Integrales propuestas

1) $\int_2^3 \frac{3dx}{\sqrt{x+2}}$

2) $\int_0^{\frac{\pi}{2}} (\sin 2x + 2\sin x)dx$

3) $\int_0^1 \frac{3dx}{(x+1)^3}$

4) $\int_2^4 x \ln x \, dx$

5) $\int_0^{\frac{\pi}{3}} x^2 \sin 2x \, dx$

6) $\int_0^2 \frac{e^x}{e^x+1} dx$

7) $\int_1^3 \frac{\sqrt{x-1}}{x} dx$

8) $\int_0^{\frac{\pi}{2}} \sin x \cdot \cos^3 x \, dx$

9) $\int_1^4 \frac{dx}{2x-1}$

10) $\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$

11) $\int_0^1 \frac{1}{1+x^2} dx$

Problemas propuestos:

1) Calcular el área del trapecio limitado por la recta $y = 2x$, el eje OX y las rectas x=2 y x=4.
2) Hallar el área limitada por la recta x+y=10, el eje OX y las ordenadas de x=2 y x=8.
3) Calcular el área limitada por la curva $y=9-x^2$ y el eje OX.
4) Calcular el área del recinto limitado por la curva $y=x^2-4x$ y el eje OX.
5) Calcular el área del recinto limitado por la curva $y=4x-x^2$ y el eje OX.
6) Área limitada por la curva $y=x^3-6x^2+8x$ y el eje OX.
7) Área limitada por la parábola $y^2=4x$ y la recta y=x.
8) Calcular el área limitada por las gráficas de las funciones $y^2=4x$ e $y=x^2$.
9) Calcular utilizando el cálculo integral el área del triángulo de vértices S(2,0), A(8,0), y C(6,4).
10) Calcular el área limitada por la curva $x \cdot y=36$, el eje OX y las rectas x=6 y x=12.
11) Hallar el área del recinto determinado por la parábola $y^2=8x$, el eje de ordenadas, y la tangente a la parábola paralela a la recta: x-y+25=0.
12) Hallar el área del recinto limitado por la parábola de ecuación $y^2=4x$, el eje de ordenadas, y la tangente a la parábola paralela a la recta x-2y+8=0
13) Calcula el área comprendida entre la gráfica de la función $f(x)=x^3+4x^2+x-6$ y el eje de abcisas en el intervalo [-1,4]
14) Área comprendida entre las gráficas de las funciones $f(x)=2^x$ y $g(x)=\frac{3x+2}{2}$
15) Calcular el área del recinto limitado por las funciones $y=x^3-5x+6$ e $y=2x$

16) Dibuja el recinto limitado por la curva $f(x)=x^3-x$ y el eje de abcisas. Calcula el área

17) Área limitada por $y = \sin x$ e $y = \cos x$ entre 0 y 2π. Haz un dibujo del recinto

18) Calcula el área comprendida entre la parábola $y=x^2-2x+2$, la tangente a ésta en el punto (3,5) y los ejes de coordenadas.

19) Área comprendida entre la gráfica de la función $f(x)= \cos 3x$, el eje de abcisas y las rectas $x= -\pi/3$ y $x=\pi/2$

20) Calcular el área comprendida entre las gráficas de las funciones $y = 4x$, $y = \frac{1}{4}x$, e $y = \frac{1}{x}$.

21) Calcular el área limitada por una circunferencia centrada en el origen de radio 2, y la bisectriz del primer cuadrante.

1) $\displaystyle\int_2^3 \frac{3dx}{\sqrt{x+2}} = \left[6\cdot\sqrt{x+2}\right]_2^3 = (6\sqrt{5}-12) = 1'416$

$\displaystyle\int \frac{3}{\sqrt{x+2}}dx = 3\int (x+2)^{-1/2}dx = 3\,\frac{(x+2)^{1/2}}{1/2} = 6\cdot\sqrt{x+2}$

2) $\displaystyle\int_0^{\pi/2} (\text{sen } 2x + 2\,\text{sen} x)\,dx = \left[-\frac{\cos(2x)}{2} - 2\cos x\right]_0^{\pi/2} = \circledast$

$\displaystyle\int (\text{sen } 2x + 2\,\text{sen} x)\,dx = \frac{1}{2}\int 2\,\text{sen } 2x\,dx + 2\int \text{sen} x\,dx =$

$\displaystyle= \frac{1}{2}\cdot(-\cos 2x) - 2\cos x$

$\displaystyle\circledast = \left(-\frac{\cos\left(2\cdot\frac{\pi}{2}\right)}{2} - \cancel{2\cos\left(\frac{\pi}{2}\right)}^{\,0}\right) - \left(-\frac{\cos(0)}{2} - 2\cos 0\right) =$

$\displaystyle= \frac{1}{2} - \left(-\frac{1}{2} - 2\right) = 3$

3) $\displaystyle\int_0^1 \frac{3dx}{(x+1)^3} = \left[\frac{-3}{2(x+1)^2}\right]_0^1 = \left(\frac{-3}{8}\right) - \left(-\frac{3}{2}\right) = \frac{9}{8}$

$\displaystyle\int \frac{3dx}{(x+1)^3} = 3\int (x+1)^{-3}dx = 3\,\frac{(x+1)^{-2}}{-2} = \frac{-3}{2(x+1)^2}$

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4) $\int_2^4 x \cdot \ln x\, dx = \left[\frac{x^2}{2}\ln x - \frac{x^2}{4}\right]_2^4 = (8\ln 4 - 4) - (2\ln 2 - 1) = \circledast$

$\int \underbrace{x}_{dv}\, \underbrace{\ln x}_{u}\, dx = \frac{x^2}{2}\cdot \ln x - \int \frac{x}{2}\, dx = \frac{x^2}{2}\cdot \ln x - \frac{x^2}{4}$

$u = \ln x \quad du = \frac{1}{x}\, dx$

$dv = x\, dx \quad v = \frac{x^2}{2}$

$\circledast = 6'704$

5) $\int_0^{\pi/3} x^2 \cdot \operatorname{sen} 2x\, dx = \left[-\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\operatorname{sen}(2x) + \frac{1}{4}\cos(2x)\right]_0^{\pi/3} = ☺$

$\int \underbrace{x^2}_{u}\, \underbrace{\operatorname{sen} 2x\, dx}_{dv} = -\frac{1}{2}x^2\cos(2x) + \int \underbrace{x}_{u}\, \underbrace{\cos(2x)dx}_{dv} = \circledast$

$u = x^2 \quad du = 2x\, dx$

$dv = \operatorname{sen}(2x) \quad v = -\frac{1}{2}\cos(2x)$

$u = x \quad du = 1\, dx$

$dv = \cos(2x)dx \quad v = \frac{1}{2}\operatorname{sen}(2x)$

$\circledast = -\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\operatorname{sen}(2x) - \frac{1}{2}\int \operatorname{sen}(2x)dx =$

$\circledast = -\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\operatorname{sen}(2x) + \frac{1}{4}\cos(2x)$

$☺ = \left(-\frac{1}{2}\cdot\left(\frac{\pi}{3}\right)^2\cdot\cos\left(\frac{2\pi}{3}\right) + \frac{1}{2}\left(\frac{\pi}{3}\right)\operatorname{sen}\left(\frac{2\pi}{3}\right) + \frac{1}{4}\cos\left(\frac{2\pi}{3}\right)\right) - \left(\frac{1}{4}\cos(0)\right) =$

$= \left(-\frac{1}{2}\cdot\frac{\pi^2}{9}\left(-\frac{1}{2}\right) + \frac{1}{2}\frac{\pi}{3}\cdot\frac{\sqrt{3}}{2} + \frac{1}{4}\left(-\frac{1}{2}\right)\right) - \frac{1}{4}\cdot 1 = \frac{\pi^2}{36} + \frac{\pi\sqrt{3}}{12} - \frac{3}{8} =$

$= 0'35$

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6) $$\int_0^2 \frac{e^x}{e^x+1}\,dx = \Big[\ln(e^x+1)\Big]_0^2 = \ln(e^2+1) - \ln 2 = 1'434$$

$$\int \frac{e^x}{e^x+1}\,dx = \ln|e^x+1|$$

7) $$\int_1^3 \frac{\sqrt{x-1}}{x}\,dx = \Big[2\sqrt{x-1} - 2\,\text{arctg}(\sqrt{x-1})\Big]_1^3 = ☺$$

$$\int \frac{\sqrt{x-1}}{x}\,dx = \int \frac{\sqrt{t^2}}{t^2+1}\cdot 2t\,dt = \int \frac{2t^2}{t^2+1}\,dt = \circledast$$

$x-1 = t^2 \Rightarrow x = t^2+1$

$dx = 2t\,dt$

$$\begin{array}{l|l} 2t^2+0t+0 & t^2+1 \\ \cline{2-2} -2t^2 \quad\ -2 & 2 \\ \hline \quad\quad\ -2 & \end{array}$$

$$\circledast = \int 2\,dt - \int \frac{2}{t^2+1}\,dt = 2t - 2\,\text{arctg}(t) = 2\sqrt{x-1} - 2\,\text{arctg}(\sqrt{x-1})$$

$$☺ = \left(2\sqrt{2} - 2\,\text{arctg}(\sqrt{2})\right) - \left(\cancelto{0}{2\sqrt{0}} - \cancelto{0}{2\,\text{arctg}\,0}\right) = 0'9178$$

