

Ejercicios resueltos

Ejemplo 1 : Integre $\int x^3 \ln x \, dx$

Solución : Hacemos

$$\begin{array}{lll} u = \ln x & \xrightarrow{\text{Al derivar}} & du = \frac{1}{x} \, dx \\ dv = x^3 \, dx & \xrightarrow{\text{Al integrar}} & v = \frac{x^4}{4} \end{array}$$

La integral se transforma en

$$\int x^3 \ln x \, dx = \frac{x^4}{4} \ln x - \int \frac{x^4}{4} \frac{1}{x} \, dx = \frac{x^4}{4} \ln x - \frac{1}{4} \int x^3 \, dx = \frac{x^4}{4} \ln x - \frac{x^4}{16} + C.$$

Por lo tanto,

$$\int x^3 \ln x \, dx = \frac{x^4}{4} \ln x - \frac{x^4}{16} + C.$$

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Ejemplo 2 : Integre $\int x^2 \cos x \, dx$

Solución : Hacemos

$$\begin{array}{lll} u = x^2 & \xrightarrow{\text{Al derivar}} & du = 2x \, dx \\ dv = \cos x \, dx & \xrightarrow{\text{Al integrar}} & v = \sin x \end{array}$$

La integral se transforma en

$$\int x^2 \cos x \, dx = x^2 \sin x - \int 2x \sin x \, dx = x^2 \sin x - 2 \int x \sin x \, dx$$

Resolvemos la nueva integral $\int x \sin x \, dx$, integramos otra vez por partes. Hacemos

$$\begin{array}{lll} u = x & \xrightarrow{\text{Al derivar}} & du = dx \\ dv = \sin x \, dx & \xrightarrow{\text{Al integrar}} & v = -\cos x \end{array}$$

y obtenemos

$$\int x \sin x \, dx = -x \cos x - \int -\cos x \, dx = -x \cos x + \int \cos x \, dx = -x \cos x + \sin x + C_1,$$

entonces,

$$\int x^2 \cos x \, dx = x^2 \sin x - 2(-x \cos x + \sin x + C_1) = x^2 \sin x + 2x \cos x - 2 \sin x + C,$$

así, la familia de primitivas es

$$\int x^2 \cos x \, dx = x^2 \sin x + 2x \cos x - 2 \sin x + C.$$

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Ejemplo 3 : Integre $\int \sin^2 x \, dx$

Solución : Observemos que

$$\int \sin^2 x \, dx = \int \sin x \sin x \, dx.$$

Integramos por partes, hacemos

$$\begin{array}{lll} u = \sin x & \xrightarrow{\text{Al derivar}} & du = \cos x \, dx \\ dv = \sin x \, dx & \xrightarrow{\text{Al integrar}} & v = -\cos x \end{array}$$

La integral se transforma en

$$\begin{aligned}\int \sin^2 x \, dx &= -\sin x \cos x - \int -\cos x \cos x \, dx = -\sin x \cos x + \int \cos^2 x \, dx \\ &= -\sin x \cos x + \int (1 - \sin^2 x) \, dx = -\sin x \cos x + \int dx - \int \sin^2 x \, dx\end{aligned}$$

como $\int dx = x + C_1$, tenemos que

$$\int \sin^2 x \, dx = -\sin x \cos x + x - \int \sin^2 x \, dx + C_1 \implies \int \sin^2 x \, dx + \int \sin^2 x \, dx = -\sin x \cos x + x + C_1,$$

de aquí,

$$2 \int \sin^2 x \, dx = -\sin x \cos x + x + C_1 \implies \int \sin^2 x \, dx = \frac{1}{2} (-\sin x \cos x + x + C_1).$$

Por lo tanto,

$$\int \sin^2 x \, dx = \frac{1}{2} \sin x \cos x + \frac{1}{2} x + C$$

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Ejemplo 4 : Integre $\int e^{\sqrt{x}} \, dx$

