1) TEOREMAS:

1.1. Comprueba que $f(x) = x^2 - 5x + 6$ cumple las hipótesis del teorema de Rolle en el intervalo $[1,4]$. Encuentra el punto donde se cumple la tesis del teorema.

1.2. Comprueba si las siguientes funciones cumplen las hipótesis del teorema de Rolle en los intervalos indicados. En caso afirmativo encuentra el punto "c" donde se cumple la tesis del teorema:

a) $f(x) = x - x^3$ en $[-1,0]$

b) $f(x) = \dfrac{x^2 - 3x}{x^2 + 1}$ en $[0,3]$

c) $f(x) = |x^2 - 4|$ en $[-2,2]$

d) $f(x) = |x+1| + |x-1|$ en $[-1,1]$

e) $f(x) = x - \dfrac{|x|}{x}$ en $[-1,1]$

f) $f(x) = x^3 + 2x^2 - 2x$ en $[-3,-2]$

g) $f(x) = \cos x$ en $[0,2\pi]$

h) $f(x) = \sqrt{1 + \operatorname{sen} x}$ en $[-\pi, 0]$

i) $f(x) = -x^4 + x^2$ en $[0,1]$

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1.3. Comprueba si las funciones siguientes satisfacen las hipótesis del teorema del valor medio en el intervalo indicado. En caso afirmativo, encuentra el valor intermedio correspondiente.

a) $f(x) = x^3 - x$ en $[-1, 2]$

b) $f(x) = x^3 - 3x + 5$ en $[-2, 0]$

c) $f(x) = (x-2)^3$ en $[0, 3]$

d) $f(x) = \sqrt{1+2x}$ en $[\frac{3}{2}, 4]$

e) $f(x) = \operatorname{sen}\left(\frac{x}{2}\right)$ en $[0, \pi]$

1.4. Encuentra un punto de la parábola $f(x) = (x-1)^2$ en el que la recta tangente sea paralela a la cuerda que une los puntos $A(2,1)$ y $B(4,9)$. Representalo.

1.5. Repite el ejercicio anterior siendo $f(x) = x^2 - 6x$, $A(0,0)$ y $B(4,-8)$

1.6. Utiliza el teorema del valor medio para demostrar que la función $f(x) = \sec^2 x - \operatorname{tg}^2 x$ es constante en cualquier intervalo de $\mathbb{R}$.

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1.7. Sea la función $f(x) = \begin{cases} x^2+bx+c & \text{si } x \leq 0 \\ \dfrac{\ln(1+x)}{x} & \text{si } x > 0 \end{cases}$

a) Determinar c para que $f(x)$ sea continua en 0.

b) Determinar b para que $f(x)$ sea derivable en 0.

c) Utilizando el Teorema del Valor Medio de Lagrange demostrar que existe un punto $c \in [0, e-1]$ tal que

$$f'(c) = \frac{2-e}{(e-1)^2}$$

2) REGLA DE L'HÔPITAL:

2.1. Calcula los siguientes límites:

1) $\lim\limits_{x \to 0} \dfrac{e^x - 1}{x}$ 2) $\lim\limits_{x \to 0} \dfrac{5x - \operatorname{sen} x}{2x}$ 3) $\lim\limits_{x \to 0} \dfrac{\operatorname{sen} x}{x}$

4) $\lim\limits_{x \to 0} \dfrac{1-\cos x}{x}$ 5) $\lim\limits_{x \to 1} \dfrac{\ln x}{x^2-5x+4}$ 6) $\lim\limits_{x \to 0} \dfrac{x - \operatorname{sen} x}{\operatorname{tg} x - x}$

7) $\lim\limits_{x \to 0} \dfrac{1-\cos x}{x^2}$ 8) $\lim\limits_{x \to \infty} \dfrac{\sqrt{1+x}}{x}$ 9) $\lim\limits_{x \to \infty} \dfrac{\ln x}{x^2}$

10) $\lim\limits_{x \to \infty} \dfrac{e^x}{x^3}$ 11) $\lim\limits_{x \to \infty} \dfrac{x^3}{e^{3x}}$ 12) $\lim\limits_{x \to \infty} \dfrac{3x^2-6x}{x^2-x-2}$

13) $\lim\limits_{x \to \infty} \dfrac{\ln x}{x}$ 14) $\lim\limits_{x \to 0^+} x \cdot \ln x$ 15) $\lim\limits_{x \to 0} \left(\dfrac{1}{\operatorname{sen} x} - \dfrac{1}{x}\right)$