8) $$\int_0^{\pi/2} \text{sen}\,x\cdot\cos^3 x\,dx = \left[-\frac{\cos^4 x}{4}\right]_0^{\pi/2} = \left(\cancelto{0}{-\frac{\cos^4(\pi/2)}{4}}\right) - \left(-\frac{\cos^4(0)}{4}\right) = \frac{1}{4}$$

$$-\int -\text{sen}\,x\cdot\cos^3 x\,dx = -\frac{\cos^4 x}{4}$$

9) $$\int_1^4 \frac{dx}{2x-1} = \left[\frac{1}{2}\ln(2x-1)\right]_1^4 = \left(\frac{1}{2}\ln(7)\right) - \left(\cancel{\frac{1}{2}\ln 1}^{\,0}\right) = 0'973$$

$$\frac{1}{2}\int \frac{2\,dx}{2x-1} = \frac{1}{2}\ln(2x-1)$$

10) $$\int_0^{\pi/2} \operatorname{sen}^2 x\,dx = \left[-\frac{\operatorname{sen}x\cos x + x}{2}\right]_0^{\pi/2} = ☺$$

$$\underbrace{\int \operatorname{sen}^2 x\,dx}_{=I} = \int \underbrace{\operatorname{sen}x}_{u}\cdot\underbrace{\operatorname{sen}x\,dx}_{dv} = -\operatorname{sen}x\cos x + \int \cos^2 x\,dx = \circledast$$

$u = \operatorname{sen}x \qquad du = \cos x\,dx$ || $\cos^2 x + \operatorname{sen}^2 x = 1$

$dv = \operatorname{sen}x\,dx \qquad v = -\cos x$

$$\circledast = -\operatorname{sen}x\cos x + \int (1-\operatorname{sen}^2 x)\,dx = -\operatorname{sen}x\cos x + x - \underbrace{\int \operatorname{sen}^2 x\,dx}_{=I}$$

$$\Rightarrow 2I = -\operatorname{sen}x\cos x + x \quad \Rightarrow I = -\frac{\operatorname{sen}x\cos x + x}{2}$$

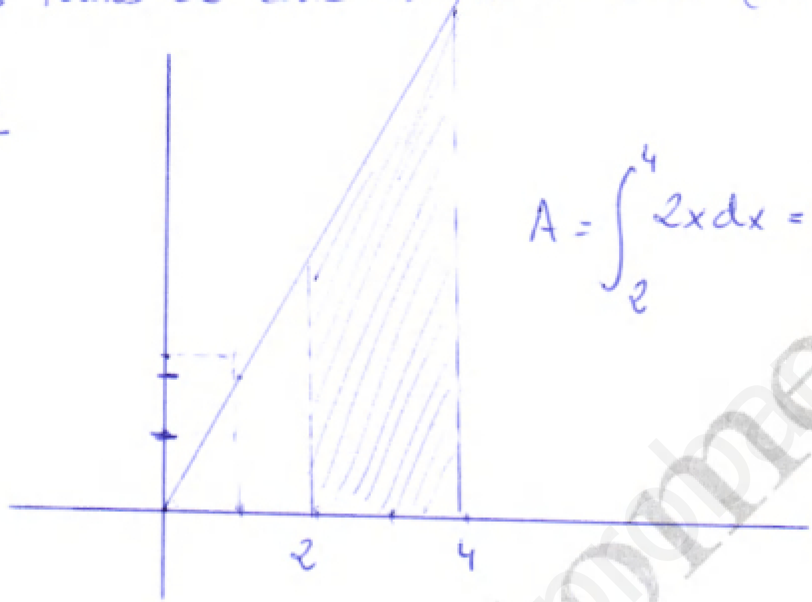
$$☺ = \left(-\frac{\cancel{\operatorname{sen}\left(\frac{\pi}{2}\right)\cdot\cos\left(\frac{\pi}{2}\right)}^{\,0} + \frac{\pi}{2}}{2}\right) - \cancel{\left(-\frac{\operatorname{sen}0\cdot\cos 0 + 0}{2}\right)}^{\,0} = \frac{\pi}{4}$$

11) $$\int_0^1 \frac{1}{1+x^2}\,dx = \left[\operatorname{arctg}(x)\right]_0^1 = \operatorname{arctg}(1) - \cancel{\operatorname{arctg}(0)}^{\,0} = 0'785$$

Problema 1:

$y = 2x \longrightarrow$ Puntos de corte $\Rightarrow 2x = 0 \quad x = 0$ (No afectará)

x	y = 2x
0	0
1	2



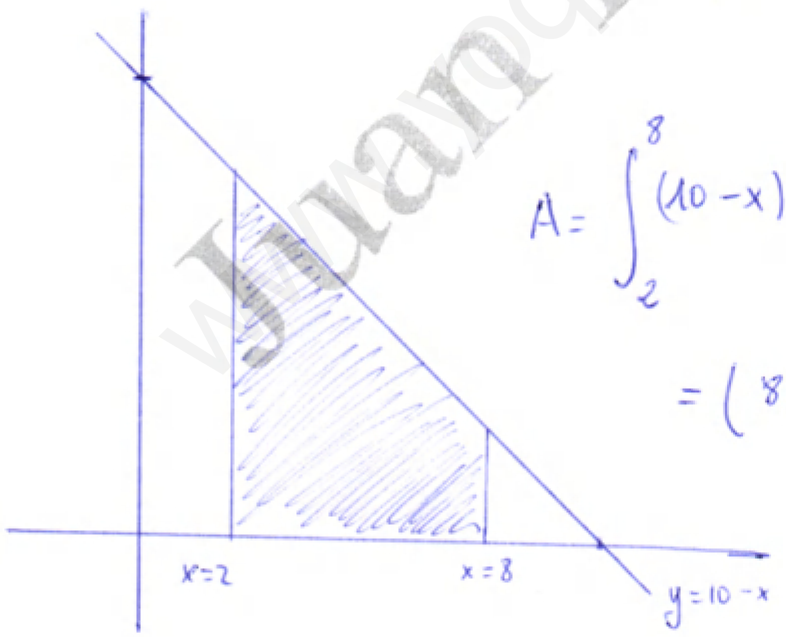
$$A = \int_2^4 2x\,dx = \left[x^2\right]_2^4 = 16 - 4 = 12\,u$$

Problema 2:

$x + y = 10 \longrightarrow y = 10 - x$

$x = 2 \;;\; x = 8$

x	y = 10 − x
0	10
10	0



$$A = \int_2^8 (10 - x)\,dx = \left[10x - \frac{x^2}{2}\right]_2^8 =$$

$$= (80 - 32) - (20 - 2) = 30\,u^2$$

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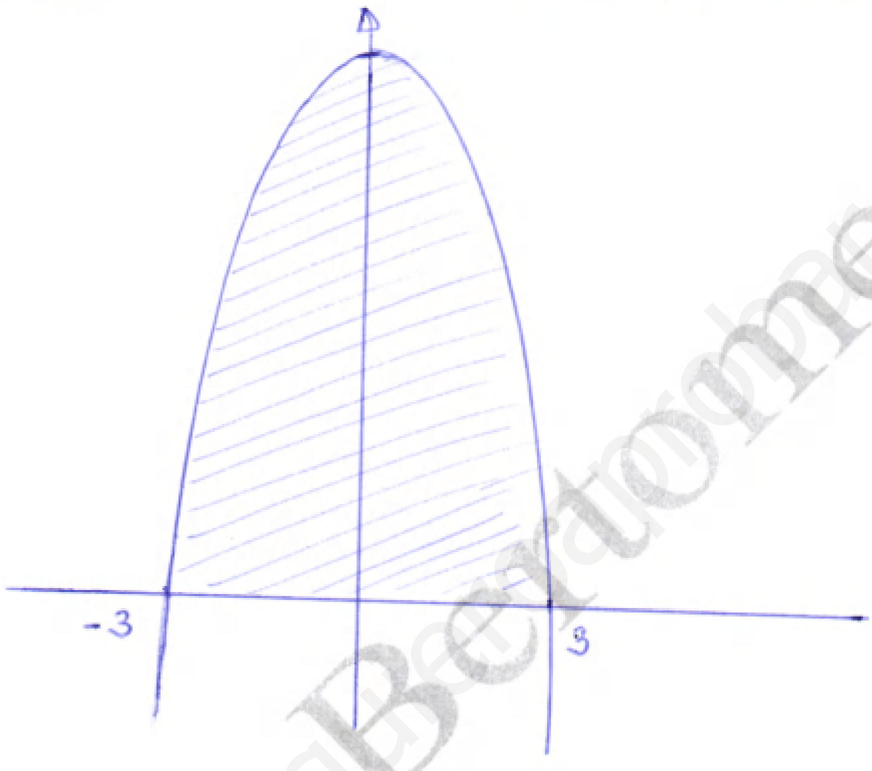
Problema 3:

$y = 9 - x^2 \longrightarrow$ Parábola Convexa $(a<0)$

$9 - x^2 = 0 \Rightarrow x = +3 \quad PC\,(3,0)$

$ \Rightarrow x = -3 \quad PC\,(-3,0)$

x	$y = 9 - x^2$
-3	0
-2	5
0	9
2	5
3	0



$$A = \int_{-3}^{3} (9 - x^2)\,dx = \left[9x - \frac{x^3}{3}\right]_{-3}^{3} = (27 - 9) - (-27 + 9) = 36\,u^2$$

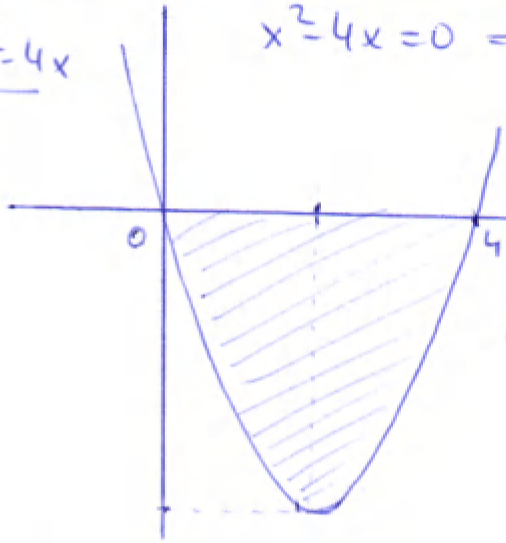
Problema 4:

