Derivar las siguientes funciones:

$$\mathbf{a)}\ y=e^{-3x}\quad \mathbf{b)}\ y=\sqrt{x-5}\quad \mathbf{c)}\ y=2tg(3x-2)\quad \mathbf{d)}\ y=\cos e^{x}\quad \mathbf{e)}\ y=5sen\frac{x}{2}\quad \mathbf{f)}\ y=\sqrt[4]{3x}$$

$$\mathbf{g)}\ y=\ln(senx)\quad \mathbf{h)}\ y=tg\frac{3}{x}\quad \mathbf{i)}\ f(x)=e^{3x}\quad \mathbf{j)}\ y=(\ln x)^2\quad \mathbf{k)}\ y=\ln x^2\quad \mathbf{l)}\ y=sen(\cos x)$$

$$\mathbf{m)}\ f(t)=\cos\left(\frac{t+8}{3}\right)\quad \mathbf{n)}\ f(t)=\frac{sen(6t)}{3}\quad \mathbf{o)}\ f(t)=e^{t^2}\quad \mathbf{p)}\ f(t)=\ln\left(\frac{1}{t}\right)\quad \mathbf{q)}\ f(x)=8tg(xe^{x})$$

$$a)\ y'=-3\cdot e^{-3x}\quad b)\ y'=\frac{1}{2\sqrt{x-5}}\quad c)\ y'=2\cdot\frac{1}{\cos^2(3x-2)}\cdot 3=\frac{6}{\cos^2(3x-2)}$$

$$d)\ y'=-sen\left(e^{x}\right)\cdot e^{x}=-e^{x}\cdot sen\left(e^{x}\right)\quad e)\ y'=5\cdot\cos\frac{x}{2}\cdot\frac{1}{2}=\frac{5}{2}\cdot\cos\frac{x}{2}$$

$$f)\ y'=\frac{3}{4\cdot\sqrt[4]{(3x)^3}}\quad g)\ y'=\frac{1}{sen\,x}\cdot\cos x=\frac{\cos x}{sen\,x}=\cot g\,x\quad h)\ y'=\frac{1}{\cos^2\frac{3}{x}}\cdot\left(\frac{-3}{x^2}\right)=\frac{-3}{x^2\cdot\cos^2\frac{3}{x}}$$

$$i)\ f'(x)=e^{3x}\cdot 3=3e^{3x}\quad j)\ y'=2\ln x\cdot\frac{1}{x}=\frac{2\ln x}{x}$$

$$k)\ y'=\frac{1}{x^2}\cdot 2x=\frac{2}{x}\quad l)\ y'=\cos(\cos x)\cdot(-sen\,x)=-sen\,x\cdot\cos(\cos x)$$

$$m)\ f'(t)=-sen\frac{t+8}{3}\cdot\frac{1}{3}=-\frac{1}{3}\cdot sen\frac{t+8}{3}\quad n)\ f'(t)=\frac{1}{3}\cdot\cos(6t)\cdot 6=2\cos(6t)$$

$$o)\ f'(t)=e^{t^2}\cdot 2t=2t\cdot e^{t^2}\quad p)\ f'(t)=\frac{1}{\frac{1}{t}}\cdot\left(-\frac{1}{t^2}\right)=-\frac{1}{t}$$

$$q)\ f'(x)=8\cdot\frac{1}{\cos^2(xe^{x})}\cdot\left(e^{x}+x\cdot e^{x}\right)=\frac{8e^{x}+8xe^{x}}{\cos^2\left(xe^{x}\right)}$$

Derivar las siguientes funciones:

$$\mathbf{a)}\ y=\ln\left(\cos x^3\right)\quad \mathbf{b)}\ y=\frac{sen^3(6x-1)}{8}\quad \mathbf{c)}\ y=\frac{x}{2}+\sqrt[3]{3x^2+5}\quad \mathbf{d)}\ y=sen^2x+\cos^2x$$

$$\mathbf{e)}\ y=\frac{1-e^{x}}{1+e^{x}}\quad \mathbf{f)}\ f(x)=x\cdot arctgx\quad \mathbf{g)}\ f(x)=\ln\frac{1}{x+\sqrt{x^2+1}}\quad \mathbf{h)}\ y=sen^2x\cdot sen\,x^2$$

$$\mathbf{i)}\ f(x)=\sqrt{x}\cdot\left(x^3-\sqrt{x}+1\right)\quad \mathbf{j)}\ y=\left(\sqrt[3]{x}+2x\right)\bullet\left(1+\sqrt[3]{x^2}+3x\right)\quad \mathbf{k)}\ y=\frac{1-x^3}{\sqrt{\pi}}\quad \mathbf{l)}\ f(x)=\frac{1-\ln x}{1+\ln x}$$

