ECUACIONES TRIGONOMÉTRICAS

1. Resolver las siguientes ecuaciones:

a) $\text{sen } x = \text{tg } x$
b) $6\cos^2 \frac{x}{2} + \cos x + 1 = 0$
c) $2\cdot\text{sen } x + 2\cdot\cos x = \sqrt{2}$
d) $\cos 2x = \text{sen } x$
e) $\text{sen } x - \cos x = 0$
f) $\cos 2x = 2\cdot\text{sen} 2x$
g) $2\cdot\text{sen}^2 x + 3\cdot\cos x = 0$
h) $\text{sen } x - \sqrt{3}\cdot\cos x = 0$
i) $\cos 2x + 1 = \cos x$
j) $\cos 2x + \text{sen } x = 4\cdot\text{sen} 2x$
k) $2 \cos^2 x + 3\cdot\cos x = 2$
l) $\cos^2 x - 3 \text{ sen}^2 x = 0$
m) $\text{sen}^2 x - 3\cdot\text{sen } x\cdot\cos x + 2\cdot\cos^2 x = 0$
n) $\text{tg } 2x = -\text{tag } x$
o) $\text{sen}^2 x - \cos^2 x = \frac{1}{2}$
p) $\text{tg} x \cdot \sec x = \sqrt{2}$
q)
r) $2\cdot\text{tg } x - 3\cdot\text{ctg } x - 1 = 0$
s) $3\cdot\cos x = 2\cdot\sec x - 5$
t) $\cos 2x = 5 - 6\cdot\cos^2 x$
u) $\text{cosec } \alpha\cdot\sec \alpha\cdot\cos^2 \alpha + \text{tg } \alpha = \text{ctg } \alpha$
v) $2\cdot\text{sen } x + 1 = \text{cosec } x$
w) $\sec x + \text{tg } x = 2$
x) $\text{sen} x + \cos ec x = \frac{13}{6}$
y) $\cos 2x = \text{sen } x$
z) $\text{tg}^2 x + 3 = 4\cdot\text{tg}$
aa) $\cos x - 2\cdot\text{sen } x\cdot\cos x = 0$
bb)
cc) $2\cdot\cos^2 x + 4\cdot\text{sen}^2 x = 3$
dd) $\sec x + 4\cdot\cos x = 5$

ECUACIONES TRIGONOMÉTRICAS

1) Resolver las siguientes ecuaciones:

a) $senx = tgx$

Solución.

$$\text{sen}x = \frac{\text{sen}x}{\cos x}$$

$$\text{sen } x \cdot \cos x = \text{sen } x \ : \quad \text{sen } x \cdot \cos x - \text{sen } x = 0 : \quad \text{sen } x \cdot (\cos x - 1) = 0 : \begin{cases} \text{sen } x = 0 \\ \cos x - 1 = 0 \end{cases}$$

Casos:

i) Si $\text{sen}x = 0 \Rightarrow x = 0; x = \pi = 180^\circ$ son soluciones de la ecuación.

ii) Si $\text{sen}x \neq 0 \Rightarrow \cos x = 1 \Rightarrow x = 0$ es la única solución de la ecuación, que implicaría además $senx = 0$, luego no puede ser.

b) $6\cos^2\left(\frac{x}{2}\right) + \cos x + 1 = 0$

Solución.

$$6\cos^2\left(\frac{x}{2}\right) + \cos x + 1 = 6\frac{\cos x + 1}{2} + \cos x + 1 = 0$$

$$3\cos x + 3 + \cos x + 1 = 0$$

$$4\cos x + 4 = 0$$

$$\cos x = -1 \Rightarrow x = \pi + 2\pi k = 180^\circ + 360^\circ k$$

c) $2senx + 2\cos x = \sqrt{2}$

Solución.

$$2\text{sen}x + 2\cos x = \sqrt{2}$$

$$\text{sen}x + \cos x = \frac{\sqrt{2}}{2}$$

elevando al cuadrado y desarrollando

$$(\text{sen}x + \cos x)^2 = \frac{1}{2} \quad : \quad \text{sen}^2 x + \cos^2 x + 2\text{sen}x\cos x = \frac{1}{2}$$