16) $\lim\limits_{x \to \infty} (e^{-x} \cdot \ln^2 x)$ 17) $\lim\limits_{x \to \infty} (e^{-x} \cdot \sqrt{x})$ 18) $\lim\limits_{x \to \infty} \left(e^x \cdot \dfrac{1}{x^2}\right)$

19) $\lim\limits_{x\to\infty} x\cdot\left(e^{1/x}-1\right)$ 20) $\lim\limits_{x\to\frac{\pi}{4}} (1-\operatorname{tg} x)\sec(2x)$ 21) $\lim\limits_{x\to 0^+} x^{\operatorname{sen} x}$

22) $\lim\limits_{x\to 0^+} x^{x}$ 23) $\lim\limits_{x\to\infty} (1+x)^{1/x}$ 24) $\lim\limits_{x\to\infty} x^{1/x}$ 25) $\lim\limits_{x\to 0} (\cos x)^{1/x}$

26) $\lim\limits_{x\to 0^+} \left(\frac{1}{x}\right)^{\operatorname{tg} x}$ 27) $\lim\limits_{x\to 0} \frac{x\cos x-\operatorname{sen} x}{x^3}$ 28) $\lim\limits_{x\to 0} \frac{e^{x}-e^{-x}-4x}{x-\operatorname{sen} x}$

29) $\lim\limits_{x\to\frac{\pi}{2}} (1+2\cos x)^{1/\cos x}$ 30) $\lim\limits_{x\to 0} \left(\frac{1}{x}-\frac{1}{\ln(1+x)}\right)$ 31) $\lim\limits_{x\to 0} \frac{2x\operatorname{sen} x}{1-\cos x}$

32) $\lim\limits_{x\to 0} \frac{\ln(\cos x)-1+\cos x}{x^2}$ 33) $\lim\limits_{x\to 0} \frac{\ln(\cos 3x)}{\ln(\cos 2x)}$

34) $\lim\limits_{x\to 0} \frac{e^{x}-x-\cos x}{\operatorname{sen}^2 x}$ 35) $\lim\limits_{x\to 0} \frac{(e^{x}-e^{-x})^2}{x^2}$ 36) $\lim\limits_{x\to 0} \frac{x\operatorname{sen} x}{1-\cos x}$

37) $\lim\limits_{x\to 0} \frac{\operatorname{sen} x-x}{\operatorname{tg} x-x}$ 38) $\lim\limits_{x\to\infty} \left[x\cdot\left(\operatorname{arctg} e^{x}-\frac{\pi}{2}\right)\right]$

39) $\lim\limits_{x\to\infty} \left(\sqrt{x+\sqrt{x}}-\sqrt{x}\right)$ 40) $\lim\limits_{x\to 0} \left(\frac{x^2+2}{2-2\operatorname{sen} x}\right)^{\frac{2}{x}}$ 41) $\lim\limits_{x\to\infty} \left(\frac{3x+1}{3x-1}\right)^{x}$

42) $\lim\limits_{x\to 1} \left(\frac{1}{\ln x}-\frac{x}{x-1}\right)$ 43) $\lim\limits_{x\to\frac{\pi}{2}^-} (\operatorname{tg} x)^{\cos x}$ 44) $\lim\limits_{x\to 0} (\cos(2x))^{\frac{1}{\operatorname{sen} x^2}}$

45) $\lim\limits_{x\to 0} \left(\frac{1}{x}-\frac{1}{\operatorname{sen} x}\right)$ 46) $\lim\limits_{x\to 0^+} x^{\operatorname{tg} x}$ 47) $\lim\limits_{x\to\infty} \left(\frac{1}{2x}\right)^{1/x}$

48) $\lim\limits_{x\to 0^+} \left(\frac{1}{x}-\frac{1}{\operatorname{arctg} x}\right)$ 49) $\lim\limits_{x\to 1^-} \left[(x^2-1)\cdot\operatorname{tg}\left(\frac{\pi x}{2}\right)\right]$ 50) $\lim\limits_{x\to\infty} \left(x\cdot\ln\frac{x+1}{x}\right)$

51) $\lim\limits_{x\to 0} \dfrac{\text{sen}\, x}{4x}$ 52) $\lim\limits_{x\to 1} \dfrac{x^2-1}{\ln x}$ 53) $\lim\limits_{x\to 0} \dfrac{x^3-x}{\sqrt{x+9}-3}$

54) $\lim\limits_{x\to 0} \dfrac{\text{tg}\, x}{x}$ 55) $\lim\limits_{x\to 1} \dfrac{4x^5-4x^2}{-x^2+1}$ 56) $\lim\limits_{x\to 0} \dfrac{x}{\text{sen}\, x}$