a) $y' = \dfrac{1}{\cos x^3} \cdot \left(-\operatorname{sen} x^3\right) \cdot 3x^2 = \dfrac{-3x^2 \operatorname{sen} x^3}{\cos x^3} = -3x^2 \cdot \operatorname{tg} x^3$

b) $y' = \dfrac{1}{8} \cdot 3\operatorname{sen}^2(6x-1) \cdot \cos(6x-1) \cdot 6 = \dfrac{9}{4} \cdot \operatorname{sen}^2(6x-1) \cdot \cos(6x-1)$

c) $y' = \dfrac{1}{2} + \dfrac{1}{3} \cdot \left(3x^2+5\right)^{-\frac{2}{3}} \cdot 6x = \dfrac{1}{2} + \dfrac{2x}{\sqrt[3]{\left(3x^2+5\right)^2}}$

d) $y' = 0$ (ya que $\operatorname{sen}^2 x + \cos^2 x = 1$)   e) $y' = \dfrac{-e^x \cdot \left(1+e^x\right) - \left(1-e^x\right) \cdot e^x}{\left(1+e^x\right)^2} = \dfrac{-2e^x}{\left(1+e^x\right)^2}$

f) $f'(x) = \operatorname{arctg} x + x \cdot \dfrac{1}{1+x^2} = \operatorname{arctg} x + \dfrac{x}{1+x^2}$

g) $f(x) = \ln 1 - \ln\left(x + \sqrt{x^2+1}\right) = 0 - \ln\left(x + \sqrt{x^2+1}\right)$

$$f'(x) = -\frac{1}{\left(x+\sqrt{x^2+1}\right)} \cdot \left(1 + \frac{2x}{2\sqrt{x^2+1}}\right) = \frac{-\left(\sqrt{x^2+1}+x\right)}{\sqrt{x^2+1} \cdot \left(x+\sqrt{x^2+1}\right)} = -\frac{1}{\sqrt{x^2+1}}$$

h) $y' = 2\operatorname{sen} x \cdot \cos x \cdot \operatorname{sen} x^2 + \operatorname{sen}^2 x \cdot \cos x^2 \cdot 2x$

i) $f(x) = \sqrt{x^7} - x + \sqrt{x}$ $f'(x) = \dfrac{7x^6}{2\sqrt{x^7}} - 1 + \dfrac{1}{2\sqrt{x}} = \dfrac{7x^3}{2\sqrt{x}} \quad 1 + \dfrac{1}{2\sqrt{x}}$

j) $y = \sqrt[3]{x} + 3x + 3 \cdot \sqrt[3]{x^4} + 2 \cdot \sqrt[3]{x^5} + 6x^2$ $y' = \dfrac{1}{3 \cdot \sqrt[3]{x^2}} + 3 + 4 \cdot \sqrt[3]{x} + \dfrac{10}{3} \cdot \sqrt[3]{x^2} + 12x$

k) $y' = \dfrac{1}{\sqrt{\pi}}\left(-3x^2\right) = \dfrac{-3x^2}{\sqrt{\pi}}$ l) $f'(x) = \dfrac{-\frac{1}{x} \cdot (1+\ln x) - (1-\ln x) \cdot \frac{1}{x}}{(1+\ln x)^2} = \dfrac{-2}{x \cdot (1+\ln x)^2}$

a) $\boldsymbol{y = x \cdot \operatorname{arcsen} x + \sqrt{1-x^2}}$ **b)** $\boldsymbol{y = \operatorname{arctg}[\ln(ax+b)]}$ **c)** $\boldsymbol{y = \ln \operatorname{arctg} \sqrt{1+x^2}}$

d) $\boldsymbol{y = \arccos \dfrac{2x-1}{\sqrt{3}}}$ **e)** $\boldsymbol{y = 2^{\frac{x}{\ln x}}}$ **f)** $\boldsymbol{y = x^3 - 3^x}$ **g)** $\boldsymbol{y = e^{\sqrt{x-1}}}$ **h)** $\boldsymbol{f(x) = e^{-x^2} \cdot \ln x}$

i) $\boldsymbol{y = x \cdot 10^{\sqrt{x}}}$ **j)** $\boldsymbol{y = \ln \dfrac{1-e^x}{e^x}}$

a) $y' = \operatorname{arcsen} x + x \cdot \dfrac{1}{\sqrt{1-x^2}} + \dfrac{1}{2 \cdot \sqrt{1-x^2}} \cdot (-2x) = \operatorname{arcsen} x$

b) $y' = \dfrac{1}{1+[\ln(ax+b)]^2} \cdot \dfrac{1}{ax+b} \cdot a = \dfrac{a}{(ax+b) \cdot \left(1+[\ln(ax+b)]^2\right)}$