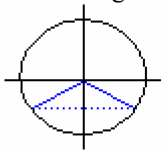
teniendo en cuenta que $\text{sen}^2x + \cos^2x = 1$, y despejando

$$2\text{sen}x\cos x = -\frac{1}{2}$$

mediante la definición de ángulo doble

$$\text{sen}(2x) = -\frac{1}{2}$$

teniendo en cuenta los ángulos asociados



$$\begin{cases} 2x = \frac{7\pi}{6} + 2\pi k = 210^\circ + 360 k \Rightarrow x = \frac{7\pi}{12} + \pi k = 105^\circ + 180^\circ k \\ 2x = \frac{11\pi}{6} + 2\pi k = 330^\circ + 360 k \Rightarrow x = \frac{11\pi}{12} + \pi k = 165^\circ + 180^\circ k \end{cases}$$

d) $\cos(2x) = senx$

Solución.

$$\cos(2x) = \text{senx}$$

por la definición de coseno del ángulo doble

$$\cos^2 x - \text{sen}^2 x = \text{senx}$$

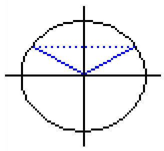
teniendo en cuenta la ecuación fundamental se despeja el $\cos^2 x$ en función del $\text{sen}^2 x$

$$\left(1 - \text{sen}^2 x\right) - \text{sen}^2 x = \text{senx}$$

ordenando se obtiene una ecuación de segundo grado en función del sen x

$$2\text{sen}^2 x + \text{senx} - 1 = 0 \Rightarrow \text{senx} = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4} : \begin{cases} \text{senx} = \dfrac{1}{2} \\ \text{senx} = -1 \end{cases}$$

Teniendo en cuenta los ángulos asociados



$$\begin{cases} senx = \dfrac{1}{2} \Rightarrow \begin{cases} x = \dfrac{\pi}{6} + 2\pi k = 30° + 360k \\ x = \dfrac{5\pi}{6} + 2\pi k = 150° + 360k \end{cases} \\ senx = -1 \Rightarrow x = \dfrac{3\pi}{2} + 2\pi k = 270° + 360k \end{cases}$$

e) $senx - \cos x = 0$

Solución.

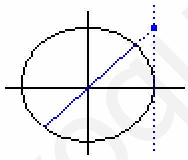
$$\text{senx} - \cos x = 0$$

$$\text{senx} = \cos x$$

dividiendo ambos miembros de la igualdad por cos x

$$\text{tgx} = 1$$

Teniendo en cuenta los ángulos asociados



$$tgx = 1 \Rightarrow \begin{cases} x = \dfrac{\pi}{4} + 2\pi k = 45° + 360k \\ x = \dfrac{5\pi}{4} + 2\pi k = 225° + 360k \end{cases}$$

f) $\cos(2x) = 2sen(2x)$

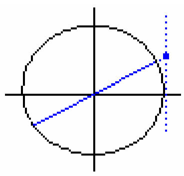
Solución.

$$\cos(2x) = 2\text{sen}(2x)$$

dividiendo toda la ecuación por 2cos(2x)

$$\text{tg}(2x) = \frac{1}{2}$$

$$2x = \arctan\left(\frac{1}{2}\right)$$



$$\begin{cases} 2x \cong 0{,}148\pi + 2\pi k = 26°33'54'' + 360k \Rightarrow x \cong 0{,}074\pi + 2\pi k = 13°16'57'' + 360k \\ 2x \cong 1{,}148\pi + 2\pi k = 206°33'54'' + 360k \Rightarrow x \cong 1{,}074\pi + 2\pi k = 103°16'57'' + 360k \end{cases}$$

g) $2sen^2x + 3\cos x = 0$

Solución.

