^{12}$$

$$11: \frac{\left(\frac{2}{5}\right)^3 \left(\frac{2}{3}\right)^2 \left(\frac{5}{6}\right)^3}{\left(\frac{4}{3}\right)^5 \cdot \left(\frac{1}{8}\right)^2} = \frac{\frac{2^3 \cdot 2^2 \cdot 5^3}{5^3 \cdot 3^2 \cdot (3 \cdot 2)^3}}{\frac{(2^2)^5 \cdot 1^2}{3^5 \cdot (2^3)^2}} = \frac{\frac{2^5}{3^2 \cdot 3^3 \cdot 2^3}}{\frac{2^{10}}{3^5 \cdot 2^6}} = \frac{\frac{2^2}{3^5}}{\frac{2^4}{3^5}} = \frac{2^2}{2^4} = \frac{1}{2^2}$$

$$= \frac{2^2}{3^5} \cdot \frac{3^5}{2^4} = \frac{2^2 \cdot 3^5}{3^5 \cdot 2^4} = 2^{-2} = \frac{1}{2^2}$$

$$12: \frac{10^{-3} \cdot 2^5 \cdot 14^2 \cdot 5}{6^5 \cdot 12^3} = \frac{(5 \cdot 2)^{-3} \cdot 2^5 \cdot (7 \cdot 2)^2 \cdot 5}{(3 \cdot 2)^5 \cdot (2^2 \cdot 3)^3} = \frac{5^{-3} \cdot 2^{-3} \cdot 2^5 \cdot 7^2 \cdot 2^2 \cdot 5}{3^5 \cdot 2^5 \cdot 2^6 \cdot 3^3} =$$

$$= \frac{7^2 \cdot 2^2 \cdot 5}{3^8 \cdot 2^9 \cdot 5^3} = \frac{7^2 \cdot 2^{-7} \cdot 5^{-2}}{3^8} = \frac{7^2}{3^8 \cdot 2^7 \cdot 5^2}$$

$$13: \frac{\left[\left(\frac{2}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^{-5}\right]^{-3}}{\left[\left(\frac{2}{3}\right)^{-5} \cdot \left(\frac{2}{3}\right)^{-8}\right]^{-2}} = \frac{\left(\frac{2}{3}\right)^{-6} \cdot \left(\frac{2}{3}\right)^{15}}{\left(\frac{2}{3}\right)^{10} \cdot \left(\frac{2}{3}\right)^{16}} = \frac{\left(\frac{2}{3}\right)^9}{\left(\frac{2}{3}\right)^{26}} = \left(\frac{2}{3}\right)^{9-26} = \left(\frac{2}{3}\right)^{-17} = \frac{3^{17}}{2^{17}}$$

$$= \left(\frac{2}{3}\right)^{15}$$

$$14: \frac{6 \cdot 12^3 \cdot 18^2 \cdot 3^2 \cdot 108^2}{27^2 \cdot 5^2 \cdot 16 \cdot 48 \cdot 36} = \frac{3 \cdot 2 \cdot (2^2 \cdot 3)^3 \cdot (2 \cdot 3^2)^2 \cdot 3^2 \cdot (3^3 \cdot 2^2)^2}{3^2 \cdot 5^2 \cdot 2^4 \cdot 2^4 \cdot 3 \cdot 2^2 \cdot 3^2} =$$

$$\begin{aligned} 12 &= 2^2 \cdot 3 \\ 18 &= 2 \cdot 3^2 \\ 108 &= 3^3 \cdot 2^2 \\ 27 &= 3^3 \\ 16 &= 2^4 \\ 48 &= 2^4 \cdot 3 \\ 36 &= 2^2 \cdot 3^2 \end{aligned}$$

$$(a^4)^m = a^{4 \cdot m} \\ a^4 \cdot a^4 = a^{4+4}$$

$$= \frac{3 \cdot 2 \cdot 2^6 \cdot 3^3 \cdot 2^2 \cdot 3^4 \cdot 3^2 \cdot 3^6 \cdot 2^4}{3^8 \cdot 2^{10}} = \frac{3^{16} \cdot 2^{13}}{3^8 \cdot 2^{10}} = 3^8 \cdot 2^3$$