c) $y' = \dfrac{1}{\operatorname{arctg}\sqrt{1+x^2}} \cdot \dfrac{1}{1+\left(\sqrt{1+x^2}\right)^2} \cdot \dfrac{1}{2\sqrt{1+x^2}} \cdot 2x = \dfrac{x}{\operatorname{arctg}\sqrt{1+x^2} \cdot \left(2+x^2\right) \cdot \sqrt{1+x^2}}$

d) $y' = \dfrac{-1}{\sqrt{1-\left(\dfrac{2x-1}{\sqrt{3}}\right)^2}} \cdot \dfrac{2}{\sqrt{3}} = \dfrac{-2}{\sqrt{3-4x^2+4x-1}} = \dfrac{-2}{\sqrt{-4x^2+4x+2}} = \dfrac{-\sqrt{2}}{\sqrt{-2x^2+2x+1}}$

e) $y' = 2^{\frac{x}{\ln x}} \cdot \dfrac{\ln x - x \cdot \dfrac{1}{x}}{(\ln x)^2} \cdot \ln 2 = \dfrac{2^{\frac{x}{\ln x}} \cdot \ln 2 \cdot (\ln x - 1)}{(\ln x)^2}$

f) $y' = 3x^2 - 3^x \cdot \ln 3$ g) $y' = e^{\sqrt{x-1}} \cdot \dfrac{1}{2 \cdot \sqrt{x-1}} = \dfrac{e^{\sqrt{x-1}}}{2 \cdot \sqrt{x-1}}$

h) $f'(x) = e^{-x^2} \cdot (-2x) \cdot \ln x + e^{-x^2} \cdot \dfrac{1}{x} = -2x \cdot e^{-x^2} \cdot \ln x + \dfrac{e^{-x^2}}{x} = e^{-x^2} \cdot \left(-2x \cdot \ln x + \dfrac{1}{x}\right)$

i) $y' = 10^{\sqrt{x}} + x \cdot 10^{\sqrt{x}} \cdot \dfrac{1}{2\sqrt{x}} \cdot \ln 10 = 10^{\sqrt{x}} \cdot \left(1 + \dfrac{x \cdot \ln 10}{2\sqrt{x}}\right)$

j) $y' = \dfrac{1}{\dfrac{1-e^x}{e^x}} \cdot \dfrac{-e^x \cdot e^x - (1-e^x)e^x}{(e^x)^2} = \dfrac{-e^x}{(1-e^x) \cdot e^x} = \dfrac{-1}{1-e^x}$

a) $y = \mathrm{cosc}^2(1-x)$ b) $y = \cos^2(\mathrm{arcsec}\,x^2)$ c) $y = \arcsin \dfrac{x+1}{x-1}$ d) $y = \mathrm{arctg}\dfrac{x+a}{1-ax}$

e) $y = \arcsin(\mathrm{sen}\,x^2) + \arccos(\cos x^2)$ f) $y = x \cdot \sqrt{a^2-x^2} + a^2 \cdot \arcsin \dfrac{x}{a}$

g) $y = \dfrac{8x^4-3}{32} \cdot \arcsin x + \dfrac{2x^3+3x}{32} \cdot \sqrt{1-x^2}$ h) $y = e^x \cdot (x-1) \cdot [(x+1) \cdot \cos x + (x-1) \cdot \mathrm{sen}\,x]$

a) $y = \dfrac{1}{\mathrm{sen}^2(1-x)}$ $y' = \dfrac{-2\,\mathrm{sen}(1-x) \cdot \cos(1-x) \cdot (-1)}{\mathrm{sen}^4(1-x)} = \dfrac{2 \cdot \cos(1-x)}{\mathrm{sen}^3(1-x)}$

b) Podemos sustituir $\mathrm{arcsec}\,x^2$ por una expresión más familiar a través de los siguientes pasos:

$$\mathrm{arcsec}\,x^2 = z \quad \to \quad \sec z = x^2 \quad \to \quad \frac{1}{\cos z} = x^2 \quad \to \quad \cos z = \frac{1}{x^2} \quad \to \quad z = \arccos \frac{1}{x^2}$$