Se transforma en una ecuación de segundo grado en función del cos x

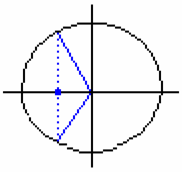
$$2\text{sen}^2 x + 3\cos x = 0$$

$$2\left(1-\cos^2 x\right) + 3\cos x = 0$$

$$2\cos^2 x - 3\cos x - 2 = 0 : \cos x = \frac{3 \pm \sqrt{9+16}}{4} = \frac{3 \pm 5}{4} : \begin{cases} \cos x = 2 \\ \cos x = \dfrac{-1}{2} \end{cases}$$

$\cos x = 2$ no tiene sentido ya que el coseno está acotado en [−1, 1]

$$\cos x = \frac{-1}{2} \Rightarrow$$



$$\begin{cases} x = \dfrac{2\pi}{3} + 2\pi k = 120^\circ + 360 k \\ x = \dfrac{4\pi}{3} + 2\pi k = 240^\circ + 360 k \end{cases}$$

h) $senx - \sqrt{3}\,cos\,x = 0$

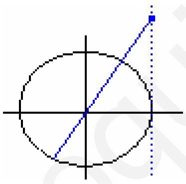
Solución.

Se transforma a tg x

$$\text{senx} - \sqrt{3}\cos x = 0$$

$$\text{senx} = \sqrt{3}\cos x \xrightarrow{\div \cos x} \frac{\text{senx}}{\cos x} = \frac{\sqrt{3}}{\cos x}$$

$$\text{tgx} = \sqrt{3}$$



$$tgx = \sqrt{3} \Rightarrow \begin{cases} x = \dfrac{\pi}{3} + 2\pi k = 60^\circ + 360 k \\ x = \dfrac{4\pi}{3} + 2\pi k = 240^\circ + 360 k \end{cases}$$

i) $\cos(2x) + 1 = \cos x$

Solución.

Se transforma en una ecuación de segundo grado en función del cos x

$$\cos(2x) + 1 = \cos x$$

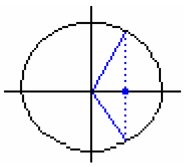
$$\cos^2 x - \text{sen}^2 x + 1 = \cos x$$

$$\cos^2 x - \left(1 - \cos^2 x\right) + 1 = \cos x$$

$$2\cos^2 x = \cos x ; \quad 2\cos^2 x - \cos x = 0$$

$$\cos x \cdot (2\cos x - 1) = 0 : \begin{cases} \cos x = 0 \Rightarrow \begin{cases} x = 0 = 360^\circ k = 2\pi k \\ x = \dfrac{3\pi}{2} + 2\pi k = 270^\circ + 360^\circ k \end{cases} \\ 2\cos x - 1 = 0 \Rightarrow \cos x = \dfrac{1}{2} \end{cases}$$

$$\cos x = \frac{1}{2} \Rightarrow$$



$$\begin{cases} x = \dfrac{\pi}{3} + 2\pi k = 60^\circ + 360 k \\ x = \dfrac{5\pi}{3} + 2\pi k = 300^\circ + 360 k \end{cases}$$

j) $cos(2x)+senx=sen(3x)$

Solución.

Aplicando transformaciones de sumas en producto se obtiene una ecuación equivalente.

$$\cos(2x)+\text{senx}=\text{sen}(3x)$$

$$\cos(2x)=\text{sen}(3x)-\text{senx}$$

$$\cos(2x)=2\cos\left(\frac{3x+x}{2}\right)\cdot\text{sen}\,\frac{3x-x}{2}$$

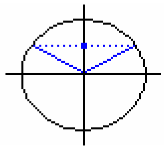
$$\cos(2x)=2\cos 2x\cdot\text{sen x}$$

$$\cos(2x)-2\cos 2x\cdot\text{sen x}=0$$

$$\cos(2x)\cdot(1-2\text{sen x})=0:\begin{cases}\cos(2x)=0\\ 1-2\text{sen x}=0\end{cases}$$

$$\cos x=0\Rightarrow x=\frac{\pi}{2}+\pi k=90°+180°\,k$$

$$senx=\frac{1}{2}$$



$$\Rightarrow\begin{cases}x=\frac{\pi}{6}+2\pi k=30°+360k\\ x=\frac{5\pi}{6}+2\pi k=150°+360k\end{cases}$$

k) $2\cos^2 x+3\cos x=2$

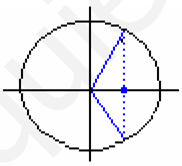
Solución.

