

42. Calcula el ángulo que forman las rectas:

a) $r: 3x - 4y = 0$ y $s: 2x + 2y + 3 = 0$

b) $r: y = x - 5$ y $s: y = 2x + 2$

OBSERVACIÓN

$$\cos(\hat{r}, \hat{s}) = |\cos(\vec{v}_r, \vec{v}_s)| = |\cos(\vec{n}_r, \vec{n}_s)|$$
$$\parallel (\vec{v}_r, \vec{v}_s) = (\vec{n}_r, \vec{n}_s) \parallel$$

a) $r: 3x - 4y = 0 \rightarrow \vec{n}_r = (3, -4)$

$s: 2x + 2y + 3 = 0 \rightarrow \vec{n}_s = (2, 2)$

$$\cos(\hat{r}, \hat{s}) = |\cos(\vec{n}_r, \vec{n}_s)| = \frac{|\vec{n}_r \cdot \vec{n}_s|}{|\vec{n}_r| \cdot |\vec{n}_s|} = \frac{|(3, -4) \cdot (2, 2)|}{\sqrt{3^2 + (-4)^2} \cdot \sqrt{2^2 + 2^2}} =$$
$$= \frac{|3 \cdot 2 + (-4) \cdot 2|}{5 \cdot \sqrt{8}} = \frac{|-2|}{5 \cdot 2\sqrt{2}} = \frac{2}{10\sqrt{2}} = \frac{1}{5\sqrt{2}}$$

$$\cos(\hat{r}, \hat{s}) = \frac{1}{5\sqrt{2}} \Rightarrow (\hat{r}, \hat{s}) = \arccos \frac{1}{5\sqrt{2}} = \boxed{81,87^\circ}$$

b) $r: y = x - 5 \rightarrow x - y - 5 = 0 \rightarrow \vec{n}_r = (1, -1)$

$s: y = 2x + 2 \rightarrow 2x - y + 2 = 0 \rightarrow \vec{n}_s = (2, -1)$

$$\cos(\hat{r}, \hat{s}) = \frac{|(1, -1) \cdot (2, -1)|}{\sqrt{1^2 + (-1)^2} \cdot \sqrt{2^2 + (-1)^2}} = \frac{|2 + 1|}{\sqrt{2} \cdot \sqrt{5}} = \frac{3}{\sqrt{10}}$$

$$(\hat{r}, \hat{s}) = \arccos \frac{3}{\sqrt{10}} = 18,43^\circ$$

43. Calcula el ángulo formado por $r: 4x + 2y - 7 = 0$ y el eje Y .

$$r: 4x + 2y - 7 = 0 \rightarrow \vec{n}_r = (4, 2)$$

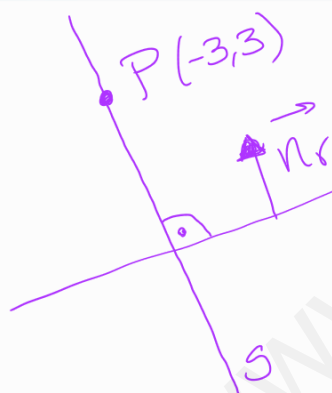
$$s: OY \rightarrow x = 0 \rightarrow \vec{n}_{OY} = (1, 0)$$

$$\cos(\hat{r}, \hat{OY}) = |\cos(\vec{n}_r, \vec{n}_{OY})| = \frac{|(4, 2) \cdot (1, 0)|}{\sqrt{4^2 + 2^2} \cdot \sqrt{1^2 + 0^2}} =$$

$$= \frac{|4 + 0|}{\sqrt{20} \cdot 1} = \frac{2\cancel{4}}{\cancel{2}\sqrt{5}} = \frac{2}{\sqrt{5}}$$

$$(\hat{r}, \hat{OY}) = \arccos\left(\frac{2}{\sqrt{5}}\right) = 26,56^\circ$$

44. Calcula la recta perpendicular a $r: x + y - 3 = 0$ y que pasa por el punto $P(-3, 3)$.



¿S?

$$r: x + y - 3 = 0$$

$$\vec{n}_r = (1, 1) = \vec{U}_3$$

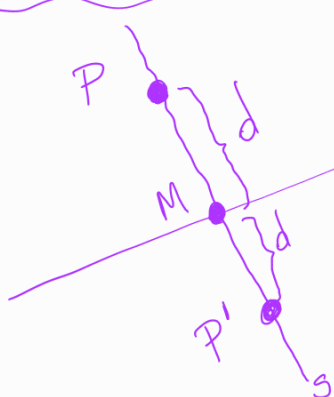
$$s: \frac{x+3}{1} = \frac{y-3}{1}$$

PUNTO SÍMÉTRICO A UNA RECTA:

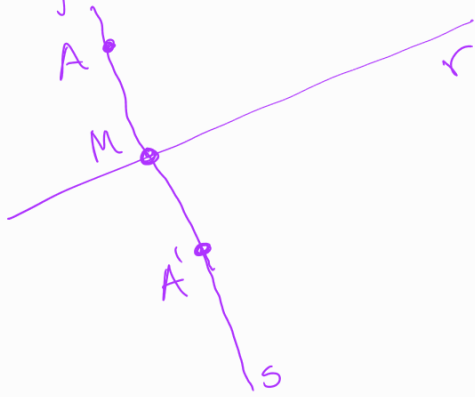
Nos dan P y r

$\parallel M$ es pto medio de PP' $\parallel$

$M = r \cap s$ tal que $s \begin{cases} P \in s \\ s \perp r \end{cases}$



Ejemplo: Sea $A=(3, 2)$ respecto $r: 2x-y+1=0$



$$M = r \cap s \quad y \quad s \begin{cases} A(3, 2) \\ s \perp r \\ \vec{U}_s = \vec{n}_r = (2, -1) \\ \vec{n}_s = (1, 2) \end{cases}$$

$$s: 1 \cdot (x-3) + 2 \cdot (y-2) = 0$$

$$\underline{x + 2y - 7 = 0}$$

$$M_{r \cap s} \begin{cases} 2x - y + 1 = 0 \rightarrow \underline{2x+1=y} \\ x + 2y - 7 = 0 \end{cases} \xrightarrow{\downarrow} x + 2(2x+1) - 7 = 0$$

$$x + 4x + 2 - 7 = 0$$

$$5x - 5 = 0$$

$$5x = 5$$

$$\underline{x = 1} \Rightarrow y = 2 \cdot 1 + 1 = \underline{3}$$

$$M = (1, 3) \begin{cases} 1 = \frac{3+x'}{2} \Rightarrow 2 = 3+x' \Rightarrow 2-3=x' \Rightarrow \underline{x' = -1} \\ 3 = \frac{2+y'}{2} \Rightarrow 6 = 2+y' \Rightarrow 6-2=y' \Rightarrow \underline{y' = 4} \end{cases}$$

$$\boxed{A' = (-1, 4)}$$