$y = x^2 - 4x \longrightarrow$ Parábola cóncava $(a>0)$

$x^2 - 4x = 0 \Rightarrow x(x-4) = 0 \Rightarrow x = 0$

$ \Rightarrow x = 4$

x	$y = x^2 - 4x$
0	0
2	-4
4	0



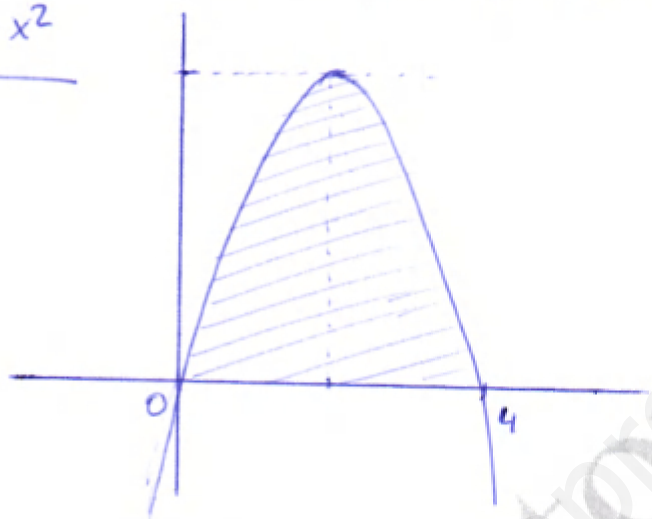
$$A = \left|\int_0^4 (x^2 - 4x)\,dx\right| = \left|\left[\frac{x^3}{3} - \frac{4x^2}{2}\right]_0^4\right| = \frac{32}{3}\,u$$

Problema 5:

$y = 4x - x^2 \longrightarrow$ Parábola convexa $(a<0)$

$4x - x^2 = 0 \Rightarrow x(4-x) = 0 \begin{cases} x = 0 \\ x = 4 \end{cases}$

x	$y = 4x - x^2$
0	0
2	4
4	0



$$A = \int_0^4 (4x - x^2)\,dx = \left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^4 = \frac{32}{3}$$

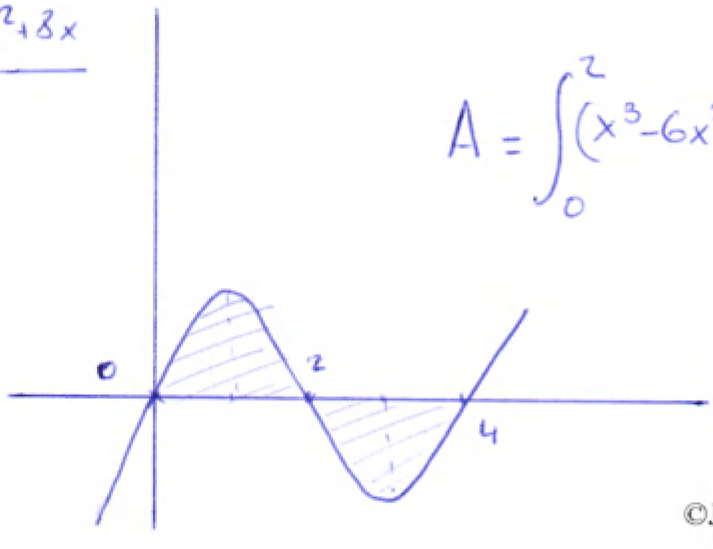
Problema 6:

$y = x^3 - 6x^2 + 8x \Rightarrow x^3 - 6x^2 + 8x = 0 \quad x(x^2 - 6x + 8) = 0$

$\begin{cases} x = 0 \\ x^2 - 6x + 8 = 0 \begin{cases} x = 4 \\ x = 2 \end{cases} \end{cases}$

x	$y = x^3 - 6x^2 + 8x$
0	0
1	3
2	0
3	-3
4	0

$$A = \int_0^2 (x^3 - 6x^2 + 8x)\,dx + \left|\int_2^4 (x^3 - 6x^2 + 8x)\,dx\right|$$



$$A = \left[\frac{x^4}{4} - 2x^3 + 4x^2 \right]_0^2 + \left| \left[\frac{x^4}{4} - 2x^3 + 4x^2 \right]_2^4 \right| =$$

$$= \left(\frac{2^4}{4} - 2 \cdot 2^3 + 4 \cdot 2^2 - 0 \right) + \left| \left(\frac{4^4}{4} - 2 \cdot 4^3 + 4 \cdot 4^2 \right) - \left(\frac{2^4}{4} - 2 \cdot 2^3 + 4 \cdot 2^2 \right) \right| =$$

$$= (4 - 16 + 16) + \left| (0 - 4) \right| = 8\, u^2$$

Problema 7:

$$\left. \begin{array}{l} y^2 = 4x \\ y = x \end{array} \right. \longrightarrow \left\{ \begin{array}{l} y = +\sqrt{4x} \\ y = -\sqrt{4x} \end{array} \right.$$

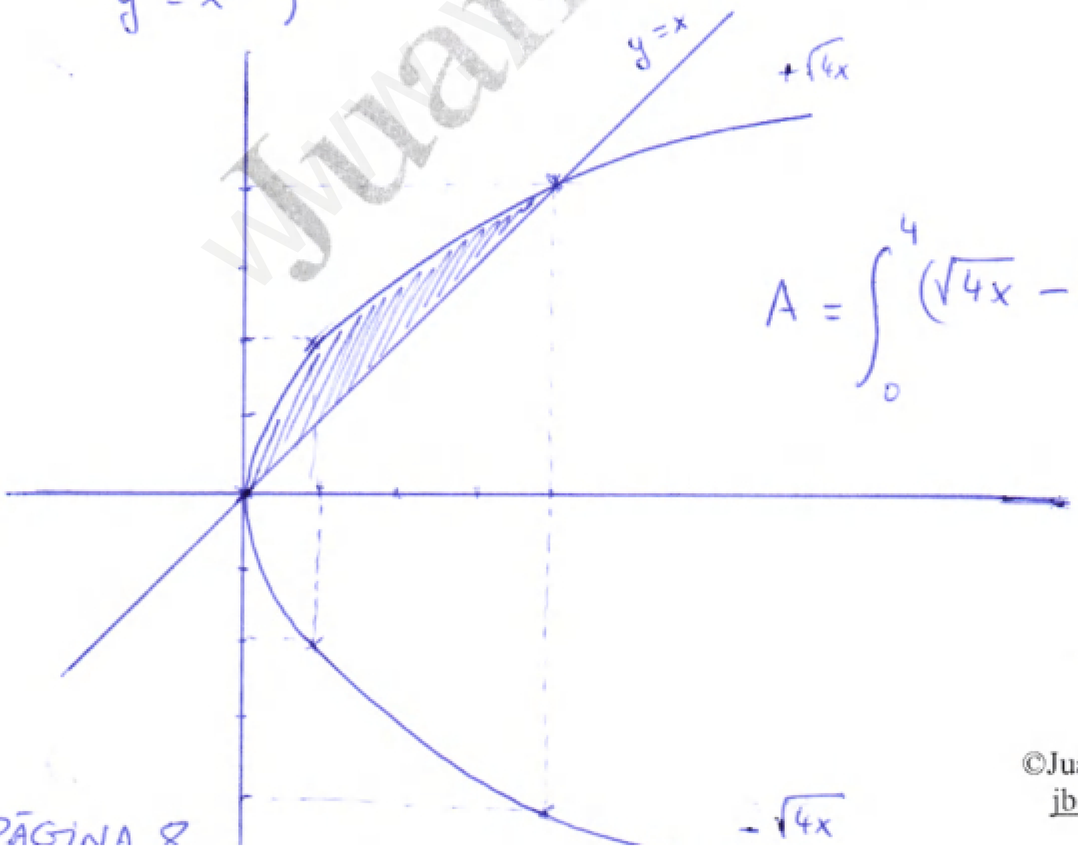
x	$\sqrt{4x}$
0	0
1	2
4	4

x	$-\sqrt{4x}$
0	0
1	-2
4	-4

x	$y = x$
0	0
1	1

Puntos de corte:

$$\left. \begin{array}{l} y^2 = 4x \\ y = x \end{array} \right\} \quad x^2 = 4x \Rightarrow x^2 - 4x = 0 \quad x(x-4) = 0 \quad \begin{array}{l} \nearrow x = 0 \\ \searrow x = 4 \end{array}$$



$$A = \int_0^4 (\sqrt{4x} - x)\, dx = (*)$$

$$(*) = \int_0^4 (2\cdot\sqrt{x} - x)\,dx = \left[2\cdot\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^4 = \left[\frac{4}{3}\sqrt{x^3} - \frac{x^2}{2}\right]_0^4 =$$

$$= \left(\frac{4}{3}\sqrt{4^3} - \frac{4^2}{2}\right) - 0 = \frac{32}{3} - 8 = \frac{8}{3}\ u^2$$