$$2\cos^2 x+3\cos x=2$$

$$2\cos^2 x+3\cos x-2=0$$

$$\cos x=\frac{-3\pm\sqrt{9+16}}{4}=\frac{-3\pm5}{4}:\begin{cases}\cos x=-2\notin[-1,1]\ \text{no tiene sentido}\\ \cos x=\frac{1}{2}\end{cases}$$

$$\cos x=\frac{1}{2}\Rightarrow$$



$$\begin{cases}x=\frac{\pi}{3}+2\pi k=60°+360k\\ x=\frac{5\pi}{3}+2\pi k=300°+360k\end{cases}$$

l) $\cos^2 x-3sen^2x=0$

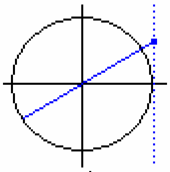
Solución.

$$\cos^2 x-3\text{sen}^2x=0$$

$$\cos^2 x=3\text{sen}^2x\xrightarrow{\div3\cos^2 x}\frac{3\text{sen}^2x}{3\cos^2 x}=\frac{\cos^2 x}{3\cos^2 x}$$

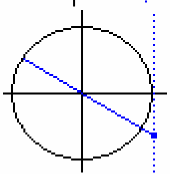
$$\text{tg}^2x=\frac{1}{3}\Rightarrow\text{tgx}=\pm\frac{\sqrt{3}}{3}$$

$$\text{tgx}=\frac{\sqrt{3}}{3}$$



$$\Rightarrow\begin{cases}x=\frac{\pi}{6}+2\pi k=30°+360°k\\ x=\frac{7\pi}{6}+2\pi k=210°+360°k\end{cases}$$

$$\text{tgx}=-\frac{\sqrt{3}}{3}$$



$$\Rightarrow\begin{cases}x=\frac{5\pi}{6}+2\pi k=150°+360°k\\ x=\frac{11\pi}{6}+2\pi k=330°+360°k\end{cases}$$

m) $sen^2x-3senx\cos x+2\cos^2 x=0$

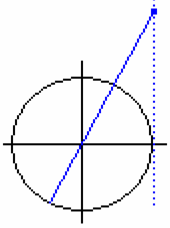
Solución.

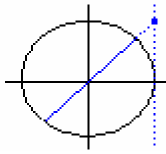
Se transforma en una ecuación de segundo grado dividiendo todos los términos de la ecuación por $\cos^2 x$.

$$\operatorname{sen}^2 x - 3\operatorname{sen}x\cos x + 2\cos^2 x = 0$$

$$\frac{\operatorname{sen}^2 x}{\cos^2 x} - \frac{3\operatorname{sen}x\cos x}{\cos^2 x} + \frac{2\cos^2 x}{\cos^2 x} = 0$$

$$\operatorname{tg}^2 x - 3\operatorname{tg} x + 2 = 0 \ : \quad \operatorname{tg} x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4\cdot 1\cdot 2}}{2\cdot 1} = \frac{3\pm 1}{2}$$

$\operatorname{tg}x = 2 \Rightarrow$  $\Rightarrow \begin{cases} x \cong \frac{7\pi}{20} + 2\pi k = 63^\circ 26' 58'' + 360^\circ k \\ x = \frac{7\pi}{6} + 2\pi k = 210^\circ + 360^\circ k \end{cases}$

$\operatorname{tg}x = 2 \Rightarrow$  $\Rightarrow \begin{cases} x = \frac{\pi}{4} + 2\pi k = 45^\circ + 360\, k \\ x = \frac{5\pi}{4} + 2\pi k = 225^\circ + 360\, k \end{cases}$

n) $tg(2x) = -tgx$

Solución.