$$\frac{a^n}{a^m} = a^{n-m}$$

$$15: \frac{15^2 \cdot 5^{-2} \cdot 5^3 \cdot 45^2}{(5^3)^2 \cdot 27 \cdot 3^{-2}} = \frac{(5 \cdot 3)^2 \cdot 5^{-2} \cdot 5^3 \cdot (3^2 \cdot 5)^2}{(5^3)^2 \cdot 3^3 \cdot 3^{-2}} = \frac{5^2 \cdot 3^2 \cdot 5^{-2} \cdot 5^3 \cdot 3^4 \cdot 5^2}{5^6 \cdot 3^3 \cdot 3^{-2}} =$$

$$= \frac{5^2 \cdot 3^2 \cdot 5^3 \cdot 3^4 \cdot 5^2 \cdot 3^2}{5^6 \cdot 3^3 \cdot 5^2} = \frac{5^7 \cdot 3^8}{5^8 \cdot 3^3} = 5^{-1} \cdot 3^5 = \frac{3^5}{5}$$

Bloque 2. Raíces.

$$1: \sqrt{\frac{27}{4}} = \sqrt{\frac{3^3}{2^2}} = \frac{3\sqrt{3}}{2}$$

$$2: \sqrt[5]{\frac{5x^{10}}{y^8}} = \frac{x^2}{y} \cdot \frac{\sqrt[5]{5}}{\sqrt[5]{y^3}} = \frac{x^2 \cdot \sqrt[5]{5}}{y \cdot \sqrt[5]{y^3}}$$

$$3: \sqrt[3]{\frac{8x^4y^{32}}{n^6}} = \frac{2 \times y^{10}}{n^2} \sqrt[3]{xy^2}$$

$$4: \sqrt[4]{\frac{x^5 \cdot 81}{y^6}} = \sqrt[4]{\frac{x^5 \cdot 3^4}{y^6}} = \frac{3x}{y} \sqrt[4]{\frac{x}{y^2}}$$

$$5: \sqrt{8a^3 \cdot b^2} = \sqrt{2^3 \cdot a^3 \cdot b^2} = \underline{2ab\sqrt{2a}}$$

$$6: \sqrt[3]{24a^5b^3} = \sqrt[3]{2^3 \cdot 3 \cdot a^5 \cdot b^3} = \sqrt[3]{2^3} \cdot \sqrt[3]{3} \cdot \sqrt[3]{a^5} \cdot \sqrt[3]{b^3} = 2b\sqrt[3]{3a^5} =$$

$$= 2ba\sqrt[3]{3a^2}$$

$$7: \sqrt{\frac{16a^3}{b^2c^3}} = \frac{\sqrt{16a^3}}{\sqrt{b^2c^3}} = \frac{\sqrt{2^4 \cdot a^3}}{\sqrt{b^2 \cdot c^3}} = \frac{2^2 a \sqrt{a}}{bc \sqrt{c}} = \frac{2^2 a}{bc} \sqrt{\frac{a}{c}}$$

$$8: \sqrt[4]{\frac{81a^5 \cdot b^2}{c^7 d^8}} = \frac{\sqrt[4]{81a^5 \cdot b^2}}{\sqrt[4]{c^7 d^8}} = \frac{\sqrt[4]{3^4 \cdot a^5 \cdot b^2}}{\sqrt[4]{c^7 \cdot d^8}} = \frac{3a \sqrt[4]{ab^2}}{cd \sqrt[4]{c^3}} = \frac{3a}{cd^2} \sqrt[4]{\frac{ab^2}{c^3}}$$

$$10: \sqrt[3]{\frac{a^3 b^9}{c^{15}}} = \frac{\sqrt[3]{a^3 b^9}}{\sqrt[3]{c^{15}}} = \frac{a b^3}{c^5 \sqrt[3]{c}}$$

$$11: \sqrt[3]{\frac{a^7 b c^4}{d^5}} = \frac{\sqrt[3]{a^7 b c^4}}{\sqrt[3]{d^5}} = \frac{a^2 c \sqrt[3]{a b c}}{d \sqrt[3]{d^2}} = \frac{a^2 \cdot c}{d} \sqrt[3]{\frac{a b c}{d}}$$

$$12: \sqrt{a^2} \sqrt{b^3 c^5} = a \sqrt{b^3 c^5} = a \sqrt[4]{b^3 c^5} = a c \sqrt[4]{b^3 c}$$

$$13: \sqrt[3]{\frac{\sqrt{x^6 \cdot a^2}}{\sqrt[3]{b^9 \cdot c}}} = \sqrt[3]{\frac{x^3 \cdot a}{b^3 \cdot \sqrt[3]{c}}} = \frac{\sqrt[3]{x^3 \cdot a}}{\sqrt[3]{b^3 \sqrt[3]{c}}} = \frac{x \sqrt[3]{a}}{b \sqrt[3]{\sqrt[3]{c}}} = \frac{x}{b} \sqrt[3]{\frac{a}{\sqrt[3]{c}}}$$

$$\frac{\sqrt{x^6 \cdot a^2}}{\sqrt[3]{b^9 \cdot c}} = \frac{\sqrt[6]{x^{12} \cdot a^4}}{\sqrt[6]{b^{18} \cdot c^2}} = \frac{x^2 a}{b^3 \sqrt[6]{c^2}} = \frac{x^2 \cdot a}{b^3 \sqrt[3]{c}}$$