Problema 8:

$$y^2 = 4x \longrightarrow \begin{cases} y = +\sqrt{4x} \\ y = -\sqrt{4x} \end{cases}$$

$$y = x^2$$

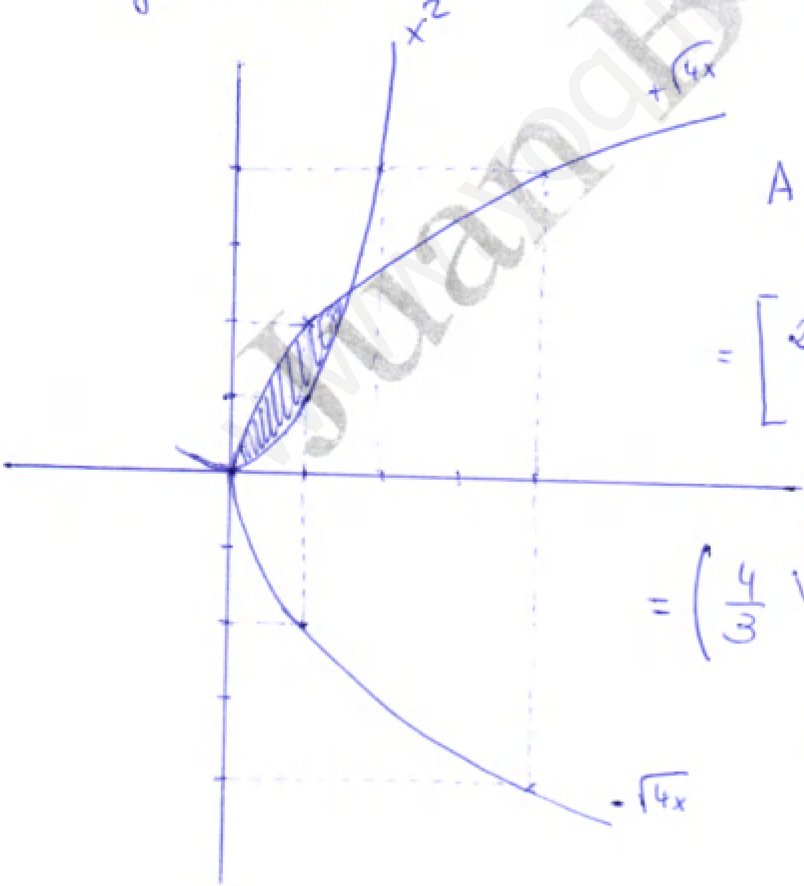
x	$y=\sqrt{4x}$
0	0
1	2
4	4

x	$y=-\sqrt{4x}$
0	0
1	-2
4	-4

x	$y=x^2$
0	0
1	1
2	4

Puntos de corte:

$$\left.\begin{array}{l} y^2 = 4x \\ y = x^2 \end{array}\right\} \quad x^4 = 4x \Rightarrow x^4 - 4x = 0 \quad x(x^3-4) = 0 \quad \begin{array}{l} x = 0 \\ x = \sqrt[3]{4} \simeq 1'59 \end{array}$$



$$A = \int_0^{\sqrt[3]{4}} (\sqrt{4x} - x^2)\,dx = \int_0^{\sqrt[3]{4}} (2\sqrt{x} - x^2)\,dx =$$

$$= \left[2\cdot\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right]_0^{\sqrt[3]{4}} = \left[\frac{4}{3}\sqrt{x^3} - \frac{x^3}{3}\right]_0^{\sqrt[3]{4}} =$$

$$= \left(\frac{4}{3}\sqrt{(\sqrt[3]{4})^3} - \frac{(\sqrt[3]{4})^3}{3}\right) - 0 =$$

$$= \frac{8}{3} - \frac{4}{3} = \frac{4}{3}\ u^2$$

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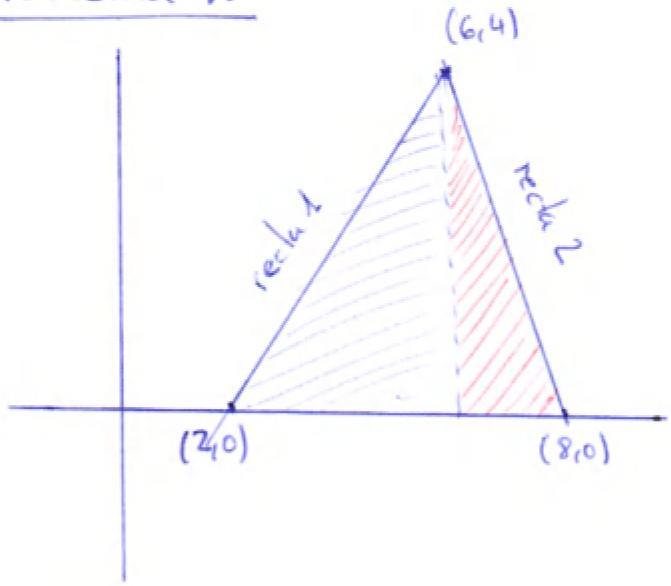
Problema 9:



Recta 1:

$$y = mx + n$$

$$\left.\begin{array}{l} (2,0) \to 0 = 2m + n \\ (6,4) \to 4 = 6m + n \end{array}\right\} \begin{array}{l} 4 = 4m \\ m = 1 \end{array}$$

$$\Rightarrow n = -2m = -2$$

$$y = x - 2$$

Recta 2:

$$y = mx + n$$

$$\left.\begin{array}{l} (6,4) \to 4 = 6m + n \\ (8,0) \to 0 = 8m + n \end{array}\right\} \begin{array}{l} -4 = 2m \Rightarrow m = -2 \\ n = -8m = +16 \end{array} \Rightarrow y = -2x + 16$$

$$A = \int_2^6 (x-2)\,dx + \int_6^8 (-2x+16)\,dx = \left[\frac{x^2}{2} - 2x\right]_2^6 + \left[-\frac{2x^3}{3} + 16x\right]_6^8$$

$$= \left[\left(\frac{6^2}{2} - 2\cdot 6\right) - \left(\frac{2^2}{2} - 2\cdot 2\right)\right] + \left[\left(\frac{-2\cdot 8^3}{3} + 16\cdot 8\right) - \left(-\frac{2\cdot 6^3}{3} + 16\cdot 6\right)\right] =$$

$$= 8 + 4 = 12\ u^2$$

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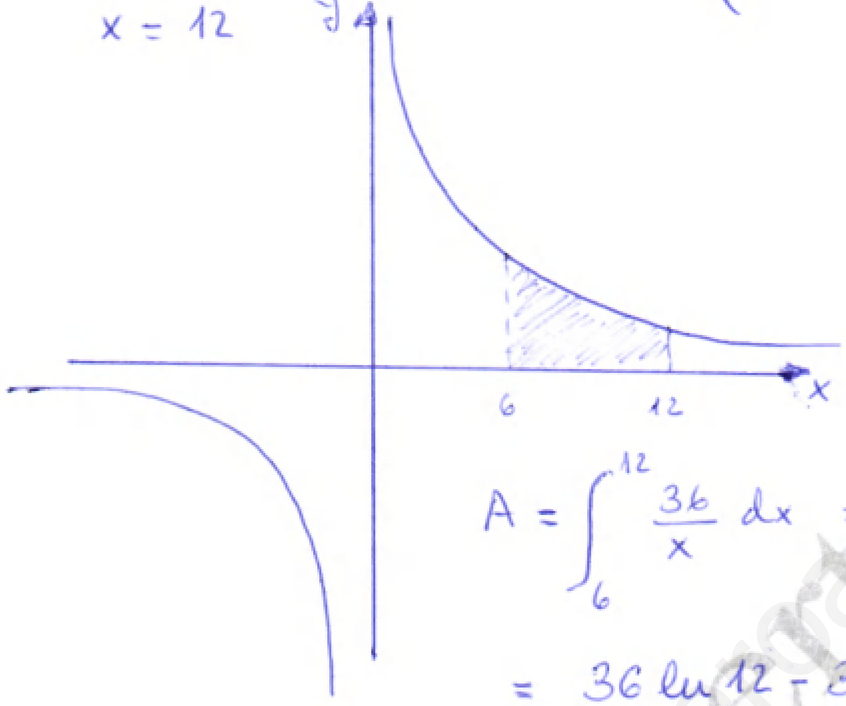
Problema 10:

$$x \cdot y = 36 \rightarrow y = \frac{36}{x} \Rightarrow \begin{cases} D = \mathbb{R} - \{0\} \\ x = 0 \quad A.V \\ \text{No corta con los ejes} \end{cases}$$

x	$y = \frac{36}{x}$
0^+	$+\infty$
0^-	$-\infty$
1	36
6	6
12	3

$x = 6$

$x = 12$



$$A = \int_6^{12} \frac{36}{x}\,dx = \left[36 \ln x\right]_6^{12} =$$

$$= 36 \ln 12 - 36 \ln 6 = 24'9533\ u^2$$

Problema 11:

$$y^2 = 8x \longrightarrow y = \sqrt{8x}$$

$$x - y + 25 = 0 \Rightarrow \left.\begin{matrix} y = x + 25 \\ y = mx + n \end{matrix}\right\} m = 1$$

Determinamos punto tangencia

$$y' = \frac{1}{2\sqrt{8x}} \cdot 8 = \frac{4}{\sqrt{8x}} = \frac{4}{2\sqrt{2x}} = \frac{2}{\sqrt{2x}} \Rightarrow y' = 1 = m$$

$$\frac{2}{\sqrt{2x}} = 1 \Rightarrow 2 = \sqrt{2x} \Rightarrow 4 = 2x \Rightarrow x = 2$$

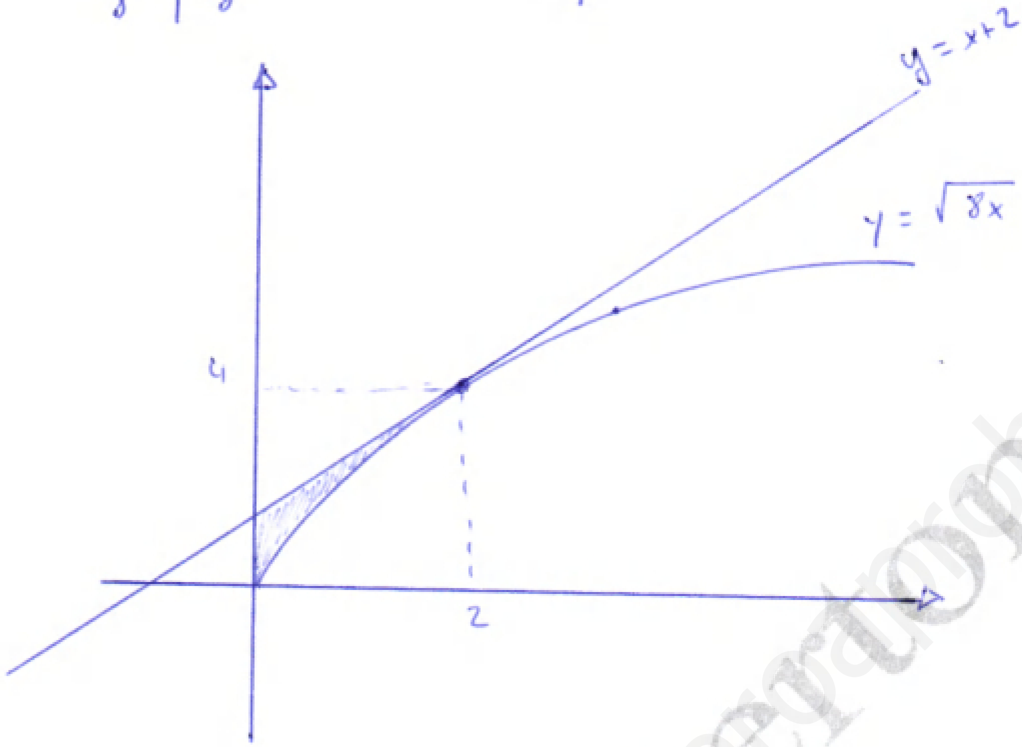
Punto $(2, \sqrt{16}) = (2, 4)$

Recta Tangente $\longrightarrow y - y_0 = m(x - x_0) \Rightarrow y = x + 2$

$$y - 4 = 1(x - 2)$$

x	$y=\sqrt{8x}$
0	0
2	4
8	8

x	$y = x+2$
0	2
2	4



$$A = \int_0^2 (x+2-\sqrt{8x})\,dx = \left[\frac{x^2}{2} + 2x - \frac{1}{8}\frac{(8x)^{3/2}}{3/2}\right]_0^2 =$$

$$= \left[\frac{x^2}{2} + 2x - \frac{2\sqrt{(8x)^3}}{24}\right]_0^2 = \frac{2^2}{2} + 2\cdot 2 - \frac{1}{12}\sqrt{16^3} = 6 - \frac{64}{12} = \frac{2}{3}\,u^2$$