La función a derivar es entonces: $y = \cos^2\left(\arccos \dfrac{1}{x^2}\right) = \left(\cos\left(\arccos \dfrac{1}{x^2}\right)\right)^2 = \dfrac{1}{x^4}$

$$y' = \frac{-4x^3}{x^8} = -\frac{4}{x^5}$$

c) $$y' = \frac{1}{\sqrt{1-\left(\frac{x+1}{x-1}\right)^2}} \cdot \frac{(x-1)-(x+1)}{(x-1)^2} = \frac{-2}{\sqrt{\frac{-4x}{(x-1)^2}} \bullet (x-1)^2} = \frac{-1}{(x-1)\cdot\sqrt{-x}}$$

d) $$y' = \frac{1}{1+\left(\frac{x+a}{1-ax}\right)^2} \cdot \frac{(1-ax)-(x+a)\cdot(-a)}{(1-ax)^2} = \frac{1+a^2}{(1-ax)^2+(x+a)^2} = \frac{1+a^2}{1+a^2x^2+x^2+a^2} =$$

$$\frac{1+a^2}{1+a^2+x^2(1+a^2)} = \frac{1+a^2}{(1+a^2)(1+x^2)} = \frac{1}{1+x^2}$$

e) $y = \arcsin(\operatorname{sen} x^2) + \arccos(\cos x^2) = x^2 + x^2 = 2x^2 \qquad y' = 4x$

f) $$y' = \sqrt{a^2-x^2} + x \cdot \frac{1}{2\sqrt{a^2-x^2}} \cdot (-2x) + a^2 \cdot \frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}} \cdot \frac{1}{a} = \sqrt{a^2-x^2} - \frac{2x^2}{2\sqrt{a^2-x^2}} +$$

$$\frac{a}{\sqrt{\frac{a^2-x^2}{a^2}}} = \sqrt{a^2-x^2} - \frac{x^2}{\sqrt{a^2-x^2}} + \frac{a^2}{\sqrt{a^2-x^2}} = \frac{2\left(a^2-x^2\right)}{\sqrt{a^2-x^2}} = \sqrt{a^2-x^2}$$

g) $$y' = x^3 \cdot \operatorname{arcsen} x + \frac{8x^4-3}{32} \cdot \frac{1}{\sqrt{1-x^2}} + \frac{6x^2+3}{32} \cdot \sqrt{1-x^2} + \frac{2x^3+3x}{32} \cdot \frac{1}{2\sqrt{1-x^2}} \cdot (-2x) =$$

$$x^3 \cdot \operatorname{arcsen} x + \frac{8x^4-3}{32\sqrt{1-x^2}} + \frac{\left(6x^2+3\right)\cdot\sqrt{1-x^2}}{32} - \frac{4x^4+6x^2}{64\sqrt{1-x^2}} =$$

$$x^3 \cdot \operatorname{arcsen} x + \frac{16x^4-6-4x^4-6x^2}{64\cdot\sqrt{1-x^2}} + \frac{\left(6x^2+3\right)\cdot\sqrt{1-x^2}}{32} =$$

$$x^3 \cdot \operatorname{arcsen} x + \frac{3\left(2x^4-1-x^2\right)}{32\sqrt{1-x^2}} + \frac{3\left(2x^2+1\right)\cdot\sqrt{1-x^2}}{32}$$

h) $y = e^x \cdot \left(x^2-1\right) \cdot \cos x + e^x \cdot (x-1)^2 \cdot \operatorname{sen} x$

$$y' = e^x\cdot\left(x^2-1\right)\cdot\cos x + e^x\cdot 2x\cdot\cos x + e^x\cdot\left(x^2-1\right)\cdot(-\operatorname{sen} x) + e^x\cdot(x-1)^2\cdot\operatorname{sen} x +$$

$$e^x\cdot 2\cdot(x-1)\cdot\operatorname{sen} x + e^x\cdot(x-1)^2\cdot\cos x = 2x^2\cdot e^x\cdot\cos x$$

a a) $\mathbf{y = \ln\sqrt{\dfrac{1+2senx}{1-2senx}}}$ **b)** $\mathbf{y = \dfrac{1}{2}\cdot\ln\left(tg\dfrac{x}{2}\right) - \dfrac{cosx}{2\,sen^2 x}}$ **c)** $\mathbf{y = \ln\dfrac{1+tg\dfrac{x}{2}}{1-tg\dfrac{x}{2}}}$

d) $\mathbf{f(x) = \dfrac{1}{3}\cdot arcsen\dfrac{3+4senx}{4+3sen\,x}}$ **e)** $\mathbf{f(x) = \dfrac{x}{a+\sqrt{a^2-x^2}}}$

f) $\mathbf{y = \sqrt{x^2+1} - \ln\left(\dfrac{1}{x} + \sqrt{\dfrac{1}{x}} + \sqrt{1+\dfrac{1}{x^2}}\right)}$

a) $y = \ln\left(\dfrac{1+2\operatorname{sen} x}{1-2\operatorname{sen} x}\right)^{\frac{1}{2}} = \dfrac{1}{2}\cdot\ln\dfrac{1+2\operatorname{sen} x}{1-2\operatorname{sen} x}$