Solución : Hacemos el cambio de variable

$$p = \sqrt{x}; \quad dp = \frac{1}{2\sqrt{x}} \implies 2p \, dp = dx$$

y la integral nos queda

$$\int e^{\sqrt{x}} \, dx = \int 2pe^p \, dp = 2 \int pe^p \, dp$$

integraremos por partes, hacemos

$$\begin{array}{ccc} u = p & \xrightarrow{\text{Al derivar}} & du = dp \\ dv = e^p \, dp & \xrightarrow{\text{Al integrar}} & v = e^p, \end{array}$$

la integral se transforma,

$$\int pe^p \, dp = pe^p - \int e^p \, dp = pe^p - e^p + C_1,$$

como $p = \sqrt{x}$, se tiene

$$\int e^{\sqrt{x}} \, dx = 2 \left(\sqrt{x} e^{\sqrt{x}} - e^{\sqrt{x}} + C_1 \right) = 2e^{\sqrt{x}} (\sqrt{x} - 1) + C,$$

es decir,

$$\int e^{\sqrt{x}} \, dx = 2e^{\sqrt{x}} (\sqrt{x} - 1) + C.$$

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Ejemplo 5 : Integre $\int \csc^3 x \, dx$

Solución : Escribimos la integral como

$$\int \csc^3 x \, dx = \int \csc^2 x \csc x \, dx.$$

Integramos por partes, hacemos

$$\begin{array}{ccc} u = \csc x & \xrightarrow{\text{Al derivar}} & du = -\csc x \cot x \, dx \\ dv = \csc^2 x \, dx & \xrightarrow{\text{Al integrar}} & v = -\cot x, \end{array}$$

La integral se transforma en

$$\int \csc^3 x \, dx = \csc x (-\cot x) - \int (-\cot x) (-\csc x \cot x) \, dx = -\csc x \cot x - \int \csc x \cot^2 x \, dx,$$

es conocido que

$$\cot^2 x = \csc^2 x - 1,$$

así,

$$\begin{aligned}\int \csc^3 x \, dx &= -\csc x \cot x - \int \csc x \cot^2 x \, dx = -\csc x \cot x - \int \csc x (\csc^2 x - 1) \, dx \\ &= -\csc x \cot x - \int \csc^3 x \, dx + \int \csc x \, dx = -\csc x \cot x - \int \csc^3 x \, dx + \ln |\csc x - \cot x| + C,\end{aligned}$$

es decir,

$$\int \csc^3 x \, dx = -\csc x \cot x - \int \csc^3 x \, dx + \ln |\csc x - \cot x| + C,$$

de aquí,

$$2 \int \csc^3 x \, dx = -\csc x \cot x + \ln |\csc x - \cot x| + C,$$

con lo que,

$$\int \csc^3 x \, dx = -\frac{1}{2} \csc x \cot x + \frac{1}{2} \ln |\csc x - \cot x| + C.$$

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Ejemplo 6 : Demuestre que

$$\int \frac{x^n \, dx}{\sqrt{1-x^2}} = -x^{n-1} \sqrt{1-x^2} + (n-1) \int x^{n-2} \sqrt{1-x^2} \, dx$$

Demostración : Escribimos la integral como

$$\int \frac{x^n \, dx}{\sqrt{1-x^2}} = \int x^{n-1} \frac{x}{\sqrt{1-x^2}} \, dx$$

Integramos por partes, hacemos

$$\begin{array}{lll}u = x^{n-1} & \xrightarrow{\text{Al derivar}} & du = (n-1) x^{n-2} \, dx \\ dv = \frac{x}{\sqrt{1-x^2}} \, dx & \xrightarrow{\text{Al integrar}} & v = -\sqrt{1-x^2}.\end{array}$$

La integral se transforma en

$$\int \frac{x^n \, dx}{\sqrt{1-x^2}} = x^{n-1} (-\sqrt{1-x^2}) - \int (-\sqrt{1-x^2}) (n-1) x^{n-2} \, dx = -x^{n-1} \sqrt{1-x^2} + (n-1) \int x^{n-2} \sqrt{1-x^2} \, dx$$

entonces,

$$\int \frac{x^n \, dx}{\sqrt{1-x^2}} = -x^{n-1} \sqrt{1-x^2} + (n-1) \int x^{n-2} \sqrt{1-x^2} \, dx.$$