Problema 12:

$$y^2 = 4x \longrightarrow y = \sqrt{4x} = 2\sqrt{x}$$

$$x - 2y + 8 = 0 \Rightarrow 2y = x + 8 \Rightarrow y = \frac{1}{2}x + 4 \quad \begin{cases} m = 1/2 \\ n = 4 \end{cases}$$

Determinamos punto tangencia:

$$m = f'(x_0) \Rightarrow y' = 2\cdot\frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{x}} \Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{2} \Rightarrow 2 = \sqrt{x}$$

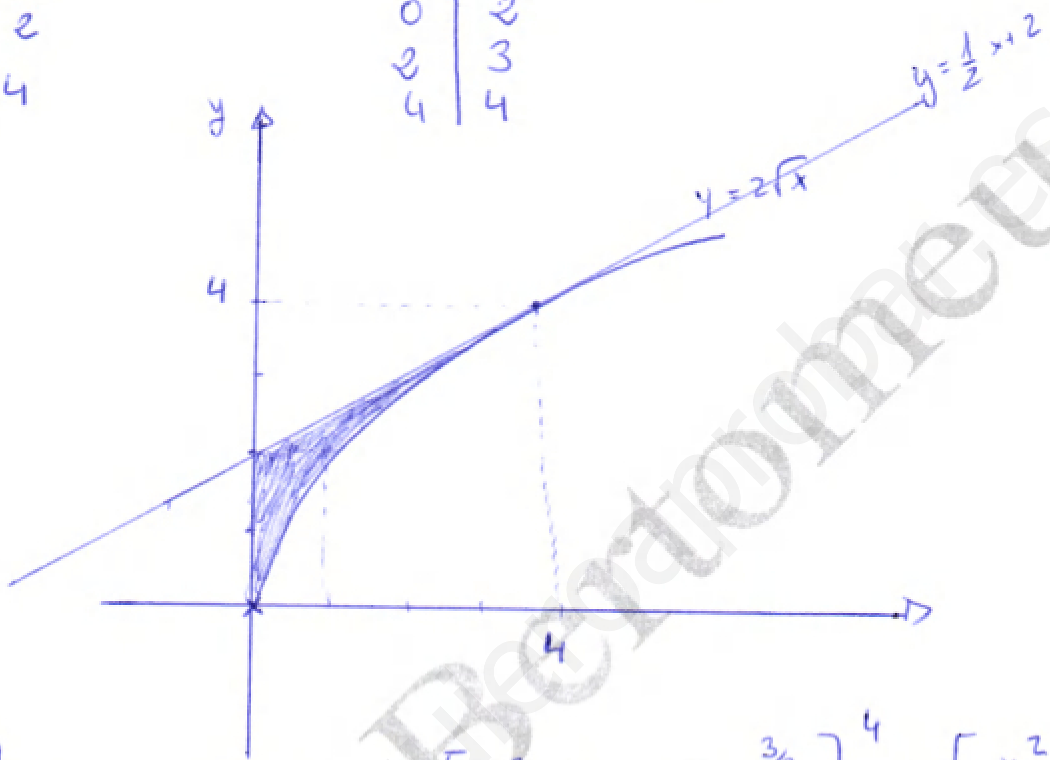
$$\Rightarrow x = 4 \Rightarrow \text{Punto } (4, 4)$$

Recta Tangente $\longrightarrow$ $y - y_0 = m(x - x_0)$

$$y - 4 = \frac{1}{2}(x - 4) \Rightarrow y = \frac{1}{2}x + 2$$

x	$y = 2\sqrt{x}$
0	0
1	2
4	4

x	$y = \frac{1}{2}x + 2$
0	2
2	3
4	4



$$A = \int_0^4 \left(\frac{1}{2}x + 2 - 2\sqrt{x}\right)dx = \left[\frac{x^2}{4} + 2x - 2\cdot\frac{x^{3/2}}{3/2}\right]_0^4 = \left[\frac{x^2}{4} + 2x - \frac{4}{3}\sqrt{x^3}\right]_0^4$$

$$= \left(\frac{4^2}{4} + 2\cdot 4 - \frac{4}{3}\sqrt{4^3}\right) - 0 = 4 + 8 - \frac{32}{3} = 12 - \frac{32}{3} = \frac{4}{3}\ u^2$$

Problema 13:

$f(x) = x^3 + 4x^2 + x - 6$ en el intervalo $[-1, 4]$

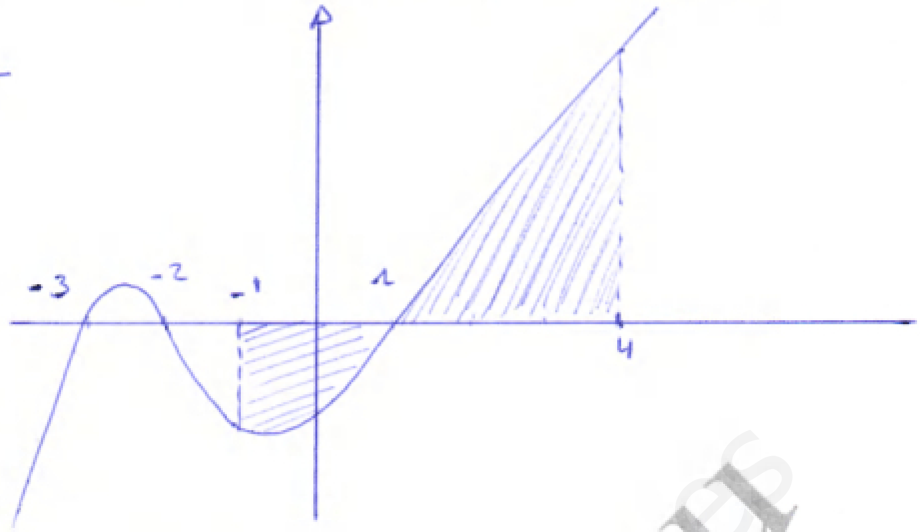
Cortes con los ejes:

$f(x) = 0 \longrightarrow x^3 + 4x^2 + x - 6 = 0$

	1	4	1	-6
1		1	5	6
	1	5	6	$\boxed{0}$

$x^2 + 5x + 6 = 0$ $\quad x = \frac{-5 \pm \sqrt{25 - 24}}{2} \begin{cases} -2 \\ -3 \end{cases}$

x	$y = x^3+4x^2+x-6$
-3	0
$-2'5$	$0'875$
-2	0
0	-6
1	0
4	126



$$A = \left| \int_{-1}^{1} (x^3+4x^2+x-6)\,dx \right| + \int_{1}^{4} (x^3+4x^2+x-6)\,dx =$$

$$= \left| \left[\frac{x^4}{4} + \frac{4x^3}{3} + \frac{x^2}{2} - 6x \right]_{-1}^{1} \right| + \left[\frac{x^4}{4} + \frac{4x^3}{3} + \frac{x^2}{2} - 6x \right]_{1}^{4} =$$

$$= \left| \left[\left(\frac{1}{4} + \frac{4}{3} + \frac{1}{2} - 6 \right) - \left(\frac{1}{4} - \frac{4}{3} + \frac{1}{2} + 6 \right) \right] \right| + \left[\left(\frac{4^4}{4} + \frac{4^4}{3} + \frac{4^2}{2} - 24 \right) - \left(\frac{1}{4} + \frac{4}{3} + \frac{1}{2} - 6 \right) \right]$$

$$= \frac{28}{3} + \frac{549}{4} = 146'583\ u^2$$

Problema 14:

$$\left. \begin{array}{l} f(x) = 2^x \\ g(x) = \dfrac{3x+2}{2} \end{array} \right\}$$