$$y' = \frac{1}{2}\cdot\frac{1}{\dfrac{1+2\operatorname{sen} x}{1-2\operatorname{sen} x}}\cdot\frac{2\cos x\cdot(1-2\operatorname{sen} x)-(1+2\operatorname{sen} x)\cdot(-2\cos x)}{(1-2\operatorname{sen} x)^2} = \frac{1}{2}\cdot\frac{1-2\operatorname{sen} x}{1+2\operatorname{sen} x}\cdot\frac{4\cos x}{(1-2\operatorname{sen} x)^2} =$$

$$\frac{4\cos x}{2-8\operatorname{sen}^2 x} = \frac{2\cos x}{1-4\operatorname{sen}^2 x}$$

b) $y' = \dfrac{1}{2}\cdot\dfrac{1}{tg\dfrac{x}{2}}\cdot\dfrac{1}{\cos^2\dfrac{x}{2}}\cdot\dfrac{1}{2} - \dfrac{-\operatorname{sen} x\cdot 2\cdot\operatorname{sen}^2 x - \cos x\cdot(2\cdot 2\operatorname{sen} x\cdot\cos x)}{4\cdot\operatorname{sen}^4 x} =$

$$\frac{1}{4\cdot\operatorname{sen}\dfrac{x}{2}\cdot\cos\dfrac{x}{2}} + \frac{2\operatorname{sen}^3 x + 4\operatorname{sen} x\cdot\cos^2 x}{4\cdot\operatorname{sen}^4 x} = \frac{1}{2\cdot\operatorname{sen} x} + \frac{\operatorname{sen}^2 x + 2\cdot\cos^2 x}{2\operatorname{sen}^3 x} =$$

$$\frac{2\operatorname{sen}^2 x + 2\cos^2 x}{2\operatorname{sen}^3 x} = \frac{1}{\operatorname{sen}^3 x}$$

c) $y' = \dfrac{1}{\dfrac{1+\operatorname{tg}\frac{x}{2}}{1-\operatorname{tg}\frac{x}{2}}} \cdot \dfrac{\dfrac{1}{\cos^2\frac{x}{2}} \cdot \dfrac{1}{2} \cdot \left(1-\operatorname{tg}\dfrac{x}{2}\right) - \left(1+\operatorname{tg}\dfrac{x}{2}\right) \cdot \left(-\dfrac{1}{\cos^2\frac{x}{2}}\right) \cdot \dfrac{1}{2}}{\left(1-\operatorname{tg}\dfrac{x}{2}\right)^2} =$

$$\frac{\dfrac{1}{\cos^2\frac{x}{2}}}{1-\operatorname{tg}^2\dfrac{x}{2}} = \frac{1}{\cos^2\dfrac{x}{2} - \operatorname{sen}^2\dfrac{x}{2}} = \frac{1}{\cos x}$$

d) $f'(x) = \dfrac{1}{3} \cdot \dfrac{1}{\sqrt{1-\left(\dfrac{3+4\operatorname{sen} x}{4+3\operatorname{sen} x}\right)^2}} \cdot \dfrac{4\cos x \cdot (4+3\operatorname{sen} x) - (3+4\operatorname{sen} x)\cdot(3\cos x)}{(4+3\operatorname{sen} x)^2} =$

$$\frac{16\cos x + 12\operatorname{sen} x \cdot \cos x - 9\cos x - 12\operatorname{sen} x \cdot \cos x}{3 \cdot \sqrt{\dfrac{16+24\operatorname{sen} x + 9\operatorname{sen}^2 x - 9 - 24\operatorname{sen} x - 16\operatorname{sen}^2 x}{(4+3\operatorname{sen} x)^2}} \cdot (4+3\operatorname{sen} x)^2} =$$

$$\frac{7\cos x}{3 \cdot \sqrt{7-7\operatorname{sen}^2 x} \cdot (4+3\operatorname{sen} x)} = \frac{7\cos x}{3\sqrt{7}\cos x \cdot (4+3\operatorname{sen} x)} = \frac{\sqrt{7}}{3(4+3\operatorname{sen} x)}$$

e) $f'(x) = \dfrac{\left(a+\sqrt{a^2-x^2}\right) - x \cdot \left(\dfrac{1}{2\sqrt{a^2-x^2}} \cdot (-2x)\right)}{\left(a+\sqrt{a^2-x^2}\right)^2} = \dfrac{a+\sqrt{a^2-x^2}+\dfrac{x^2}{\sqrt{a^2-x^2}}}{\left(a+\sqrt{a^2-x^2}\right)^2} =$

