

Radicales

49) Simplifica:

$$a) \sqrt[5]{2^3 \sqrt{2} \sqrt[3]{2}} = \sqrt[5]{2} \cdot \sqrt[15]{2} \cdot \sqrt[30]{2} = 2^{\frac{1}{5}} \cdot 2^{\frac{1}{15}} \cdot 2^{\frac{1}{30}} = 2^{\frac{1}{5} + \frac{1}{15} + \frac{1}{30}} = 2^{\frac{9}{30}} = 2^{\frac{3}{10}} = \sqrt[10]{2^3}$$

Reducimos a índice común para multiplicar, m.c.m.(2,5,30)=30

$$b) \sqrt[12]{2 \cdot \sqrt[3]{15} \cdot \sqrt[5]{20}} = \sqrt[12]{2} \cdot \sqrt[6]{3 \cdot 5} \cdot \sqrt[30]{2^2 \cdot 5} = \sqrt[30]{3^{15} \cdot 2^{30} \cdot 3^5 \cdot 5^5 \cdot 2^2 \cdot 5} = \sqrt[30]{3^{20} \cdot 2^{32} \cdot 5^6} = 2 \cdot \sqrt[15]{2 \cdot 3^{10} \cdot 5^3}$$

Simplificamos índice y exponentes : 2.

50) Reduce a un único radical:

$$a) \sqrt[3]{729} = \sqrt[3]{3^6} = 3 ; b) \sqrt[3]{\sqrt[4]{4096 \cdot x^{12} \cdot y^{18}}} = \sqrt[6]{2^{12} \cdot x^{12} \cdot y^{18}} = \sqrt[9]{2^9} = 2$$

51b) Simplifica:

$$\sqrt{\frac{\sqrt{x^3} \cdot \sqrt{x^5}}{\sqrt{x^5} \cdot \sqrt{x^3}}} = \frac{\sqrt[2]{x^3} \cdot \sqrt[2]{x^5}}{\sqrt[2]{x^5} \cdot \sqrt[2]{x^3}} = \frac{\sqrt[4]{x^3} \cdot \sqrt[8]{x^5}}{\sqrt[4]{x^5} \cdot \sqrt[8]{x^3}} = \frac{x^{\frac{3}{4}} \cdot x^{\frac{5}{8}}}{x^{\frac{5}{4}} \cdot x^{\frac{3}{8}}} = \frac{x^{\frac{3}{4} + \frac{5}{8}}}{x^{\frac{5}{4} + \frac{3}{8}}} = \frac{x^{\frac{11}{8}}}{x^{\frac{13}{8}}} = x^{-\frac{2}{8}} = x^{-\frac{1}{4}} = \frac{1}{\sqrt[4]{x}}$$

2 · 2 = 4
2 · 2 · 2

3 | 2 → expon. sale
1 | 1 → expon. dentro

$$52b) 2 \cdot \sqrt{75} + 7 \cdot \sqrt{27} - \sqrt{48} = 2 \cdot \sqrt{5^2 \cdot 3} + 7 \cdot \sqrt{3^3} - \sqrt{3 \cdot 2^4} = 2 \cdot 5 \cdot \sqrt{3} + 7 \cdot 3 \cdot \sqrt{3} - 2^2 \sqrt{3} = (10 + 21 - 4) \sqrt{3} = 27 \sqrt{3}$$

$$d) 3 \sqrt{32} - 5 \sqrt{18} + \sqrt{18} = 3 \cdot \sqrt{2^5} - 5 \cdot \sqrt{2 \cdot 3^2} + \sqrt{2 \cdot 3^2} = 3 \cdot 2^2 \cdot \sqrt{2} - 5 \cdot 3 \cdot \sqrt{2} + 3 \cdot \sqrt{2} = (12 - 15 + 3) \sqrt{2} = 0 \sqrt{2} = 0$$

$$\begin{aligned}
 (53b) \quad \frac{2 \cdot \sqrt[5]{\sqrt[3]{2} + \sqrt[3]{16}}}{3 \cdot \sqrt[5]{128} \cdot \sqrt[3]{4}} &= \frac{2 \cdot \sqrt[5]{\sqrt[3]{2} + 2 \cdot \sqrt[3]{2}}}{3 \cdot \sqrt[5]{2^7} \cdot \sqrt[3]{2^2}} = \frac{2 \cdot \sqrt[5]{3 \cdot \sqrt[3]{2}}}{3 \cdot 2 \sqrt[5]{2^7} \cdot \sqrt[3]{2^2}} = \\
 &= \frac{\sqrt[5]{3^3 \cdot 2}}{3 \cdot \sqrt[15]{2^6 \cdot 2^{10}}} = \frac{\sqrt[15]{3^3 \cdot 2}}{3 \cdot \sqrt[15]{2^{16}}} = \frac{1}{3} \cdot \sqrt[15]{\frac{3^3 \cdot 2}{2^{16}}} = \frac{1}{3} \cdot \sqrt[15]{\frac{3^3}{2^{15}}} = \frac{1}{3 \cdot 2} \cdot \sqrt[15]{3^3} = \boxed{\frac{\sqrt[5]{3}}{6}}
 \end{aligned}$$

$$\begin{aligned}
 (53c) \quad \sqrt[5]{\frac{\sqrt{5} + 3\sqrt{20}}{7^4 \sqrt{5}}} &= \sqrt[5]{\frac{\sqrt{5} + 3\sqrt{2^2 \cdot 5}}{7^4 \sqrt{5}}} = \sqrt[5]{\frac{\sqrt{5} + 3 \cdot 2\sqrt{5}}{7^4 \sqrt{5}}} = \sqrt[5]{\frac{7\sqrt{5}}{7^4 \sqrt{5}}} = \\
 &= \sqrt[5]{\frac{4\sqrt{5^2}}{5}} = \boxed{\sqrt[20]{5}}
 \end{aligned}$$

$$(54d) \text{ Racionaliza: } \frac{7}{5 \cdot \sqrt[7]{3^4}} = \frac{7}{5 \cdot \sqrt[7]{3^4}} \cdot \frac{\sqrt[7]{3^3}}{\sqrt[7]{3^3}} = \frac{7 \cdot \sqrt[7]{3^3}}{5 \cdot \sqrt[7]{3^{4+3}}} = \frac{7 \sqrt[7]{3^3}}{15}$$