En $x=0$: $2^0 = 1$; $\frac{3\cdot 0+2}{2} = 1$

En $x=1$: $2^1 = 2$; $\frac{3\cdot 1+2}{2} = \frac{5}{2}$

En $x=2$: $2^2 = 4$; $\frac{3\cdot 2+2}{2} = 4$

Puntos de corte $x=0$ y $x=2$

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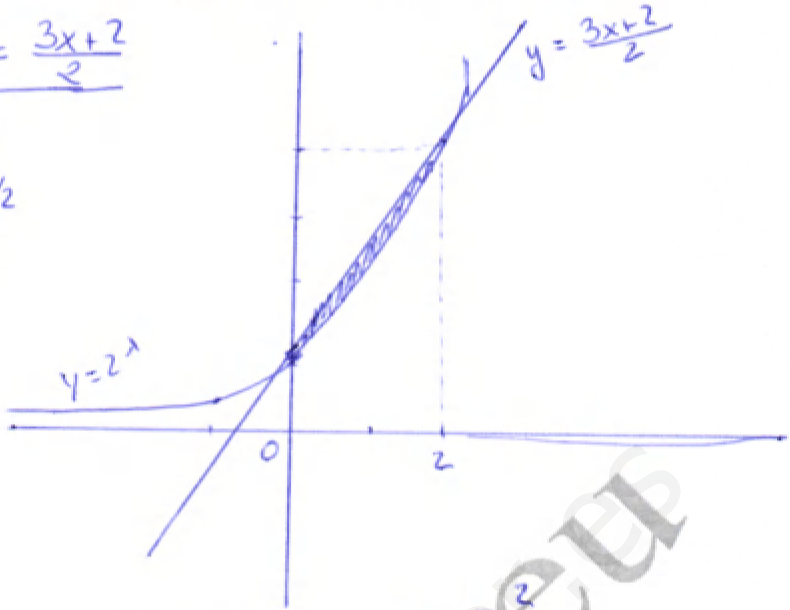
x	$y = 2^x$
-1	1/2
0	1
1	2
2	4

x	$y = \frac{3x+2}{2}$
0	1
1	5/2
2	4



$$A = \int_0^2 \left(\frac{3}{2}x + 1 - 2^x\right) dx = \left[\frac{3x^2}{4} + x - \frac{2^x}{\ln 2}\right]_0^2 =$$

$$= \left(3 + 2 - \frac{4}{\ln 2}\right) - \left(-\frac{1}{\ln 2}\right) = 0'672\,u^2$$

Problema 15:

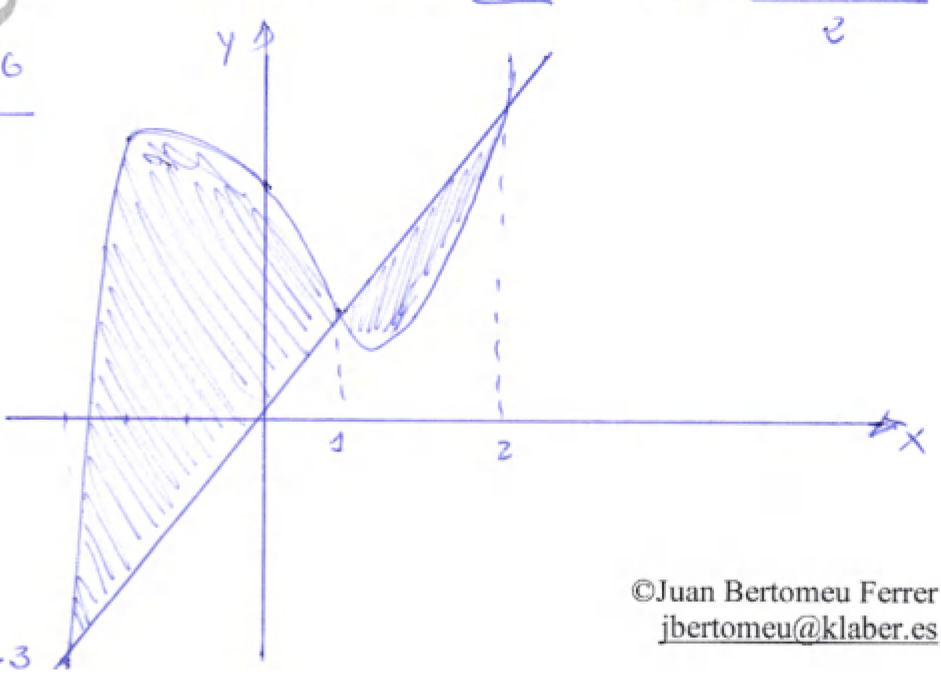
$$\left.\begin{array}{l} y = x^3 - 5x + 6 \\ y = 2x \end{array}\right\} \quad x^3 - 5x + 6 = 2x \;;\; x^3 - 7x + 6 = 0$$

	1	0	-7	6
1		1	1	-6
	1	1	-6	0

$x^2 + x - 6 = 0$

$x = \frac{-1 \pm \sqrt{1+24}}{2} = \begin{cases} x = 2 \\ x = -3 \end{cases}$

x	$y = x^3 - 5x + 6$
-3	-6
-2	8
0	6
1	2
2	4



$$A=\int_{-3}^{1}(x^3-5x+6-2x)\,dx+\int_{1}^{2}(2x-(x^3-5x+6))\,dx=$$

$$=\int_{-3}^{1}(x^3-7x+6)\,dx+\int_{1}^{2}(-x^3+7x-6)\,dx=\left[\frac{x^4}{4}-\frac{7x^2}{2}+6x\right]_{-3}^{1}+\left[-\frac{x^4}{4}+\frac{7x^2}{2}-6x\right]_{1}^{2}$$

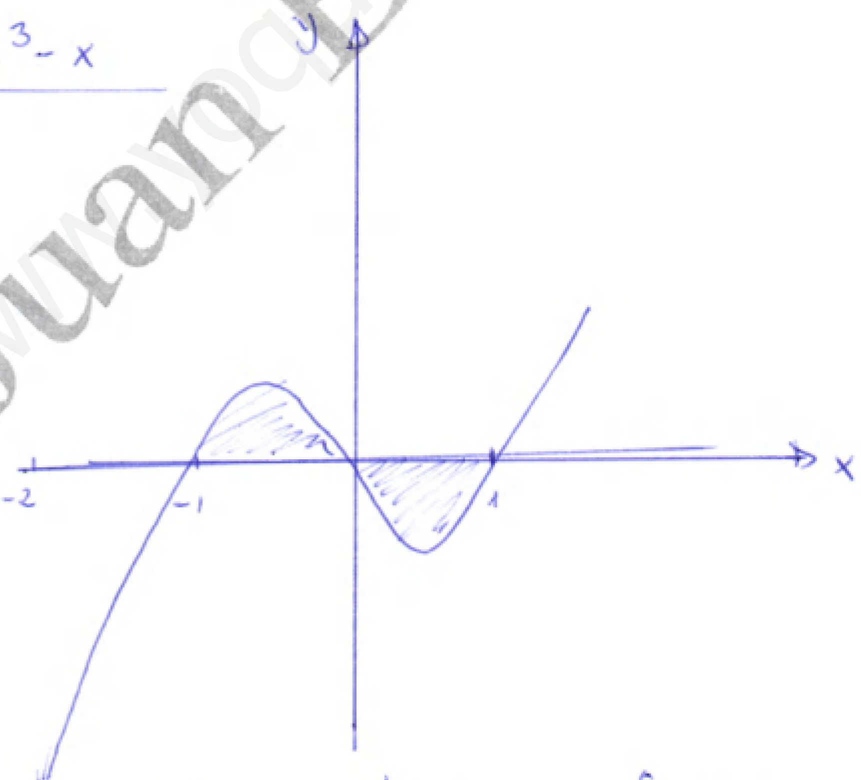
$$=\left[\left(\frac{1}{4}-\frac{7}{2}+6\right)-\left(\frac{(-3)^4}{4}-\frac{7(-3)^2}{2}+6\cdot(-3)\right)\right]+\left[\left(-\frac{2^4}{4}+\frac{7\,2^2}{2}-6\cdot2\right)-\left(-\frac{1}{4}+\frac{7}{2}-6\right)\right]=$$

$$=32+0'75=32'75\,u^2$$

Problema 16:

$y=x^3-x$; $x^3-x=0\Rightarrow x(x^2-1)=0 \begin{cases} x=0 \\ x^2-1=0 \begin{cases} x=+1 \\ x=-1\end{cases}\end{cases}$

x	$y=x^3-x$
-2	-6
-1	0
0	0
1	0



$$A=\int_{-1}^{0}(x^3-x)\,dx+\left|\int_{0}^{1}(x^3-x)\,dx\right|=\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_{-1}^{0}+\left|\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_{0}^{1}\right|=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$$

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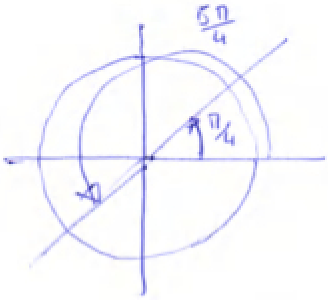
Problema 17:

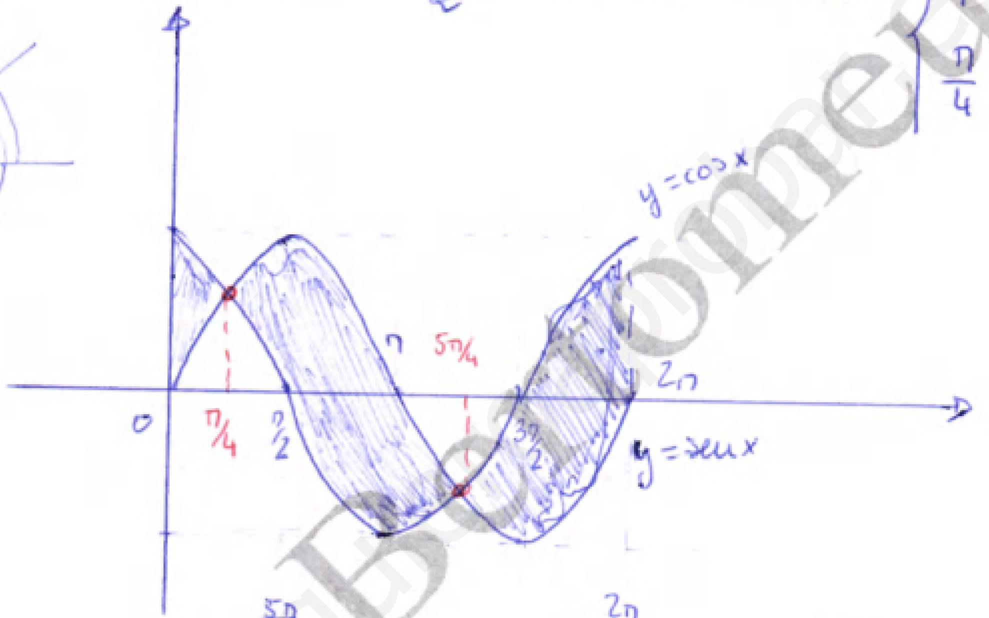
$$\left.\begin{array}{l} y = \operatorname{sen} x \\ y = \cos x \\ x \in [0, 2\pi] \end{array}\right\}$$