57) $\lim\limits_{x\to 0} \dfrac{e^{-x}-e^x}{3\,\text{sen}\, x}$ 58) $\lim\limits_{x\to 7} \dfrac{x^2-9x+14}{\ln(x^2-6x-6)}$ 59) $\lim\limits_{x\to 0} \left(\dfrac{1}{4x} - \dfrac{2}{x\cdot e^x}\right)$

60) $\lim\limits_{x\to 0^+} (\text{tg}\, x)^x$ 61) $\lim\limits_{x\to \infty} \sqrt[x-2]{\dfrac{x^2+1}{x+3}}$ 62) $\lim\limits_{x\to 0} \dfrac{\text{sen}\, x}{1-\cos x}$

63) $\lim\limits_{x\to 0} \dfrac{2\,\text{sen}\, x}{x^2-3x}$ 64) $\lim\limits_{x\to 0} \dfrac{x-\text{sen}\, x}{x\cdot \text{sen}\, x}$ 65) $\lim\limits_{x\to 0} \dfrac{x^4-\left(\frac{1}{3}\right)x^3}{x-\text{tg}\, x}$

66) $\lim\limits_{x\to 0} \dfrac{x-2\,\text{sen}\, x}{\text{tg}\, x-2\,\text{sen}\, x}$ 67) $\lim\limits_{x\to 0} \dfrac{\ln(e^x+x^3)}{x}$ 68) $\lim\limits_{x\to 0^+} \dfrac{\ln(1+x)}{\sqrt[4]{x^3}}$

69) $\lim\limits_{x\to 0} \dfrac{\ln(\cos(3x))}{x^2}$ 70) $\lim\limits_{x\to 0} \dfrac{e^x-e^{\text{sen}\, x}}{1-\cos x}$ 71) $\lim\limits_{x\to 0} \dfrac{1+x-e^x}{\text{sen}^2 x}$

72) $\lim\limits_{x\to 2} \dfrac{2^x-x^2}{9-3^x}$ 73) $\lim\limits_{x\to 0} (e^x-x)^{1/x}$ 74) $\lim\limits_{x\to 0} (\text{cotg}\, x)^{\text{sen}\, x}$

75) $\lim\limits_{x\to 0} \left(\dfrac{2}{\ln(1+x)} - \dfrac{2}{x}\right)$ 76) $\lim\limits_{x\to \frac{\pi}{2}} (x-\pi/2)\cdot \text{tg}\, x$ 77) $\lim\limits_{x\to \infty} x\cdot e^{2/x}$

78) $\lim\limits_{x\to 0} (\text{arcsen}\, x\cdot \text{cotg}\, x)$ 79) $\lim\limits_{x\to \infty} x\cdot \text{arctg}\left(\dfrac{2}{x}\right)$ 80) $\lim\limits_{x\to \infty} (\ln x)^{e^{-x}}$

1) TEOREMAS:

1.1) $f(x) = x^2 - 5x + 6$ es continua y derivable en $\mathbb{R}$, y por tanto, y en concreto, lo es en $[1,4]$. Además

$$\left.\begin{array}{l} f(1) = 2 \\ f(4) = 2 \end{array}\right\} f(1) = f(4) \Rightarrow \exists\, c \in \,]1,4[\, / \, f'(c) = 0$$

Veamoslo:

$f'(x) = 2x - 5$

$f'(x) = 0 \rightarrow 2x - 5 = 0 \Rightarrow x = \frac{5}{2}$ que efectivamente pertenece a $]1,4[$

1.2) a) $f(x) = x - x^3 \rightarrow$ Continua y Derivable en $\mathbb{R}$

$$\left.\begin{array}{l} f(-1) = 0 \\ f(0) = 0 \end{array}\right\} f(-1) = f(0) \Rightarrow \exists\, c \in \,]-1,0[\, / \, f'(c) = 0$$

$f'(x) = 1 - 3x^2$

$f'(x) = 0 \rightarrow 1 - 3x^2 = 0 \left\langle \begin{array}{l} x = 0'577 \\ x = -0'577 \rightarrow x \in \,]-1,0[\end{array}\right.$

b) $f(x) = \dfrac{x^2 - 3x}{x^2 + 1}$; $Dom(f(x)) = \mathbb{R} \rightarrow$ Continua en $\mathbb{R}$.

$$f'(x) = \frac{(2x-3)(x^2+1) - (x^2-3x)\,2x}{(x^2+1)^2} = \frac{3x^2 + 2x - 3}{(x^2+1)^2}$$

$\hookrightarrow Dom(f'(x)) = \mathbb{R} \longrightarrow$ Derivable en $\mathbb{R}$