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Ejercicios

1. Calcular las siguientes integrales

1. $\int x e^x \, dx$
2. $\int \frac{x}{e^x} \, dx$
3. $\int x 2^{-x} \, dx$
4. $\int x \sin x \, dx$
5. $\int t \cos t \, dt$
6. $\int x e^{2x} \, dx$
7. $\int \frac{x^2}{e^{3x}} \, dx$
8. $\int x^2 3^x \, dx$
9. $\int x^2 \sin x \, dx$
10. $\int t^3 \sin t \, dt$
11. $\int \ln x \, dx$
12. $\int \arctan x \, dx$
13. $\int \arcsen x \, dx$
14. $\int 4x \ln 2x \, dx$
15. $\int \sqrt{x} \ln x \, dx$
16. $\int x \arctan x \, dx$
17. $\int x \arcsen x \, dx$
18. $\int x^3 e^{x^2} \, dx$
19. $\int \cos^2 x \, dx$
20. $\int \theta \cos 3\theta \, d\theta$
21. $\int x^5 \cos(x^3) \, dx$
22. $\int (t^2 + 5t + 6) \cos 2t \, dt$
23. $\int \sec^3 \theta \, d\theta$
24. $\int e^x \sin x \, dx$
25. $\int \sin 3x \cos 5x \, dx$
26. $\int x \sin x \cos x \, dx$

27. $\int x^2 \ln x \, dx$ 28. $\int \frac{\ln x}{\sqrt{x}} \, dx$ 29. $\int e^{5x} \cos 2x \, dx$ 30. $\int \cos\left(\frac{x}{2}\right) \cos\left(\frac{x}{3}\right) \, dx$
 31. $\int z^2 e^{3z} \, dz$ 32. $\int t^2 e^{-t/2} \, dt$ 33. $\int e^{at} \cos bt \, dt$ 34. $\int (x^2 - 2x + 5) e^{-x} \, dx$
 35. $\int \frac{x \, dx}{\sin^2 x}$ 36. $\int x \ln\left(\frac{1-x}{1+x}\right) \, dx$ 37. $\int x^2 \arctan 3x \, dx$ 38. $\int 5^x \sin 5x \, dx$
 39. $\int \ln^2 x \, dx$ 40. $\int e^{\sqrt{x}} \, dx$ 41. $\int e^{ax} \sin bx \, dx$ 42. $\int \ln\left(x\sqrt{1+x^2}\right) \, dx$
 43. $\int \sin(\ln x) \, dx$ 44. $\int y^3 e^{-y^2} \, dy$ 45. $\int \frac{x \cos x}{\sin^2 x} \, dx$ 46. $\int 3^x \cos x \, dx$
 47. $\int x^5 e^{x^2} \, dx$ 48. $\int \frac{\ln^2 t}{t^2} \, dt$ 49. $\int \frac{\ln(\ln x)}{x} \, dx$ 50. $\int (x^2 - 2x + 3) \ln x \, dx$
 51. $\int t^3 e^t \, dt$ 52. $\int \sqrt{x^2 + 1} \, dx$ 53. $\int x \tan^2 2x \, dx$ 54. $\int x (\arctan x)^2 \, dx$
 55. $\int \frac{\ln x}{x^3} \, dx$ 56. $\int \frac{\arcsin \sqrt{\theta}}{\sqrt{1-\theta}} \, d\theta$ 57. $\int \frac{\sin^2 x}{e^x} \, dx$ 58. $\int \cos x \cos^2(3x) \, dx$
 59. $\int x \csc^2 x \, dx$ 60. $\int x \tan^{-1} x \, dx$ 61. $\int \cos^2(\ln x) \, dx$ 62. $\int \cos t \ln(\sin t) \, dt$
 63. $\int (\ln x)^2 \, dx$ 64. $\int \sin \sqrt{x} \, dx$ 65. $\int x^2 \cos 3x \, dx$ 66. $\int x \cos^2 x \sin x \, dx$
 67. $\int \sec^5 \theta \, d\theta$ 68. $\int \frac{x \, dx}{\cos^3(x^2)}$ 69. $\int \frac{x e^x}{(x+1)^2} \, dx$ 70. $\int (\arcsin x)^2 \, dx$
 71. $\int x^3 \ln x \, dx$ 72. $\int t \sin 4t \, dt$ 73. $\int x^2 \sin 2x \, dx$ 74. $\int \sec^5(ax+b) \, dx$
 75. $\int x 5^x \, dx$ 76. $\int \theta \sec^2 \theta \, d\theta$ 78. $\int \sec^3(ax+b) \, dx$
 79. $\int z \cos 2z \, dz$ 80. $\int x \sin^2 x \, dx$ 81. $\int e^{-\theta} \cos 3\theta \, d\theta$ 82. $\int x a^x \, dx$
 83. $\int \frac{\ln x \, dx}{\sqrt{1-x}}$ 84. $\int \arccos z \, dz$ 85. $\int \sin 2t \sin 4t \, dt$ 86. $\int \sin 2t \ln(\cos^7 t) \, dt$
 87. $\int \frac{x e^{2x} \, dx}{\sqrt{1-e^{2x}}}$ 88. $\int t^3 \arctan 2t \, dt$ 89. $\int \frac{x \arcsin x}{\sqrt{1-x^2}} \, dx$ 90. $\int \frac{x \arcsin x}{\sqrt{(1-x^2)^3}} \, dx$
 91. $\int \frac{x \ln x \, dx}{\sqrt{1-x^2}}$ 92. $\int \sin 2x \ln(\sin^4 x \cos^5 x) \, dx$ 93. $\int \sin 2x \ln\left(\frac{\cos^{1/2} x}{\sin^{1/3} x}\right) \, dx$
 94. $\int \sin(2ax) \ln(\tan ax) \, dx$ 95. $\int \sin 2x \ln(\sin^5 x) \, dx$ 96. $\int \sin x \ln\left(\cot \frac{x}{2}\right) \, dx$
 97. $\int \cos 2x \ln(\sin x + \cos x) \, dx$ 98. $\int \cos x \ln(\sin^{-2} x \cos^3 x) \, dx$
 99. $\int \sin x \ln(\sin^4 x \cos^5 x) \, dx$ 100. $\int \frac{\arcsin^4 t \ln(\arcsin^3 t)}{\sqrt{1-t^2}} \, dt$