Se transforma la igualdad en función de sen x y cos x.

$$\operatorname{tg}(2x) = -\operatorname{tg}x$$

$$\frac{\operatorname{sen}(2x)}{\cos(2x)} = -\frac{\operatorname{sen} x}{\cos x}$$

$$\frac{2\operatorname{sen}x\cos x}{\cos^2 x - \operatorname{sen}^2 x} = -\frac{\operatorname{sen}x}{\cos x}$$

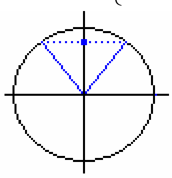
$$2\operatorname{sen} x\cdot\cos^2 x = -\operatorname{sen} x\cdot\cos^2 x + \operatorname{sen}^3 x$$

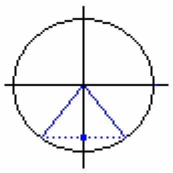
$$\operatorname{sen}^3 x - 3\operatorname{sen} x\cdot\cos^2 x = 0$$

$$\operatorname{sen}^3 x - 3\operatorname{sen} x\cdot\left(1 - \operatorname{sen}^2 x\right) = 0$$

$$4\operatorname{sen}^3 x - 3\operatorname{sen} x = 0$$

$$\operatorname{sen} x\cdot\left(4\operatorname{sen}^2 x - 3\right) = 0 : \begin{cases} \operatorname{sen} x = 0 : x = 0 = \pi k = 180^\circ\, k \\ 4\operatorname{sen}^2 x - 3 = 0 : \operatorname{sen} x = \pm\sqrt{\frac{3}{4}} = \frac{\pm\sqrt{3}}{2} \end{cases}$$

$\operatorname{sen} x = \frac{\sqrt{3}}{2}$  $\Rightarrow \begin{cases} x = \frac{\pi}{3} + 2\pi k = 60^\circ + 360^\circ k \\ x = \frac{2\pi}{3} + 2\pi k = 120^\circ + 360^\circ k \end{cases}$

$\operatorname{sen} x = -\frac{\sqrt{3}}{2}$  $\Rightarrow \begin{cases} x = \frac{4\pi}{3} + 2\pi k = 240^\circ + 360^\circ k \\ x = \frac{5\pi}{3} + 2\pi k = 360^\circ + 300^\circ k \end{cases}$

o) $sen^2 x - \cos^2 x = \frac{1}{2}$

Solución.

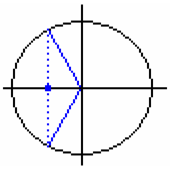
Teniendo en cuenta la definición de coseno del ángulo doble, se transforma la expresión

$$\text{sen}^2 x - \cos^2 x = \frac{1}{2}$$

$$-\cos(2x) = \frac{1}{2}$$

$$\cos(2x) = -\frac{1}{2}$$

$$\cos(2x) = -\frac{1}{2}$$



$$\begin{cases} 2x = \frac{2\pi}{3} + 2\pi k = 120^\circ + 360^\circ k \Longrightarrow x = \frac{\pi}{3} + \pi k = 60^\circ + 180^\circ k \\ 2x = \frac{4\pi}{3} + 2\pi k = 240^\circ + 360^\circ k \Longrightarrow x = \frac{2\pi}{3} + \pi k = 120^\circ + 180^\circ k \end{cases}$$

p) $tgx \cdot \sec x = \sqrt{2}$

Solución.

$$\text{tgx} \sec x = \sqrt{2}$$

$$\frac{\text{senx}}{\cos x} \cdot \frac{1}{\cos x} = \sqrt{2}$$

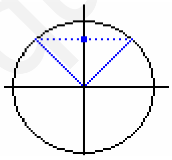
$$\frac{\text{senx}}{1 - \text{sen}^2 x} = \sqrt{2}$$

$$\sqrt{2}\text{sen}^2 x + \text{senx} - \sqrt{2} = 0$$

Ecuación de segundo grado en función de sen x

$$\text{senx} = \frac{-1 \pm \sqrt{1+8}}{2\sqrt{2}} = \frac{-1 \pm 3}{2\sqrt{2}} : \begin{cases} \text{senx} = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \text{senx} = \frac{-4}{2\sqrt{2}} = \frac{-2}{\sqrt{2}} = -\sqrt{2} \notin [-1, 1] \Rightarrow \text{No válida} \end{cases}$$

$$\text{sen } x = \frac{\sqrt{2}}{2}$$



$$\Rightarrow \begin{cases} x = \frac{\pi}{4} + 2\pi k = 45^\circ + 360^\circ k \\ x = \frac{3\pi}{4} + 2\pi k = 135^\circ + 360^\circ k \end{cases}$$

r) $2tgx - 3\cot gx - 1 = 0$

Solución.