$7-4 = 3$ $3^4 \cdot 3^3 = 3^{4+3} = 3^7$

$$\begin{aligned}
 (57c) \text{ Opera y racionaliza: } \frac{7 \cdot \sqrt{2}}{3\sqrt{2} \cdot \sqrt{2}} - \frac{3 \cdot \sqrt{5}}{2\sqrt{5} \cdot \sqrt{5}} &= \frac{7\sqrt{2}}{3\sqrt{2}^2} - \frac{3\sqrt{5}}{2\sqrt{5}^2} = \\
 &= \frac{7\sqrt{2}}{6} - \frac{3\sqrt{5}}{10} = \frac{35\sqrt{2} - 9\sqrt{5}}{30}
 \end{aligned}$$

$$\begin{aligned}
 (58a) \text{ Racionaliza } \frac{\sqrt{2} - \sqrt{3}}{-\sqrt{2} - \sqrt{3}} &= \frac{\sqrt{2} - \sqrt{3}}{-\sqrt{2} - \sqrt{3}} \cdot \frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} + \sqrt{3}} = \frac{(\sqrt{2} - \sqrt{3}) \cdot (\sqrt{2} + \sqrt{3}) \cdot (-1)}{(-\sqrt{2})^2 - \sqrt{3}^2} = \\
 &= \frac{-(\sqrt{2}^2 + \sqrt{3}^2 - 2 \cdot \sqrt{2} \cdot \sqrt{3})}{2 - 3} = \frac{-(2 + 3 - 2\sqrt{6})}{-1} = 5 - 2\sqrt{6}
 \end{aligned}$$

$$\textcircled{58b} \frac{a}{\sqrt{a+\sqrt{b}}} = \frac{a}{\sqrt{a+\sqrt{b}}} \cdot \frac{\sqrt{a+\sqrt{b}}}{\sqrt{a+\sqrt{b}}} = \frac{a \cdot \sqrt{a+\sqrt{b}}}{\sqrt{a+\sqrt{b}} \cdot \sqrt{a+\sqrt{b}}} \cdot \frac{a-\sqrt{b}}{a-\sqrt{b}} =$$

$$= \frac{a \cdot (a-\sqrt{b}) \cdot \sqrt{a+\sqrt{b}}}{a^2 - \sqrt{b}^2} = \frac{a \cdot (a-\sqrt{b}) \cdot \sqrt{a+\sqrt{b}}}{a^2 - b}$$

$$\textcircled{58d} \sqrt[3]{\frac{2\sqrt{5}}{\sqrt{5}+3\sqrt{20}}} = \sqrt[3]{\frac{2\sqrt{5} \cdot (\sqrt{5}-3\sqrt{20})}{(\sqrt{5}+3\sqrt{20})(\sqrt{5}-3\sqrt{20})}} = \sqrt[3]{\frac{2 \cdot \sqrt{5}^2 - 6\sqrt{100}}{\sqrt{5}^2 - 3^2 \sqrt{20}^2}} =$$

$$= \sqrt[3]{\frac{2 \cdot 5 - 6 \cdot 10}{5 - 9 \cdot 20}} = \sqrt[3]{\frac{-50}{-175}} = \sqrt[3]{\frac{2}{7}} = \sqrt[3]{\frac{2}{7}} \cdot \sqrt[3]{\frac{7^2}{7^2}} = \frac{\sqrt[3]{2 \cdot 7^2}}{7}$$

2ª forma:

$$\sqrt[3]{\frac{2\sqrt{5}}{\sqrt{5}+3\sqrt{20}}} = \sqrt[3]{\frac{2\sqrt{5}}{\sqrt{5}+6\sqrt{5}}} = \sqrt[3]{\frac{2\sqrt{5}}{7\sqrt{5}}} = \sqrt[3]{\frac{2}{7}} =$$

1º Agrupo los nros en un binomio para poder aplicar caso C.

$$\textcircled{125a} \text{Racionaliza: } \frac{2}{3+\sqrt{5}-\sqrt{3}} = \frac{2}{(3+\sqrt{5})-\sqrt{3}} \cdot \frac{(3+\sqrt{5})+\sqrt{3}}{(3+\sqrt{5})+\sqrt{3}} =$$

$$= \frac{2 \cdot (3+\sqrt{5}+\sqrt{3})}{(3+\sqrt{5})^2 - \sqrt{3}^2} = \frac{2 \cdot (3+\sqrt{5}+\sqrt{3})}{9+5+6\sqrt{5}-3} = \frac{2 \cdot (3+\sqrt{5}+\sqrt{3})}{11+6\sqrt{5}} \cdot \frac{11-6\sqrt{5}}{11-6\sqrt{5}} =$$

$$= \frac{(6+2\sqrt{5}+2\sqrt{3})(11-6\sqrt{5})}{11^2 - 6^2 \cdot 5} = \frac{66 - 36\sqrt{5} + 22\sqrt{3} - 12 \cdot 5 + 22\sqrt{3} - 12\sqrt{15}}{121 - 36 \cdot 5} =$$

$$= \frac{6 - 14\sqrt{5} + 22\sqrt{3} - 12\sqrt{15}}{-59}$$