$$\begin{aligned}
101. \quad & \int \cos bx \ln (\sin^n bx \cos^m bx) \, dx & 102. \quad & \int \sin bx \ln (\sin^n bx \cos^m bx) \, dx \\
103. \quad & \int x \sinh^2 (x^2) \, dx & 104. \quad & \int_{-1}^1 \cosh^2 x \, dx & 105. \quad & \int \sinh \sqrt{x} \, dx \\
106. \quad & \int 3^t \sinh 3t \, dt & 107. \quad & \int e^{at} \cosh bt \, dt & 108. \quad & \int x^5 \cosh (x^3) \, dx \\
109. \quad & \int \csc^3 x \, dx & 110. \quad & \int \frac{x^3 \, dx}{\sqrt{1-x^2}} & 111. \quad & \int \frac{x^5 \, dx}{\sqrt{1-x^2}} & 112. \quad & \int \frac{x^7 \, dx}{\sqrt{1-x^2}}
\end{aligned}$$

2. Demostrar la fórmula de reducción

$$\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$

3. Demostrar la fórmula de reducción

$$\int (\ln x)^n \, dx = x (\ln x)^n - n \int (\ln x)^{n-1} \, dx$$

4. Demostrar la fórmula de reducción

$$\int x^n e^x \, dx = x^n e^x - n \int x^{n-1} e^x \, dx$$

5. Demostrar la fórmula de reducción

$$\int (x^2 + a^2)^n \, dx = \frac{x(x^2 + a^2)^n}{2n+1} + \frac{2na^2}{2n+1} \int (x^2 + a^2)^{n-1} \, dx,$$

con $n \neq -\frac{1}{2}$.

6. Demostrar la fórmula de reducción

$$\int \sec^n x \, dx = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx,$$

con $n \neq 1$.