$$\frac{a\cdot\sqrt{a^2-x^2}+a^2-x^2+x^2}{\sqrt{a^2-x^2}\cdot\left(a+\sqrt{a^2-x^2}\right)^2} = \frac{a\cdot\left(\sqrt{a^2-x^2}+a\right)}{\sqrt{a^2-x^2}\cdot\left(a+\sqrt{a^2-x^2}\right)^2} = \frac{a}{\sqrt{a^2-x^2}\cdot\left(a+\sqrt{a^2-x^2}\right)}$$

f) $y' = \dfrac{1}{2\sqrt{x^2+1}} \cdot 2x - \dfrac{1}{\dfrac{1}{x}+\sqrt{\dfrac{1}{x}}+\sqrt{1+\dfrac{1}{x^2}}} \cdot \left(\left(-\dfrac{1}{x^2}\right) + \dfrac{1}{2\sqrt{\dfrac{1}{x}}}\left(-\dfrac{1}{x^2}\right) + \dfrac{1}{2\sqrt{\dfrac{x^2+1}{x^2}}} \cdot \left(-\dfrac{2}{x^3}\right)\right) =$

$$\frac{x}{\sqrt{x^2+1}}-\frac{1}{\frac{1}{x}+\sqrt{\frac{1}{x}}+\sqrt{1+\frac{1}{x^2}}}\cdot\left(-\frac{1}{x^2}+\frac{\sqrt{x}}{2}\left(-\frac{1}{x^2}\right)+\frac{x}{2\sqrt{x^2+1}}\cdot\left(-\frac{2}{x^3}\right)\right)=$$

$$\frac{x}{\sqrt{x^2+1}}-\frac{1}{\frac{1}{x}+\sqrt{\frac{1}{x}}+\sqrt{1+\frac{1}{x^2}}}\cdot\left(-\frac{1}{x^2}\right)\cdot\left(1+\frac{\sqrt{x}}{2}+\frac{1}{\sqrt{x^2+1}}\right)$$

Calcular las derivadas de las funciones

a) $\mathbf{y=\operatorname{sen}^2 x+\operatorname{sen} x^2}$ **b)** $\mathbf{y=\ln\sqrt{\frac{1-x}{1+x}}}$ **c)** $\mathbf{y=x^2\operatorname{arcsen} x^2+3\operatorname{cosec}^2(x+1)}$

a) $y'=2\operatorname{sen} x\cdot\cos x+\cos x^2\cdot 2x=2\cos x\cdot(\operatorname{sen} x+x\cdot\cos x)$

b) $y=\ln\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}}=\frac{1}{2}\cdot\ln\left(\frac{1-x}{1+x}\right)$ $\qquad y'=\frac{1}{2}\cdot\frac{1}{\frac{1-x}{1+x}}\cdot\frac{-(1+x)-(1-x)}{(1+x)^2}=\frac{-2}{2\left(1-x^2\right)}=\frac{-1}{1-x^2}$

c) $y=x^2\cdot\operatorname{arcsen} x^2+\frac{3}{\operatorname{sen}^2(x+1)}$

$$y'=2x\cdot\operatorname{arcsen} x^2+x^2\cdot\frac{1}{\sqrt{1-x^4}}\cdot 2x+\frac{-3\cdot 2\operatorname{sen}(x+1)\cdot\cos(x+1)}{\operatorname{sen}^4(x+1)}=$$

$$2x\cdot\operatorname{arcsen} x^2+\frac{2x^3}{\sqrt{1-x^4}}-\frac{6\cdot\cos(x+1)}{\operatorname{sen}^3(x+1)}$$

Calcular las derivadas de las funciones

a) $\mathbf{y=\operatorname{sen}^3\sqrt{\operatorname{tg}^2\frac{2\pi^2+5}{3}}}$ **b)** $\mathbf{y=\operatorname{sen}^3 x\cdot\left(\sqrt{\operatorname{tg}^2\frac{2\pi^2+5}{3}}\right)}$ **c) Calcular** $\mathbf{y''(0)}$ **en** $\mathbf{y=\frac{x^2-x}{e^x}}$

a) $y'=0$ por ser una constante

b) $y'=3\cdot\operatorname{sen}^2 x\cdot\cos x\cdot\sqrt{\operatorname{tg}^2\frac{2\pi^2+5}{3}}+\operatorname{sen}^3 x\cdot 0=3\cdot\operatorname{sen}^2 x\cdot\cos x\cdot\sqrt{\operatorname{tg}^2\frac{2\pi^2+5}{3}}$

$$c)\quad y' = \frac{(2x-1)\cdot e^x - \left(x^2 - x\right)\cdot e^x}{\left(e^x\right)^2} = \frac{e^x\,(2x-1-x^2+x)}{e^{2x}} = \frac{3x-1-x^2}{e^x}$$

$$y'' = \frac{(3-2x)\cdot e^x - \left(3x-1-x^2\right)\cdot e^x}{\left(e^x\right)^2} = \frac{4-5x+x^2}{e^x} \quad \Rightarrow \quad y''(0) = 4$$