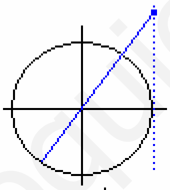
$$2\text{tg } x - 3\text{cotg } x - 1 = 0$$

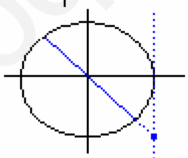
$$2\text{tg } x - 3\frac{1}{\text{tg } x} - 1 = 0$$

Multiplicando la igualdad por tg x, se transforma en una ecuación de segundo grado en función de tg x

$$2\text{tg}^2 x - \text{tg } x - 3 = 0$$

$$\text{tg } x = \frac{1 \pm \sqrt{1+24}}{4} : \begin{cases} \text{tg } x = \dfrac{3}{2} \\ \text{tg } x = -1 \end{cases}$$

$\text{tg } x = \frac{3}{2} \Rightarrow$  $\Rightarrow x \cong \frac{31\pi}{100} + \pi k = 56^\circ\, 18' 35'' + 180^\circ k$

$\text{tg } x = -1$  $\Rightarrow x = \frac{3\pi}{4} + \pi k = 135^\circ + 180^\circ k$

s) $3\cos x = 2\sec x - 5$

Solución.

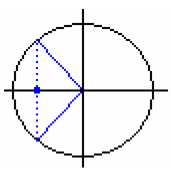
$$3\cos x = 2\sec x - 5$$

$$3\cos x = \frac{2}{\cos x} - 5$$

Multiplicando toda la ecuación por cos x se obtiene una ecuación de segundo grado

$$3\cos^2 x + 5\cos x - 2 = 0$$

$$\cos x = \frac{-5 \pm \sqrt{25+24}}{6} : \begin{cases} \cos x = \dfrac{1}{3} \\ \cos x = -2 \notin [-1, 1] \Rightarrow \text{No válida} \end{cases}$$

$\cos x = -\frac{1}{3}$  $\begin{cases} x \cong \frac{39\pi}{100} + 2\pi k = 70^\circ 31' 44'' + 360^\circ k \\ x \cong \frac{161\pi}{100} + 2\pi k = 289^\circ 28' 16'' + 360^\circ k \end{cases}$

t) $\cos(2x) = 5 - 6\cos^2 x$

Solución.

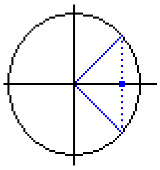
$$\cos(2x) = 5 - 6\cos^2 x$$

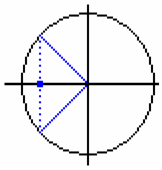
$$\cos^2 x - \text{sen}^2 x = 5 - 6\cos^2 x$$

$$\cos^2 x - \left(1 - \cos^2 x\right) = 5 - 6\cos^2 x$$

$$8\cos^2 x = 4$$

$$\cos^2 x = \frac{1}{2} \Rightarrow \cos x = \pm\frac{\sqrt{2}}{2}$$

$\cos x = \frac{\sqrt{2}}{2}$  $\Rightarrow \begin{cases} x = \frac{\pi}{4} + 2\pi k = 45^\circ + 360^\circ k \\ x = \frac{7\pi}{4} + 2\pi k = 315^\circ + 360^\circ k \end{cases}$

$\cos x = -\frac{\sqrt{2}}{2}$  $\Rightarrow \begin{cases} x = \frac{3\pi}{4} + 2\pi k = 135^\circ + 360^\circ k \\ x = \frac{5\pi}{4} + 2\pi k = 225^\circ + 360^\circ k \end{cases}$

u) $\cos ec\alpha \cdot \sec\alpha \cdot \cos^2\alpha + tg\alpha = \cot g\alpha$

Solución.

$$\cos ec\ x \cdot \sec x \cdot \cos^2 x + \text{tg}\ x = \text{cotg}\ x$$

$$\frac{1}{\text{sen}\ x} \cdot \frac{1}{\cos x} \cdot \cos^2 x + \text{tg}\ x = \text{cotg}\ x$$

$$\frac{\cos x}{\text{sen}\ x} + \text{tg}\ x = \text{cotg}\ x$$

$$\text{cotg}\ x + \text{tg}\ x = \text{cotg}\ x$$

$$\text{tg}\ x = 0 \Rightarrow x = 0 = 180^\circ k = \pi k$$

v) $2senx + 1 = \cos ecx$

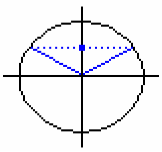
Solución.