Respuestas: Ejercicios

$$\begin{aligned}
1.1. \quad & (x-1)e^x + C; & 1.2. \quad & -(x+1)e^{-x} + C; & 1.3. \quad & -(1+x \ln 2) \frac{2^{-x}}{\ln^2 2} + C; & 1.4. \quad & \sin x - x \cos x + C; \\
1.5. \quad & \cos t + t \sin t + C; & 1.6. \quad & (2x-1) \frac{e^{2x}}{4} + C; & 1.7. \quad & -\left(\frac{2}{27} + \frac{2}{9}x + \frac{1}{3}x^2\right) e^{-3x} + C; & 1.8. \quad & 3^x \left(\frac{2}{\ln^3 3} - \frac{2x}{\ln^2 3} + \frac{x^2}{\ln 3}\right) + C; \\
1.9. \quad & 2 \cos x + 2x \sin x - x^2 \cos x + C; & 1.10. \quad & 6t \cos t - 6 \sin t - t^3 \cos t + 3t^2 \sin t + C; & 1.11. \quad & x(\ln x - 1) + C; \\
1.12. \quad & x \arctan x - \frac{1}{2} \ln(x^2 + 1) + C; & 1.13. \quad & x \arcsen x + \sqrt{1-x^2} + C; & 1.14. \quad & (2 \ln x + 2 \ln 2 - 1)x^2 + C; \\
1.15. \quad & \frac{2}{3}(\ln x - 1)x^{3/2} + C; & 1.16. \quad & \frac{1}{2} \arctan x - \frac{1}{2}x + \frac{1}{2}x^2 \arctan x + C; & 1.17. \quad & \frac{1}{2}x^2 \arcsen x - \frac{1}{4} \arcsen x + \frac{1}{4}x\sqrt{1-x^2} + C; \\
1.18. \quad & \frac{1}{2}e^{x^2}(x^2 - 1) + C; & 1.19. \quad & \frac{1}{2} \cos x \sin x + \frac{1}{2}x + C; & 1.20. \quad & \frac{1}{9} \cos 3\theta + \frac{1}{3}\theta \sin 3\theta + C; \\
1.21. \quad & \frac{1}{3} \cos(x^3) + \frac{1}{3}x^3 \sin(x^3) + C; & 1.22. \quad & \frac{5}{4} \cos 2t + \frac{11}{4} \sin 2t + \frac{1}{2}t \cos 2t + \frac{5}{2}t \sin 2t + \frac{1}{2}t^2 \sin 2t + C; \\
1.23. \quad & \frac{1}{2} \sec \theta \tan \theta + \frac{1}{2} \ln |\sec \theta + \tan \theta| + C; & 1.24. \quad & \frac{e^x}{2} (\sin x - \cos x) + C; & 1.25. \quad & \frac{5}{16} \sin 3x \sin 5x + \frac{3}{16} \cos 3x \cos 5x + C; \\
1.26. \quad & \frac{1}{2} x \sin^2 x + \frac{1}{4} \cos x \sin x - \frac{1}{4} x + C; & 1.27. \quad & \frac{1}{3} x^3 \left(\ln x - \frac{1}{3}\right) + C; & 1.28. \quad & 2\sqrt{x}(\ln x - 2) + C; \\
1.29. \quad & \frac{e^{5x}}{29} (5 \cos 2x + \sin 2x) + C; & 1.30. \quad & -\frac{12}{5} \cos\left(\frac{x}{2}\right) \sin\left(\frac{x}{3}\right) + \frac{18}{5} \sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{3}\right) + C; & 1.31. \quad & \frac{1}{3} e^{3z} \left(\frac{2}{9} - \frac{2}{3}z + z^2\right) + C; \\
1.32. \quad & -2e^{-t/2} (8 + 4t + t^2) + C; & 1.33. \quad & \frac{e^{at}}{a^2 + b^2} (a \cos bt + b \sin bt) + C; & 1.34. \quad & -e^{-x} (x^2 + 5) + C; \\
1.35. \quad & -x \cot x + \ln |\sin x| + C; & 1.36. \quad & \frac{1}{2} x^2 \ln\left(\frac{1-x}{x+1}\right) - x + \tanh^{-1} x + C; & 1.37. \quad & \frac{1}{3} x^3 \arctan 3x - \frac{1}{18} x^2 + \frac{1}{162} \ln(x^2 + \frac{1}{9}) + C; \\
1.38. \quad & \frac{5^x}{\ln^2 5 + 25} ((\ln 5) \sin 5x - 5 \cos 5x) + C; & 1.39. \quad & x(\ln^2 x - 2 \ln x + 2) + C; & 1.40. \quad & 2e^{\sqrt{x}} (\sqrt{x} - 1) + C;
\end{aligned}$$