Calcular las derivadas de las funciones

$$\textbf{a)}\ \ y = \ln\sqrt{\frac{e^{3x-5}}{\operatorname{sen}\frac{\pi x}{3}}} \qquad \textbf{b)}\ \ y = \log\left(\operatorname{tg}^2\frac{\sqrt{2x-1}}{3x-1}\right) \qquad \textbf{c)}\ \ y = \ln\left(\operatorname{sen}^2 x + \operatorname{arcsen}^2 x\right)$$

$$\textbf{d)}\ \ y = \log_a \frac{x^2 - 2\sqrt{x} + 3}{\operatorname{sen}^2(3x+5)} \qquad \textbf{e)}\ \ y = \ln\left(\frac{10^{3x+1}}{2x+8}\right)^x \qquad \textbf{f)}\ \ y = \frac{\operatorname{sen}\frac{\pi}{6}\cdot e^x \cdot \cos\frac{\pi}{6}\cdot e^x}{\sqrt{x^2+1}}$$

$$a)\quad y = \ln\sqrt{\frac{e^{3x+5}}{\operatorname{sen}\frac{\pi x}{3}}} = \ln\left(\frac{e^{3x+5}}{\operatorname{sen}\frac{\pi x}{3}}\right)^{\frac{1}{2}} = \frac{1}{2}\cdot\ln\frac{e^{3x+5}}{\operatorname{sen}\frac{\pi x}{3}}$$

$$y' = \frac{1}{2}\cdot\frac{1}{\dfrac{e^{3x-5}}{\operatorname{sen}\frac{\pi x}{3}}}\cdot\frac{e^{3x-5}\cdot 3\cdot\operatorname{sen}\frac{\pi x}{3} - e^{3x-5}\cdot\cos\frac{\pi x}{3}\cdot\frac{\pi}{3}}{\operatorname{sen}^2\frac{\pi x}{3}} = \frac{e^{3x-5}\left(3\operatorname{sen}\frac{\pi x}{3} - \cos\frac{\pi x}{3}\cdot\frac{\pi}{3}\right)}{2\left(\dfrac{e^{3x-5}}{\operatorname{sen}\frac{\pi x}{3}}\right)\cdot\operatorname{sen}^2\frac{\pi x}{3}} =$$

$$\frac{e^{3x-5}\left(3\operatorname{sen}\frac{\pi x}{3} - \cos\frac{\pi x}{3}\cdot\frac{\pi}{3}\right)}{2\,e^{3x-5}\cdot\operatorname{sen}\frac{\pi x}{3}} = \frac{3}{2} - \frac{\pi}{6}\cdot\operatorname{cotg}\frac{\pi x}{3}$$

$$b)\quad y = \log\left(\operatorname{tg}\frac{\sqrt{2x-1}}{3x-1}\right)^2 = 2\log\left(\operatorname{tg}\frac{\sqrt{2x-1}}{3x-1}\right)$$

$$y' = 2 \cdot \frac{1}{\operatorname{tg} \frac{\sqrt{2x-1}}{3x-1}} \cdot \frac{1}{\cos^2 \frac{\sqrt{2x-1}}{3x-1}} \cdot \frac{\frac{1}{2\sqrt{2x-1}} \cdot 2 \cdot (3x-1) - 3 \cdot \sqrt{2x-1}}{(3x-1)^2} \cdot \log e =$$

$$\frac{2 \cdot \left[3x-1-3(2x-1)\right] \cdot \log e}{\operatorname{tg} \frac{\sqrt{2x-1}}{3x-1} \cdot \cos^2 \frac{\sqrt{2x-1}}{3x-1} \cdot (3x-1)^2 \cdot \sqrt{2x-1}} = \frac{(-6x+4) \cdot \log e}{\operatorname{sen} \frac{\sqrt{2x-1}}{3x-1} \cdot \cos \frac{\sqrt{2x-1}}{3x-1} \cdot (3x-1)^2 \cdot \sqrt{2x-1}}$$

c) $$y' = \frac{1}{\operatorname{sen}^2 x + \operatorname{arcsen}^2 x} \cdot \left(2 \operatorname{sen} x \cdot \cos x + 2 \operatorname{arcsen} x \cdot \frac{1}{\sqrt{1-x^2}}\right) =$$

$$\frac{2\left[\operatorname{sen} x \cdot \cos x \cdot \sqrt{1-x^2} + \operatorname{arcsen} x\right]}{\left[\operatorname{sen}^2 x + \operatorname{arcsen}^2 x\right] \cdot \sqrt{1-x^2}} = \frac{\left(1-x^2\right) \cdot \operatorname{sen} 2x + 2 \cdot \sqrt{1-x^2} \cdot \operatorname{arcsen} x}{\left(1-x^2\right) \cdot \left(\operatorname{sen}^2 x + \operatorname{arcsen}^2 x\right)}$$