$mcm(2, 3) = 6$

$$14: \sqrt[3]{\frac{a^5 b^6 c}{c^5 b}} = \sqrt[3]{\frac{a^5 b^5}{c^4}} = \frac{a \cdot b}{c} \sqrt[3]{\frac{a^2 b^2}{c}}$$

$$15: \sqrt[4]{\frac{81 a^5 b^2}{c^7 d^8}} = \sqrt[4]{\frac{3^4 \cdot a^5 b^2}{c^7 d^8}} = \frac{3 a}{c \cdot d^2} \sqrt[4]{\frac{a \cdot b^2}{c^3}}$$

Bloque 3: Raíces

$$1: a b \sqrt{c d} = \sqrt{a^2 b^2 c d}$$

$$2: a b \sqrt[3]{a b^2} = \sqrt[3]{a^3 b^3 a b^2} = \sqrt[3]{a^4 b^5}$$

$$3: \frac{a b^2}{c} \sqrt[3]{\frac{2 c^2}{b}} = \sqrt[3]{\frac{a^3 b^6}{c^3} \frac{2 c^2}{b}} = \sqrt[3]{2 a^3 c^{-1} b^5} = \sqrt[3]{\frac{2 a^3 b^5}{c}}$$

$$4: \frac{a b c}{d} \sqrt{\frac{c d}{a}} = \sqrt{\frac{a^2 b^2 c^2 c d}{d^2 a}} = \sqrt{\frac{a b^2 c^3}{d}}$$

$$5: 2 a b \sqrt[5]{\frac{b}{4 a}} = \sqrt[5]{\frac{2^5 \cdot a^5 \cdot b^5 \cdot b}{2^2 a}} = \sqrt[5]{2^3 a^4 b^6}$$

$$6: \sqrt[3]{b c^2} \sqrt{\frac{a}{c}} = \sqrt[3]{\sqrt[6]{b^2 c^4 a}} = \sqrt[6]{b^2 a c^3}$$

$$7: \sqrt{a b} \sqrt{\frac{c}{a b}} = \sqrt{\sqrt{\frac{c a^2 b^2}{a b}}} = \sqrt[4]{c a b}$$

$$8: c a^2 \sqrt[4]{\frac{a b^2}{c^3}} = \sqrt[4]{\frac{c^4 \cdot a^8 \cdot a b^2}{c^3}} = \sqrt[4]{c a^9 b^2}$$

$$9: -2 a b \sqrt[3]{\frac{b}{4 a}} = \sqrt[3]{-2^3 a^3 b^3} \frac{b}{4 a} = \sqrt[3]{\frac{-2^3 a^3 \cdot b^3 \cdot b}{4 a}} =$$

$$= \sqrt[3]{-2^3 a^2 b^4}$$

$$10: \frac{\sqrt{x^2 y^3}}{x} = \sqrt{\frac{x^2 y^3}{x^2}} = \sqrt{y^3}$$

$$11: \frac{3}{2} \sqrt[3]{\frac{4}{9}} = \sqrt[3]{\frac{3^3 \cdot 2^2}{2^3 \cdot 3^2}} = \sqrt[3]{3 \cdot 2^{-1}} = \sqrt[3]{\frac{3}{2}}$$

$$12: \frac{3}{2} \sqrt[3]{\frac{4}{3}} = \sqrt[3]{\frac{3^3 \cdot 2^2}{2^3 \cdot 3}} = \sqrt[3]{\frac{3^2}{2}}$$

Bloque IV: Sumas y restas de raíces.

$$1: \sqrt{125} - 2\sqrt{5} + \sqrt{20} = \sqrt{5^3} - 2\sqrt{5} + \sqrt{2^2 \cdot 5} = 5\sqrt{5} - 2\sqrt{5} + 2\sqrt{5} = (5-2+2)\sqrt{5} = 5\sqrt{5}$$