- 1.41. $\frac{e^{ax}}{a^2+b^2} (a \sin bx - b \cos bx) + C$; 1.42. $\arctan x - 2x + x \ln(x\sqrt{x^2+1}) + C$; 1.43. $\frac{1}{2}x (\sin(\ln x) - \cos(\ln x)) + C$;
- 1.44. $-\frac{1}{2}e^{-y^2} (1+y^2) + C$; 1.45. $-x \csc x + \ln|\csc x - \cot x| + C$; 1.46. $\frac{3^x}{\ln^2 3+1} (\sin x + (\ln 3) \cos x) + C$;
- 1.47. $(1-x^2+\frac{1}{2}x^4)e^{x^2} + C$; 1.48. $-\frac{1}{t}(2+2\ln t+\ln^2 t) + C$; 1.49. $(\ln(\ln x)-1)\ln x + C$;
- 1.50. $\frac{1}{2}x^2-3x-\frac{1}{9}x^3+(3x-x^2+\frac{1}{3}x^3)\ln x + C$; 1.51. $(6t-6-3t^2+t^3)e^t + C$; 1.52. $\frac{1}{2}x\sqrt{x^2+1}+\frac{1}{2}\sinh^{-1}x + C$;
- 1.53. $\frac{x}{2}\tan 2x-\frac{1}{4}\ln|\sec 2x|-\frac{x^2}{2} + C$; 1.54. $\frac{x^2+1}{2}\arctan^2 x - x\arctan x + \frac{1}{2}\ln(x^2+1) + C$; 1.55. $-(\frac{1}{2}+\ln x)\frac{1}{2x^2} + C$;
- 1.56. $2\sqrt{\theta}-2\sqrt{1-\theta}\arcsin\sqrt{\theta} + C$; 1.57. $e^{-x}(\frac{1}{10}\cos 2x-\frac{1}{5}\sin 2x-\frac{1}{2}) + C$;
- 1.58. $\frac{1}{2}\sin x+\frac{3}{35}\cos x\sin 6x-\frac{1}{70}\sin x\cos 6x + C$; 1.59. $-x\cot x+\ln|\sin x| + C$; 1.60. $\frac{1}{2}\arctan x-\frac{x}{2}+\frac{x^2}{2}\arctan x + C$;
- 1.61. $\frac{1}{2}x+\frac{1}{10}x\cos(2\ln x)+\frac{1}{5}x\sin(2\ln x) + C$; 1.62. $(\ln(\sin t)-1)\sin t + C$; 1.63. $x(\ln^2 x-2\ln x+2) + C$;
- 1.64. $2\sin\sqrt{x}-2\sqrt{x}\cos\sqrt{x} + C$; 1.65. $\frac{2}{9}\cos 3x-\frac{2}{27}\sin 3x+\frac{1}{3}x^2\sin 3x + C$; 1.66. $\frac{1}{3}\sin x-\frac{1}{9}\sin^3 x-\frac{x}{3}\cos^3 x + C$;
- 1.67. $\frac{1}{4}\tan\theta\sec^3\theta+\frac{3}{8}\sec\theta\tan\theta+\frac{3}{8}\ln|\sec\theta+\tan\theta| + C$; 1.68. $\frac{1}{4}\sec(x^2)\tan(x^2)+\frac{1}{4}\ln|\sec(x^2)+\tan(x^2)| + C$;
- 1.69. $\frac{e^x}{x+1} + C$; 1.70. $x\arcsin^2 x-2x+2\sqrt{1-x^2}\arcsin x + C$; 1.71. $(4\ln x-1)\frac{x^4}{16} + C$;
- 1.72. $\frac{1}{16}\sin 4t-\frac{1}{4}t\cos 4t + C$; 1.73. $\frac{1}{4}\cos 2x+\frac{1}{2}x\sin 2x-\frac{1}{2}x^2\cos 2x + C$;
- 1.74. $\frac{1}{4a}\tan(ax+b)\sec^3(ax+b)+\frac{3}{8a}\sec(ax+b)\tan(ax+b)+\frac{3}{8a}\ln|\sec(ax+b)+\tan(ax+b)| + C$;
- 1.75. $5^x\left(\frac{x}{\ln 5}-\frac{1}{\ln^2 5}\right) + C$; 1.76. $\theta\tan\theta-\ln|\sec\theta| + C$; 1.77. ;
- 1.78. $\frac{1}{2a}\sec(ax+b)\tan(ax+b)+\frac{1}{2a}\ln|\sec(ax+b)+\tan(ax+b)| + C$; 1.79. $\frac{1}{4}\cos 2z+\frac{1}{2}z\sin 2z + C$;