d) $$y' = \frac{\left[\left(2x - 2 \cdot \frac{1}{2\sqrt{x}}\right) \cdot \operatorname{sen}^2 (3x+5) - \left(x^2 - 2\sqrt{x} + 3\right) \cdot 2 \operatorname{sen}(3x+5) \cdot \cos(3x+5) \cdot 3\right] \cdot \log_a e}{\left(\frac{x^2 - 2\sqrt{x} + 3}{\operatorname{sen}^2 (3x+5)}\right) \cdot \operatorname{sen}^4 (3x+5)} =$$

$$\frac{\left[\left(2x - 2 \cdot \frac{1}{2\sqrt{x}}\right) \cdot \operatorname{sen}^2 (3x+5) - \left(x^2 - 2\sqrt{x} + 3\right) \cdot 2 \operatorname{sen}(3x+5) \cdot \cos(3x+5) \cdot 3\right] \cdot \log_a e}{\left(x^2 - 2\sqrt{x} + 3\right) \cdot \operatorname{sen}^2 (3x+5)} =$$

$$\left[\frac{2 \cdot \sqrt{x^3} - 1}{\sqrt{x} \cdot \left(x^2 - 2\sqrt{x} + 3\right)} - 6 \operatorname{cotg}(3x+5)\right] \cdot \log_a e$$

e) $$y = \ln\left(\frac{10^{3x+1}}{2x+8}\right)^x = x \cdot \ln\left(\frac{10^{3x+1}}{2x+8}\right)$$

$$y' = \ln\left(\frac{10^{3x+1}}{2x+8}\right) + x \cdot \frac{1}{\frac{10^{3x+1}}{2x+8}} \cdot \frac{10^{3x+1} \cdot \ln 10 \cdot 3 \cdot (2x+8) - 2 \cdot 10^{3x+1}}{(2x+8)^2} =$$

$$\ln\left(\frac{10^{3x+1}}{2x+8}\right)+\frac{x\cdot\ln 10\cdot 3\cdot(2x+8)-2x}{(2x+8)}$$

f) $$y'=\operatorname{sen}\frac{\pi}{6}\cdot\cos\frac{\pi}{6}\cdot\frac{e^{2x}\cdot 2\cdot\sqrt{x^2+1}-e^{2x}\cdot\frac{1}{2\sqrt{x^2+1}}\cdot 2x}{x^2+1}=$$

$$\operatorname{sen}\frac{\pi}{6}\cdot\cos\frac{\pi}{6}\cdot\frac{e^{2x}\cdot 2\cdot\left(x^2+1\right)-x\cdot e^{2x}}{\left(x^2+1\right)\cdot\sqrt{x^2+1}}=\frac{\sqrt{3}\cdot e^{2x}\cdot\left[2x^2-x+2\right]}{4\cdot\sqrt{\left(x^2+1\right)^3}}$$

Calcular las derivadas de las funciones

a) $\mathbf{f(x)=arctg\frac{x}{\sqrt{1-x^2}}}$ **b)** $\mathbf{f(x)=\sqrt{x+\sqrt{x+\sqrt{x}}}}$ **c)** $\mathbf{f(x)=\sqrt{a^2-x^2}+a\cdot arcsen\left(\frac{x}{a}\right)}$

a) $$f'(x)=\frac{1}{1+\left(\frac{x}{\sqrt{1-x^2}}\right)^2}\cdot\frac{\sqrt{1-x^2}-x\cdot\frac{1}{2\sqrt{1-x^2}}\cdot(-2x)}{1-x^2}=$$

$$\frac{1-x^2+x^2}{\left(1+\frac{x^2}{1-x^2}\right)\cdot\left(1-x^2\right)\cdot\sqrt{1-x^2}}=\frac{1}{\frac{1}{1-x^2}\cdot\left(1-x^2\right)\cdot\sqrt{1-x^2}}=\frac{1}{\sqrt{1-x^2}}$$

b) $$f'(x)=\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\cdot\left(1+\frac{1}{2\sqrt{x+\sqrt{x}}}\cdot\left(1+\frac{1}{2\sqrt{x}}\right)\right)$$

c) $$f'(x)=\frac{1}{2\sqrt{a^2-x^2}}\cdot(-2x)+a\cdot\frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\cdot\frac{1}{a}=\frac{-x}{\sqrt{a^2-x^2}}+\frac{1}{\sqrt{\frac{a^2-x^2}{a^2}}}=\frac{-x+a}{\sqrt{a^2-x^2}}$$