

1.3.1. Ecuaciones Logarítmicas

Problema 60 Resolver las ecuaciones:

1. $\log x + \log 50 = \log 1000$

2. $2 \log x^3 = \log 8 + 3 \log x$

Solución:

1.

$$\log x + \log 50 = \log 1000$$

$$\log(50x) = \log 1000$$

$$50x = 1000$$

$$x = \frac{1000}{50} = 20$$

2.

$$2 \log x^3 = \log 8 + 3 \log x$$

$$6 \log x = \log 8 + 3 \log x$$

$$6 \log x - 3 \log x = \log 8$$

$$3 \log x = \log 8$$

$$\log x^3 = \log 2^3$$

$$x^3 = 2^3$$

$$x = 2$$

Problema 61 Resolver las ecuaciones:

$$1. \ 3 \log x + 2 \log x^2 = \log 128$$

$$2. \ 3 \log x^2 = 4 + 4 \log x$$

Solución:

1.

$$3 \log x + 2 \log x^2 = \log 128$$

$$3 \log x + 4 \log x = \log 128$$

$$7 \log x = \log 128$$

$$\log x^7 = \log 2^7$$

$$x^7 = 2^7$$

$$x = 2$$

2.

$$3 \log x^2 = 4 + 4 \log x$$

$$6 \log x - 4 \log x = 4$$

$$2 \log x = 4$$

$$\log x = 2$$

$$\log x = \log 10^2$$

$$x = 10^2 = 100$$

Problema 62 Halla las soluciones de:

$$\log(3x^2 - 2) = 1 + \log(x - 1)$$

Solución:

$$\log(3x^2 - 2) = \log 10 + \log(x - 1)$$

$$\log(3x^2 - 2) = \log 10(x - 1)$$

$$3x^2 - 2 = 10(x - 1)$$

$$3x^2 - 10x + 8 = 0 \implies x = \frac{10 \pm \sqrt{100 - 96}}{6}$$

$$x = \frac{10 \pm 2}{6} \implies x = 2, \ x = \frac{4}{3}$$

Problema 63 Halla las soluciones de:

$$\log(x^2 + 6x + 7) = 1 + \log(x + 1)$$

Solución:

$$\log(x^2 + 6x + 7) = \log 10 + \log(x + 1)$$

$$\log(x^2 + 6x + 7) = \log 10(x + 1)$$

$$x^2 + 6x + 7 = 10(x + 1)$$

$$x^2 - 4x - 3 = 0 \implies x = 3, x = 1$$

Problema 64 Hallar las soluciones reales de:

$$\log(3x^2 - 2) = 1 + \log(x - 1)$$

Solución:

$$\log(3x^2 - 2) = 1 + \log(x - 1) \implies \log(3x^2 - 2) = \log 10 + \log(x - 1) \implies$$

$$\log(3x^2 - 2) = \log 10(x - 1) \implies (3x^2 - 2) = 10(x - 1) \implies 3x^2 - 10x + 8 = 0$$

$$\implies \begin{cases} x = 2 \\ x = \frac{4}{3} \end{cases}$$

Problema 65 Hallar las soluciones reales de:

$$\log(x^2 + 2699) = 2 + \log(x + 2)$$

Solución:

$$\log(x^2 + 2699) = 2 + \lg(x + 2) \implies \log(x^2 + 2699) = \log 100 + \log(x + 2) \implies$$

$$\log(x^2 + 2699) = \log 100(x + 2) \implies (x^2 + 2699) = 100(x + 2) \implies$$

$$x^2 - 100x + 2499 = 0 \implies \begin{cases} x = 51 \\ x = 49 \end{cases}$$

Problema 66 Calcular

$$\log(x^2 - 1) + 2 = 1 + 2 \log(x + 1)$$

Solución:

$$\log(x^2 - 1) + 2 = 1 + 2 \log(x + 1) \implies \log(x^2 - 1) + 1 = 2 \lg(x + 1) \implies$$

$$\lg 10(x^2 - 1) = \lg(x + 1)^2 \implies 10(x^2 - 1) = (x + 1)^2 \implies 9x^2 - 2x - 11 = 0$$

$$\implies \begin{cases} x = -1 \\ x = \frac{11}{9} \end{cases}$$

La solución $x = -1$ no es válida.

Problema 67 Resolver la siguiente ecuación:

$$\log(1 + x^2) - 1 = \log(x - 2)$$

Solución:

$$\log(1 + x^2) - 1 = \log(x - 2) \implies \log(1 + x^2) - \log 10 = \log(x - 2) \implies$$

$$\log\left(\frac{1 + x^2}{10}\right) = \log(x - 2)$$

$$\frac{1 + x^2}{10} = x - 2 \implies 1 + x^2 = 10x - 20 \implies x^2 - 10x + 21 = 0 \implies$$

$$x = 7, \quad x = 3$$

Problema 68 Resolver las ecuaciones:

1. $\log \frac{10}{x} = 2 - 2 \log x$

2. $3 \log x - 2 = 2 \log x$

Solución:

1.

$$\log 10 - \log x = 2 - 2 \log x$$

$$1 - \log x = 2 - 2 \log x$$

$$2 \log x - \log x = 2 - 1$$

$$\log x = 1 \implies x = 10$$

2.

$$3 \log x - 2 = 2 \log x$$

$$3 \log x - 2 \log x = 2$$

$$\log x = 2 \implies x = 10^2 = 100$$

Problema 69 Resolver las ecuaciones:

1. $\log 10(x + 2) - \log(x^2) = 1$

2. $\log x + \log x^2 = 3$

Solución:

1.