$$2\text{sen}\ x + 1 = \text{cosec}\ x$$

$$2\text{sen}\ x + 1 = \frac{1}{\text{sen}\ x}$$

$$2\text{sen}^2 x + \text{sen}\ x - 1 = 0$$

$$\text{sen}x = \frac{-1 \pm \sqrt{1+8}}{4} : \begin{cases} \text{sen}\ x = \frac{1}{2} \\ \text{sen}\ x = -1 \Rightarrow x = \frac{3\pi}{2} + 2\pi k = 270^\circ + 360^\circ k \end{cases}$$

$senx = \frac{1}{2}$  $\Rightarrow \begin{cases} x = \frac{\pi}{6} + 2\pi k = 30^\circ + 360k \\ x = \frac{5\pi}{6} + 2\pi k = 150^\circ + 360k \end{cases}$

w) $\sec x + tgx = 2$

Solución.

$$\sec x + \operatorname{tg} x = 2$$

$$\frac{1}{\cos x} + \frac{\operatorname{sen} x}{\cos x} = 2$$

$$\frac{1+\operatorname{sen} x}{\cos x} = 2$$

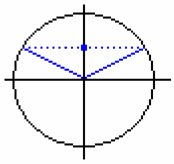
$$1 + \operatorname{sen} x = 2\cos x$$

$$1 + 2\operatorname{sen} x + \operatorname{sen}^2 x = 4\cos^2 x$$

$$1 + 2\operatorname{sen} x + \operatorname{sen}^2 x = 4\left(1 - \operatorname{sen}^2 x\right)$$

$$5\operatorname{sen}^2 x + 2\operatorname{sen} x - 3 = 0$$

$$\operatorname{sen} x = \frac{-2 \pm \sqrt{4+60}}{10} : \begin{cases} \operatorname{sen} x = \dfrac{6}{10} = \dfrac{3}{5} \\ \operatorname{sen} x = -1 \Rightarrow x = \dfrac{3\pi}{4} + 2\pi k = 270^{\circ} + 360^{\circ} k \end{cases}$$

$\operatorname{sen} x = \frac{3}{5}$  $\Rightarrow \begin{cases} x \cong \frac{\pi}{5} + 2\pi k = 36^{\circ}\, 52'\, 12'' + 360^{\circ} k \\ x \cong \frac{4\pi}{5} + 2\pi k = 143^{\circ}\, 7'\, 48'' + 360^{\circ} k \end{cases}$

x) $senx + \cos ecx = \dfrac{13}{6}$

Solución.

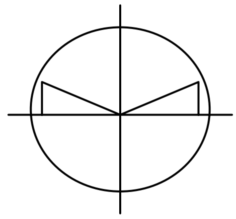
$$senx + \cos ecx = \frac{13}{6}$$

$$senx + \frac{1}{senx} = \frac{13}{6}$$

$$sen^2 x - \frac{13}{6} senx + 1 = 0$$

$$senx = \frac{\frac{13}{6} \pm \sqrt{\frac{169}{36} - 4}}{2} = \frac{\frac{13}{6} \pm \sqrt{\frac{25}{36}}}{2} = \frac{\frac{13}{6} \pm \frac{5}{6}}{2} = \frac{13 \pm 5}{12} .Casos:$$

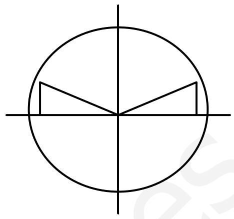
$$\begin{cases} senx = \dfrac{18}{12} \Rightarrow no \quad tiene \quad solución \\ senx = \dfrac{2}{3} \Rightarrow x \cong \dfrac{23\pi}{100} = 41^{\circ}48'37'' ; x \cong \dfrac{77\pi}{100} = 138^{\circ}11'23'' \end{cases}$$



y) $\cos(2x) = senx$

Solución.