$$\operatorname{sen} x = \cos x \Rightarrow \operatorname{sen} x = \sqrt{1-\operatorname{sen}^2 x} \Rightarrow \operatorname{sen}^2 x = 1 - \operatorname{sen}^2 x$$

$$\Rightarrow 2\operatorname{sen}^2 x = 1 \Rightarrow \operatorname{sen}^2 x = \frac{1}{2} \Rightarrow \operatorname{sen} x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \operatorname{sen} x = \frac{\sqrt{2}}{2} \Rightarrow x = \operatorname{arcsen}\left(\frac{\sqrt{2}}{2}\right) = \begin{cases} \frac{\pi}{4} \\ \frac{\pi}{4} + \pi = \frac{5\pi}{4} \end{cases}$$





$$A = \int_0^{\pi/4} (\cos x - \operatorname{sen} x)\,dx + \int_{\pi/4}^{\frac{5\pi}{4}} (\operatorname{sen} x - \cos x)\,dx + \int_{\frac{5\pi}{4}}^{2\pi} (\cos x - \operatorname{sen} x)\,dx =$$

$$= \Big[\operatorname{sen} x + \cos x\Big]_0^{\pi/4} + \Big[-\cos x - \operatorname{sen} x\Big]_{\pi/4}^{5\pi/4} + \Big[\operatorname{sen} x + \cos x\Big]_{5\pi/4}^{2\pi} =$$

$$= \left[\left(\operatorname{sen}\frac{\pi}{4} + \cos\frac{\pi}{4}\right) - \left(\operatorname{sen} 0 + \cos 0\right)\right] + \left[\left(-\cos\frac{5\pi}{4} - \operatorname{sen}\frac{5\pi}{4}\right) - \left(-\cos\frac{\pi}{4} - \operatorname{sen}\frac{\pi}{4}\right)\right] +$$

$$+ \left[\left(\operatorname{sen} 2\pi + \cos 2\pi\right) - \left(\operatorname{sen}\frac{5\pi}{4} + \cos\frac{5\pi}{4}\right)\right] = \left(\sqrt{2} - \cancel{1}\right) + \left(\sqrt{2} + \sqrt{2}\right) + \left(\cancel{1} + \sqrt{2}\right)$$

$$= 4\sqrt{2} = 5'65\ u^2$$

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Problema 18:

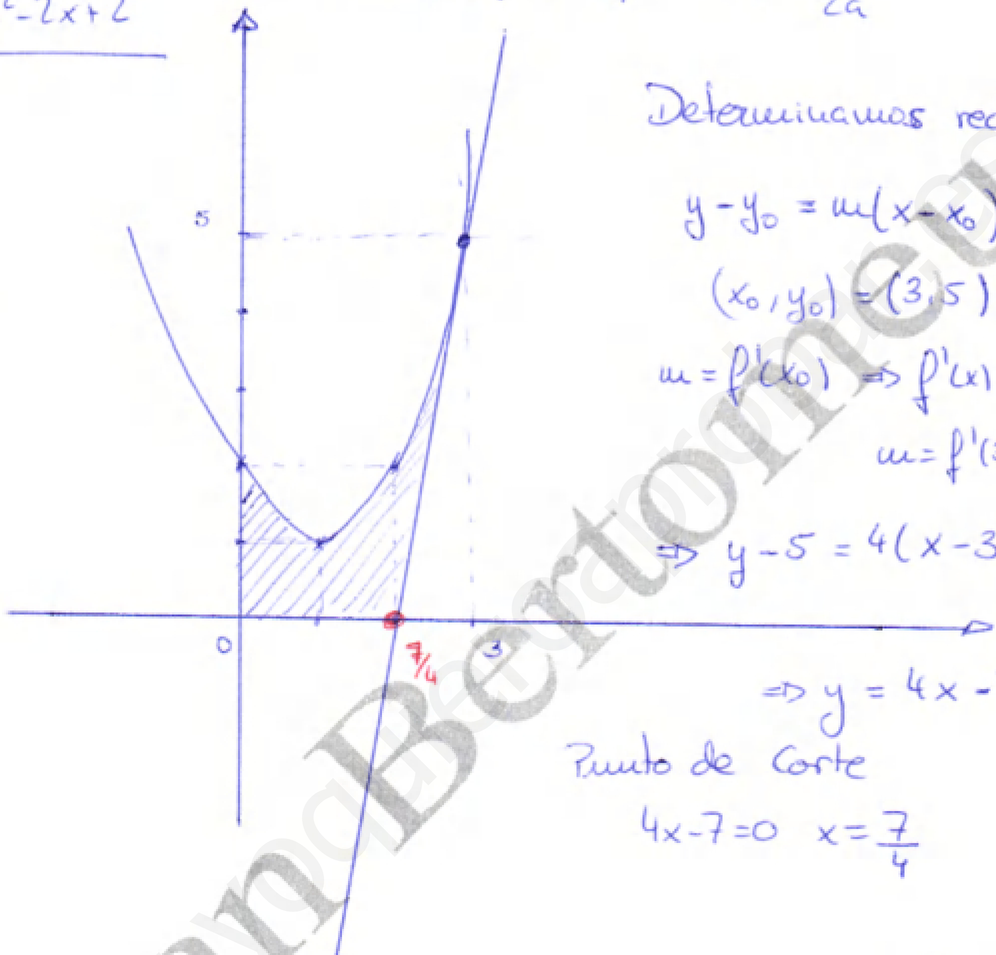
$$y = x^2 - 2x + 2$$

→ Parábola Cóncava

→ $x^2 - 2x + 2 = 0 \quad x = \frac{2 \pm \sqrt{4-8}}{2} = \nexists$ no corta

→ vértice $(1, 1)$ $\quad x_v = -\frac{b}{2a}$

x	$y = x^2 - 2x + 2$
0	2
1	1
2	2
3	5



Determinamos recta tangente

$y - y_0 = m(x - x_0)$

$(x_0, y_0) = (3, 5)$

$m = f'(x_0) \Rightarrow f'(x) = 2x - 2$

$m = f'(3) = 4$

$\Rightarrow y - 5 = 4(x - 3)$

$\Rightarrow y = 4x - 7$

Punto de corte

$4x - 7 = 0 \quad x = \frac{7}{4}$

$$A = \int_0^3 (x^2 - 2x + 2)\,dx - \int_{7/4}^3 (4x - 7)\,dx = \left[\frac{x^3}{3} - x^2 + 2x\right]_0^3 - \left[2x^2 - 7x\right]_{7/4}^3$$

$$= \left[\left(\frac{3^3}{3} - 3^2 + 2\cdot 3\right) - 0\right] - \left[(2\cdot 3^2 - 7\cdot 3) - \left(2\cdot\left(\tfrac{7}{4}\right)^2 - 7\left(\tfrac{7}{4}\right)\right)\right] =$$

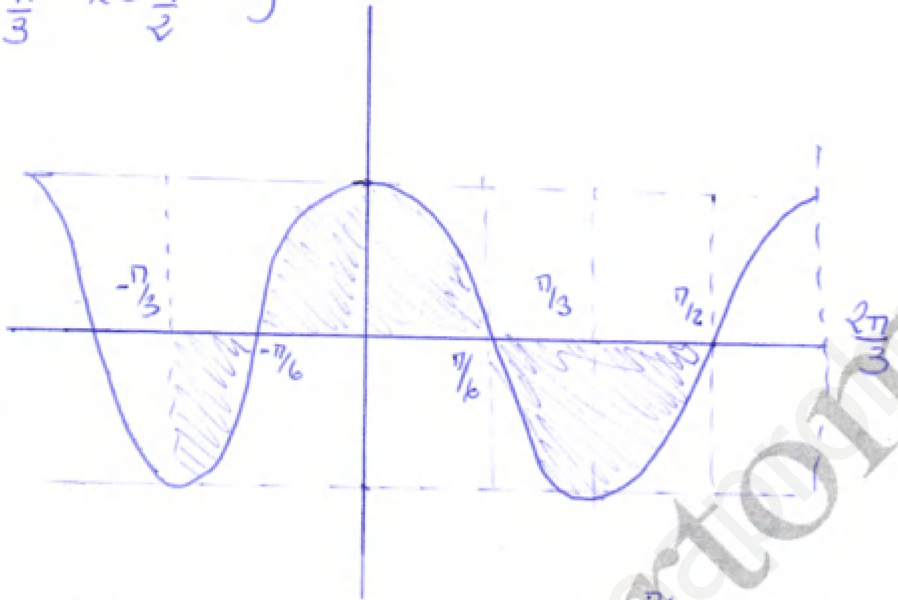
$$= 6 - 3'125 = 2'875 = \frac{23}{8}\ u^2$$