$$\log \frac{10(x+2)}{x^2} = \log 10$$

$$\frac{10(x+2)}{x^2} = 10$$

$$10x + 20 = 10x^2$$

$$x^2 - x - 2 = 0 \implies x = 2, \quad x = -1$$

2.

$$\log x + 2 \log x = 3$$

$$3 \log x = 3$$

$$\log x = 1 \implies x = 10$$

Problema 70

$$\log(3x+1) - 2 \log x = 2$$

Solución:

$$\log \left(\frac{3x+1}{x^2} \right) = \log 100 \implies 100x^2 - 3x - 1 = 0 \implies$$

$$\begin{cases} x = 0,116187 \\ x = -0,0861187 \text{ No Vale} \end{cases}$$

Problema 71

$$\log(2x+1) - 2 \log x = 1$$

Solución:

$$\log \left(\frac{2x+1}{x^2} \right) = \log 10 \implies 10x^2 - 2x - 1 = 0 \implies$$

$$\begin{cases} x = 0,43166 \\ x = -0,23166 \text{ No Vale} \end{cases}$$

Problema 72

$$2 \log(x+1) - \log x = 1$$

Solución:

$$\log \left(\frac{(x+1)^2}{x} \right) = \log 10 \implies x^2 - 8x + 1 = 0 \implies$$

$$\begin{cases} x = 0,127 \\ x = 7,873 \end{cases}$$

Problema 73

$$\log x - \log(1 - x) = 2$$

Solución:

$$\log\left(\frac{x}{1-x}\right) = \log 100 \Rightarrow 101x = 100 \Rightarrow x = \frac{100}{101}$$

Problema 74

$$\log(x+1) - \log(x^2-1) = 1$$

Solución:

$$\log\left(\frac{x+1}{x^2-1}\right) = \log 10 \Rightarrow 10x^2 - x - 11 = 0 \Rightarrow \begin{cases} x = 1, 1 \\ x = -1 \text{ No Vale} \end{cases}$$

Problema 75

$$\log x - \log(1 - x) = 2$$

Solución:

$$\log\left(\frac{x}{1-x}\right) = \log 100 \Rightarrow 101x = 100 \Rightarrow x = \frac{100}{101}$$

Problema 76 Resolver las ecuaciones:

1. $\log x^2 - \log(x-1) + 1 = 2 \log x$

2. $\log(x+1) - 2 \log(x-1) = 1$

Solución:

1. $\log x^2 - \log(x-1) + 1 = 2 \log x \Rightarrow \log \frac{10x^2}{x-1} = \log x^2 \Rightarrow$

$$x^2(11-x) = 0 \Rightarrow x = 11 \text{ y } x = 0 \text{ (no vale).}$$

2. $\log(x+1) - 2 \log(x-1) = 1 \Rightarrow \log \frac{x+1}{(x-1)^2} = \log 10 \Rightarrow$

$$10x^2 - 21x + 9 = 0 \Rightarrow x = \frac{3}{2} \text{ y } x = \frac{3}{5} \text{ no vale}$$

Problema 77 Resolver las ecuaciones:

1. $\log(10x^2 - 2) - 1 = \log(x+1) + \log x$

2. $\log(3x^2 - 2) - 2 \log(1-x) = 1$

Solución:

1. $\log(10x^2 - 2) - 1 = \log(x + 1) + \log x \implies \log \frac{10x^2 - 2}{10} = \log x(x + 1)$
 $\implies 10x^2 - 2 = 10x(x + 1) \implies x = -\frac{1}{5}$
2. $\log(3x^2 - 2) - 2\log(1 - x) = 1 \implies \log \frac{3x^2 - 2}{(1 - x)^2} = \log 10 \implies$
 $7x^2 - 20x + 12 = 0 \implies x = \frac{6}{7}, \quad x = 2 \text{ (no vale)}$

Problema 78 Resolver las ecuaciones:

1. $2\log(x - 1) + 1 = \log(x^2 - 1)$
2. $\log(10(x^3 + 2x)) - 2\log(x + 1) = 1 + \log x$

Solución:

1. $2\log(x - 1) + 1 = \log(x^2 - 1) \implies \log 10(x - 1)^2 = \log(x^2 - 1)$
 $\implies 9x^2 - 20x + 11 = 0 \implies x = \frac{11}{9} \text{ y } x = 1 \text{ (no vale).}$
2. $\log(10(x^3 + 2x)) - 2\log(x + 1) = 1 + \log x \implies$
 $\log \frac{10(x^3 + 2x)}{(x + 1)^2} = \log 10x \implies 2x^2 - x = 0 \implies$
 $x = \frac{1}{2} \text{ y } x = 0 \text{ (no vale).}$

Problema 79 Resolver las ecuaciones:

1. $\log(x - 1) + \log(x + 1) = 2\log x - 1$
2. $\log x^2 + 3\log x = 2$

Solución:

1. $\log(x - 1) + \log(x + 1) = 2\log x - 1 \implies \log(x^2 - 1) = \log(x - 1)^2$
 $\implies 9x^2 = 10 \implies x = \frac{\sqrt{10}}{3}, \quad x = -\frac{\sqrt{10}}{3} \text{ (no vale)}$
2. $\log x^2 + 3\log x = 2 \implies \log x^5 = \log 100 \implies x = \sqrt[5]{100} = 2,51188$

Problema 80 Resolver la siguiente ecuación:

$$\log(2 + x) - \log x = 1 + \log(1 - x)$$

Solución:

$$\log(2 + x) - \log x = 1 + \log(1 - x) \implies \log \frac{2 + x}{x} = \log 10 + \log(1 - x)$$

$$\log \frac{2 + x}{x} = \log(10(1 - x)) \implies 10x^2 - 9x + 2 = 0 \implies x = \frac{1}{2}, \quad x = \frac{2}{5}$$