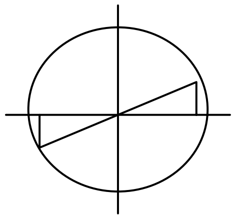
$$\cos(2x) = senx$$
$$\cos^2 x - sen^2x = senx$$
$$(1 - sen^2x) - sen^2x = senx$$
$$2sen^2x + senx - 1 = 0$$
$$senx = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4}.Casos:$$
$$\begin{cases} senx = \frac{1}{2} \Rightarrow x = \frac{\pi}{6} = 30^{\text{o}}\,; x = \frac{5\pi}{6} = 150^{\text{o}} \\ senx = -1 \Rightarrow x = \frac{3\pi}{2} = 270^{\text{o}} \end{cases}$$



z) $tg^2x + 3 = 4tgx$

Solución.

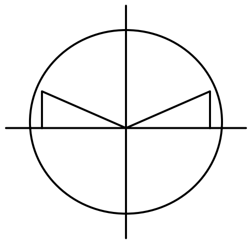
$$tg^2x + 3 = 4tgx$$
$$tg^2x - 4tgx + 3 = 0$$
$$tgx = \frac{4 \pm \sqrt{16-12}}{2} = \frac{4 \pm 2}{2}.Casos:$$
$$\begin{cases} tgx = 3 \Rightarrow x \cong \frac{2\pi}{5} = 71^{\text{o}}33'54'';x \cong \frac{7\pi}{5} = 251^{\text{o}}33'54'' \\ tgx = 1 \Rightarrow x = \frac{\pi}{4} = 45^{\text{o}}\,; x = \frac{5\pi}{4}\,225^{\text{o}} \end{cases}$$



aa) $\cos x - 2senx\cdot\cos x = 0$

Solución.

$$\cos x - 2senx\cdot\cos x = 0$$
$$\cos x(1 - 2senx) = 0.Casos:$$
$$\begin{cases} \cos x = 0 \Rightarrow x = \frac{\pi}{2} = 90^{\text{o}};x = \frac{3\pi}{2} = 270^{\text{o}} \\ 1 - 2senx = 0 \Rightarrow senx = \frac{1}{2} \Rightarrow x = \frac{\pi}{6} = 30^{\text{o}};x = \frac{5\pi}{6} = 150^{\text{o}} \end{cases}$$



cc) $2\cos^2 x + 4sen^2 x = 3$

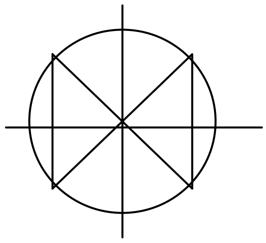
SOLUCIÓN:

$$2\cos^2 x + 4sen^2 x = 3$$

$$2\left(1 - sen^2 x\right) + 4sen^2 x = 3$$

$$2sen^2 x = 1$$

$$sen^2 x = \frac{1}{2}$$



$$senx = \pm\frac{\sqrt{2}}{2} \Rightarrow x = \frac{\pi}{4} = 45^{\circ}; x = \frac{3\pi}{4} = 135^{\circ}; x = \frac{5\pi}{4} = 225^{\circ}; x = \frac{7\pi}{4} = 315^{\circ}$$

dd) $\sec x + 4\cos x = 5$

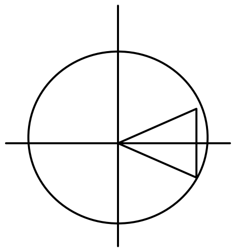
Solución.

$$\sec x + 4\cos x = 5$$

$$\frac{1}{\cos x} + 4\cos x = 5$$

$$4\cos^2 x - 5\cos x + 1 = 0$$

$$\cos x = \frac{5 \pm \sqrt{25 - 16}}{8} = \frac{5 \pm 3}{8}.Casos:$$



$$\begin{cases} \cos x = 1 \Rightarrow x = 0 \\ \cos x = \dfrac{1}{4} \Rightarrow x \cong \dfrac{21\pi}{50} = 75^{\circ}31'21''; x \cong \dfrac{-21\pi}{50} = -75^{\circ}31'21'' \end{cases}$$