Problema 19:

$$\left.\begin{array}{l} f(x) = \cos 3x \\ x = \frac{-\pi}{3} \quad x = \frac{\pi}{2} \end{array}\right\}$$

$$\left.\begin{array}{l} y = A\cos\left(\frac{2\pi}{T}x + \varphi_0\right) \\ y = 1\cos(3x + 0) \end{array}\right\} \quad \frac{2\pi}{T} = 3 \Rightarrow T = \frac{2\pi}{3}$$

$$A = 1 \; ; \; \varphi_0 = 0$$



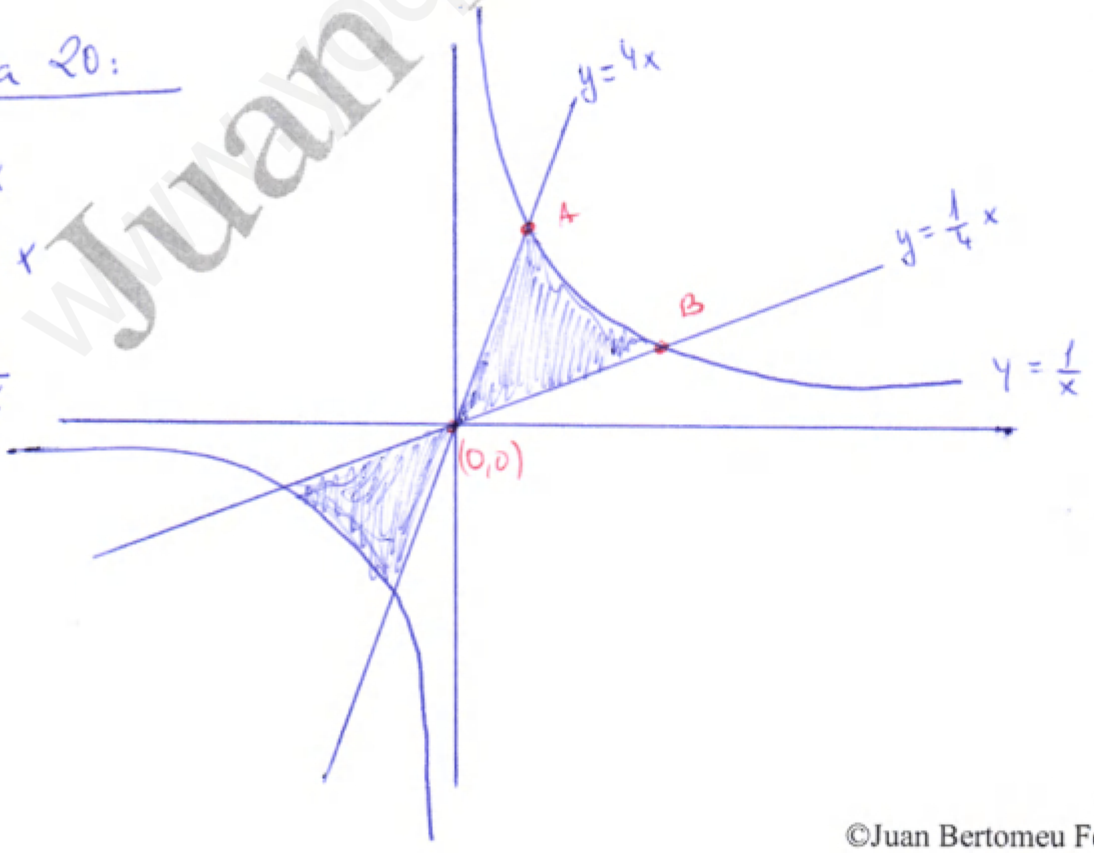
$$A = 5 \cdot \int_0^{\pi/6} \cos 3x \, dx = \frac{5}{3}\Big[\operatorname{sen} 3x\Big]_0^{\pi/6} = \frac{5}{3}\left[\operatorname{sen}\frac{\pi}{2} - \operatorname{sen} 0\right] = \frac{5}{3}\, u^2$$

Problema 20:

$y = 4x$

$y = \frac{1}{4}x$

$y = \frac{1}{x}$



Punto A:

$$\left.\begin{array}{l} y = 4x \\ y = \frac{1}{x} \end{array}\right\} \quad 4x = \frac{1}{x} \Rightarrow 4x^2 = 1 \Rightarrow x^2 = \frac{1}{4} \begin{array}{l} \nearrow x = +1/2 \\ \searrow x = -1/2 \end{array}$$

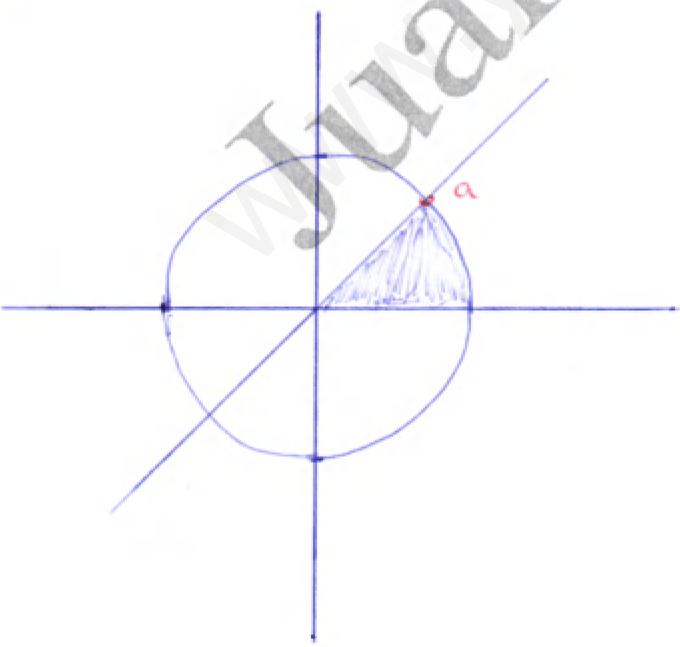
Punto B:

$$\left.\begin{array}{l} y = \frac{1}{4}x \\ y = \frac{1}{x} \end{array}\right\} \quad \frac{1}{4}x = \frac{1}{x} \Rightarrow x^2 = 4 \begin{array}{l} \nearrow x = +2 \\ \searrow x = -2 \end{array}$$

$$A = 2 \cdot \left[\int_0^{1/2} \left(4x - \frac{1}{4}x\right) dx + \int_{1/2}^{2} \left(\frac{1}{x} - \frac{1}{4}x\right) dx \right] =$$

$$2 \cdot \left[\left[\frac{7x^2}{8} \right]_0^{1/2} + \left[\ln x - \frac{x^2}{8} \right]_{1/2}^{2} \right] = 2 \cdot \left[\frac{7}{32} + 0'9175 \right] = 2'2725 \, u^2$$

Problema 21:



Ec. Circunferencia:

$(x-a)^2 + (y-b)^2 = r^2$

radio = 2

Centro $(a,b) = (0,0)$

$\Rightarrow \quad x^2 + y^2 = 4$

Bisectriz del primer cuadrante

$y = x$

Punto de corte a:

$$\left.\begin{array}{l} x^2+y^2=4 \;;\; y^2=4-x^2 \;;\; y=\sqrt{4-x^2} \\ y=x \end{array}\right\} \quad \begin{array}{l} x=\sqrt{4-x^2} \\ x^2=4-x^2 \Rightarrow 2x^2=4 \\ x=\sqrt{2} \end{array}$$

$$A=\int_0^{\sqrt{2}} x\,dx+\int_{\sqrt{2}}^{2}\left(\sqrt{4-x^2}\right)dx$$

Hacemos a parte

$$\int\sqrt{4-x^2}\cdot dx=\int\sqrt{4-4\operatorname{sen}^2 t}\cdot 2\cos t\,dt=\int\sqrt{4(1-\operatorname{sen}^2 t)}\cdot 2\cos t\,dt$$

$x=2\operatorname{sen} t$
$dx=2\cos t\,dt$

$$=\int 4\cos^2 t\,dt=\int \underbrace{4\cos t}_{u}\cdot\underbrace{\cos t\,dt}_{dv}=$$

$u=4\cos t \quad du=-4\operatorname{sen} t$
$dv=\cos t\,dt \quad v=\operatorname{sen} t$

$$=4\cos t\operatorname{sen} t+\int 4\operatorname{sen}^2 t\,dt=$$

$$=4\cos t\operatorname{sen} t+\int 4(1-\cos^2 t)\,dt=4\cos t\operatorname{sen} t+4t-\int 4\cos^2 t\,dt$$

$$\Rightarrow \overset{I}{\int 4\cos^2 t\,dt}=4\cos t\operatorname{sen} t+4t-\overset{I}{\int 4\cos^2 t\,dt}$$

$$\Rightarrow 2\int 4\cos^2 t\,dt=4\cos t\operatorname{sen} t+4t$$

$$\Rightarrow \int 4\cos^2 t\,dt=2\cos t\operatorname{sen} t+2t$$

Deshacemos el cambio

$$\operatorname{sen} t = \frac{x}{2} \longrightarrow t = \operatorname{arcsen}\left(\frac{x}{2}\right)$$

$$\cos t = \sqrt{1-\operatorname{sen}^2 t} = \sqrt{1-\frac{x^2}{4}} = \sqrt{\frac{4-x^2}{4}} = \frac{\sqrt{4-x^2}}{2}$$

$$\Rightarrow \int \sqrt{4-x^2}\,dx = \cancel{2}\cdot\frac{\sqrt{4-x^2}}{\cancel{2}}\cdot\frac{x}{2} + 2\operatorname{arcsen}\left(\frac{x}{2}\right) =$$

$$= \frac{x}{2}\cdot\sqrt{4-x^2} + 2\operatorname{arcsen}\left(\frac{x}{2}\right)$$

$$\Rightarrow A = \int_0^{\sqrt{2}} x\,dx + \int_{\sqrt{2}}^{2} \left(\sqrt{4-x^2}\right)dx =$$

$$= \left[\frac{x^2}{2}\right]_0^{\sqrt{2}} + \left[\frac{x}{2}\sqrt{4-x^2} + 2\operatorname{arcsen}\left(\frac{x}{2}\right)\right]_{\sqrt{2}}^{2} =$$

$$= 1 + \left[\left(0 + 2\operatorname{arcsen}(1)\right) - \left(1 + 2\operatorname{arcsen}\left(\frac{\sqrt{2}}{2}\right)\right)\right] =$$

$$= \cancel{1} + 2\cdot\frac{\pi}{2} \cancel{-1} - 2\cdot\frac{\pi}{4} = \pi - \frac{\pi}{2} = \frac{\pi}{2}\ u^2$$

Mucho más fácil hubiera sido razonar que el area pedida corresponde con $\frac{1}{8}$ del área de la circunferencia

$$A = \frac{1}{8}\cdot\pi\cdot r^2 = \frac{1}{8}\cdot\pi\cdot 2^2 = \frac{1}{8}\pi\cdot 4 = \frac{\pi}{2}\ u^2$$

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