$\downarrow$
 $125 = 5^3$
 $20 = 2^2 \cdot 5$

$$2: \sqrt{75} - \sqrt{8} + 3\sqrt{12} - 2\sqrt{32} = \sqrt{5^2 \cdot 3} - \sqrt{2^3} + 3\sqrt{2^2 \cdot 3} - 2\sqrt{2^5} =$$

$$= 5\sqrt{3} - 2\sqrt{2} + 6\sqrt{3} - 4\sqrt{2} = (5+6)\sqrt{3} + (-2-4)\sqrt{2} =$$

$$= 11\sqrt{3} - 6\sqrt{2}$$

$$3: \sqrt{4a^2b} - \sqrt{9ab^2} + 3\sqrt{a} - 2a\sqrt{b} = 2a\sqrt{b} - 3b\sqrt{a} + 3\sqrt{a} - 2a\sqrt{b} =$$

$$= (2a - 2a)\sqrt{b} - 3b\sqrt{a} + 3\sqrt{a} = (-b+1)3\sqrt{a}$$

no se puede operar $\rightarrow$ sólo podría sacar factor común.

$$4: 2\sqrt{a} + \sqrt{8b} - 3\sqrt{2b} + 3\sqrt{a^3} = 2\sqrt{a} + 2\sqrt{2b} - 3\sqrt{2b} + 3a\sqrt{a} =$$

$$= (2-3)\sqrt{2b} + 2\sqrt{a} + 3a\sqrt{a} = -\sqrt{2b} + (2+3a)\sqrt{a}$$

no se puede operar.
sólo podría sacar factor común: $2\sqrt{a} + 3a\sqrt{a} = (2+3a)\sqrt{a}$

$$5: \sqrt{\frac{16}{3}} - 2\sqrt{\frac{4}{3}} + 3\sqrt{\frac{1}{27}} - 2\sqrt{\frac{25}{3}} = 2^2\sqrt{\frac{1}{3}} - 2^2\sqrt{\frac{1}{3}} + \frac{3}{3}\sqrt{\frac{1}{3}} -$$

$$- 2 \cdot 5\sqrt{\frac{1}{3}} = (4-4+1-10)\sqrt{\frac{1}{3}} = -9\sqrt{\frac{1}{3}}$$

$$6: \sqrt{\frac{8}{9}} - 3\sqrt{\frac{2}{9}} - 2\sqrt{\frac{2}{16}} + \sqrt{32} = \frac{2}{3}\sqrt{2} - \frac{3}{3}\sqrt{2} - \frac{2}{2}\sqrt{2} + 2^2\sqrt{2} =$$

$$= \left(\frac{2}{3} - 1 - \frac{1}{2} + 4\right)\sqrt{2} = \frac{4-6-3+24}{6}\sqrt{2} = \frac{19}{6}\sqrt{2}$$

$$7: \sqrt{125} - \sqrt{\frac{5}{4}} + 2\sqrt{\frac{45}{9}} - 2\sqrt{5} = \sqrt{5^3} - \sqrt{\frac{5}{2^2}} + 2\sqrt{\frac{3^2 \cdot 5}{3^2}} - 2\sqrt{5} =$$

$$= 5\sqrt{5} - \frac{1}{2}\sqrt{5} + 2\sqrt{5} - 2\sqrt{5} = \left(5 - \frac{1}{2}\right)\sqrt{5} = \frac{9}{2}\sqrt{5}$$

$$8: \sqrt{50} + \sqrt{8} + \sqrt{2} - 3\sqrt{2} = 5\sqrt{2} + 2\sqrt{2} + \sqrt{2} - 3\sqrt{2} = (5+2+1-3)\sqrt{2} =$$

$$= 5\sqrt{2}$$

$$9: \sqrt{18} + \sqrt{20} - 2\sqrt{8} + \sqrt{45} = 3\sqrt{2} + 2\sqrt{5} - 2^2\sqrt{2} + 3\sqrt{5} =$$

$$= (3-4)\sqrt{2} + (2+3)\sqrt{5} = -\sqrt{2} + 5\sqrt{5}$$

$$10: 3\sqrt{a} - \sqrt{4b} + 3\sqrt{4a} + 3\sqrt{b} = 3\sqrt{a} - 2\sqrt{b} + 3 \cdot 2\sqrt{a} + 3\sqrt{b} =$$

$$= (3+6)\sqrt{a} + (-2+3)\sqrt{b} = 9\sqrt{a} + \sqrt{b}$$