- 1.80. $\frac{1}{4}x^2-\frac{1}{8}\cos 2x-\frac{1}{4}x\sin 2x + C$; 1.81. $\frac{e^{-\theta}}{10}(3\sin 3\theta-\cos 3\theta) + C$; 1.82. $a^x\left(\frac{x}{\ln a}-\frac{1}{\ln^2 a}\right) + C$;
- 1.83. $-2\sqrt{1-x}(\ln x)-4\tanh^{-1}\sqrt{1-x}+4\sqrt{1-x} + C$; 1.84. $z\arccos z-\sqrt{1-z^2} + C$;
- 1.85. $-\frac{1}{3}\sin 2t\cos 4t+\frac{1}{6}\cos 2t\sin 4t + C$; 1.86. $7\cos^2 t\left(\frac{1}{2}-\ln|\cos t|\right) + C$; 1.87. $(1-x)\sqrt{1-e^{2x}}+\frac{1}{2}\ln\left|\frac{\sqrt{1-e^{2x}}-1}{\sqrt{1-e^{2x}}+1}\right| + C$;
- 1.88. $\frac{1}{32}t-\frac{1}{24}t^3-\frac{1}{64}\arctan 2t+\frac{1}{4}t^4\arctan 2t + C$; 1.89. $-\sqrt{1-x^2}\arcsin x+x + C$; 1.90. $\frac{\arcsin x}{\sqrt{1-x^2}}-\tanh^{-1}x + C$;
- 1.91. $(1-\ln x)\sqrt{1-x^2}+\ln\left|\frac{1-\sqrt{1-x^2}}{x}\right| + C$; 1.92. $2\sin^2 x(2\ln|\sin x|-1)+5\cos^2 x\left(\frac{1}{2}-\ln|\cos x|\right) + C$;
- 1.93. $\frac{1}{2}\cos^2 x\left(\frac{1}{2}-\ln|\cos x|\right)+\frac{1}{3}\sin^2 x\left(\frac{1}{2}-\ln|\sin x|\right) + C$; 1.94. $\frac{1}{a}\sin^2(ax)\ln|\tan(ax)|+\frac{1}{a}\ln|\cos(ax)| + C$;
- 1.95. $5\sin^2 x(\ln|\sin x|-\frac{1}{2}) + C$; 1.96. $-2\ln\left|\cos\left(\frac{x}{2}\right)\right|+2\sin^2\left(\frac{x}{2}\right)\ln\left|\cot\left(\frac{x}{2}\right)\right| + C$;
- 1.97. $\frac{1}{2}(\sin x+\cos x)^2\left(\ln(\sin x+\cos x)-\frac{1}{2}\right) + C$; 1.98. $\sin x\left(\ln\left|\frac{\cos^3 x}{\sin^2 x}\right|-1\right)-3\ln|\sec x-\tan x| + C$;
- 1.99. $4\ln|\csc x-\cot x|+9\cos x-5\cos x\ln(\sin^4 x\cos^5 x) + C$; 1.100. $\frac{3}{4}\arcsin^5 x(\ln|\arcsin x-\frac{1}{5}|) + C$;
- 1.101. $-\frac{m+n}{b}\sin bx+\frac{1}{b}\ln(\sin^n bx\cos^m bx)\sin bx-\frac{m}{b}\ln\left|\frac{1-\sin bx}{\cos bx}\right| + C$;
- 1.102. $\frac{m+n}{b}\cos bx-\frac{1}{b}\ln(\sin^n bx\cos^m bx)\cos bx+\frac{n}{b}\ln\left|\frac{1-\cos bx}{\sin bx}\right| + C$; 1.103. $\frac{1}{8}\sinh(2x^2)-\frac{x^2}{4} + C$; 1.104. $\frac{\sinh 2}{2}+1$;
- 1.105. $2\sqrt{x}\cosh\sqrt{x}-2\sinh\sqrt{x} + C$; 1.106. $\frac{3^t}{9-\ln^2 3}(3\cosh 3t-(\ln 3)\sinh 3t) + C$; 1.107. $\frac{e^{at}}{b^2-a^2}(b\sinh bt-a\cosh bt) + C$;
- 1.108. $\frac{x^3}{3}\sinh(x^3)-\frac{1}{3}\cosh(x^3) + C$; 1.109. $-\frac{1}{2}\csc x\cot x+\frac{1}{2}\ln|\csc x-\cot x| + C$; 1.110. $-\frac{1}{3}\sqrt{1-x^2}(2+x^2) + C$;
- 1.111. $-\frac{1}{15}\sqrt{1-x^2}(4x^2+3x^4+8) + C$; 1.112. $-\frac{1}{35}\sqrt{1-x^2}(8x^2+6x^4+5x^6+16) + C$;