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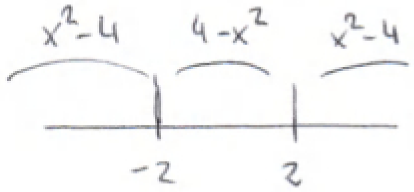
$$\left.\begin{array}{l} f(0)=0 \\ f(3)=0 \end{array}\right\} f(0)=f(3) \Rightarrow \exists c \in]0,3[\;/ f'(c)=0$$

$$f'(x) = \frac{3x^2+2x-3}{(x^2+1)^2}$$

$$f'(x)=0 \rightarrow 3x^2+2x-3=0 \left\langle \begin{array}{l} x=0'721 \rightarrow x \in]0,3[\\ x=-1'387 \end{array}\right.$$

c) $f(x) = |x^2-4|$

$$x^2-4=0 \left\langle \begin{array}{l} x=2 \\ x=-2 \end{array}\right.$$



$$\Rightarrow f(x) = \begin{cases} x^2-4 & \text{si } x \le -2 \\ 4-x^2 & \text{si } -2 < x < 2 \\ x^2-4 & \text{si } x \geq 2 \end{cases}$$

Al estar definida $f(x)$ por funciones polinómicas, los únicos puntos en los que puede ser discontinua o no derivable será en los puntos de encuentro. Así.

En $x=-2$:

$f(-2)=0$

$$\lim_{x\to -2} f(x) \rightarrow \left\{\begin{array}{l} \lim_{x\to -2^-} x^2-4 = 0 \\ \lim_{x\to -2^+} 4-x^2=0 \end{array}\right\} \Rightarrow \lim_{x\to -2} f(x)=0 \Rightarrow \text{Continua en } x=-2$$

En $x=2$:

$f(2)=0$

$$\lim_{x\to 2} f(x) \longrightarrow \left\{\begin{array}{l} \lim_{x\to 2^-} 4-x^2=0 \\ \lim_{x\to 2} x^2-4=0 \end{array}\right\} \Rightarrow \lim_{x\to 2} f(x)=0 \Rightarrow \text{Continua en } x=2$$

Veamos la derivabilidad:

$$f'(x) = \begin{cases} 2x & \text{si } x < -2 \\ -2x & \text{si } -2 < x < 2 \\ 2x & \text{si } x > 2 \end{cases}$$

En $x = -2$:

$$\left.\begin{array}{l} f'(-2)^- = 2 \cdot (-2) = -4 \\ f'(-2)^+ = -2 \cdot (-2) = +4 \end{array}\right\} f'(-2)^- \neq f'(-2)^+ \Rightarrow \text{No derivable en } x = -2$$

En $x = 2$:

$$\left.\begin{array}{l} f'(2)^- = -2 \cdot (2) = -4 \\ f'(2)^+ = 2 \cdot 2 = 4 \end{array}\right\} f'(2)^- \neq f'(2)^+ \Rightarrow \text{No derivable en } x = 2$$

En resumen:

$f(x) = |x^2 - 4| \to$ Continua en $\mathbb{R}$

$\quad \hookrightarrow$ Derivable $\forall x \in \left]-\infty, -2\right[\cup \underbrace{\left]-2, 2\right[}_{\text{El intervalo pedido!}} \cup \left]2, +\infty\right[$ ✓

$$\left.\begin{array}{l} f(-2) = 0 \\ f(2) = 0 \end{array}\right\} \exists\, c \in \left]-2, 2\right[\,/\, f'(c) = 0$$

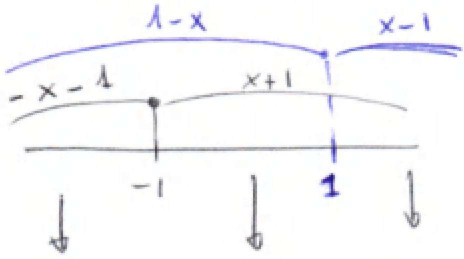
$f'(x) = -2x \quad \forall x \in \left]-2, 2\right[$

$f'(x) = 0 \to -2x = 0 \Rightarrow x = 0 \Rightarrow x \in \left]-2, 2\right[$

d) $f(x) = |x+1| + |x-1|$

$x + 1 = 0 \Rightarrow x = -1$

$x - 1 = 0 \Rightarrow x = 1$



$$\Rightarrow f(x) = \begin{cases} -2x & \text{si } x \leq -1 \\ 2 & \text{si } -1 < x \leq 1 \\ 2x & \text{si } x > 1 \end{cases}$$

$-x-1$	$1-x$	$x-1$
$-x+1$	$1+x$	$x+1$
$-2x$	2	$2x$

Al estar definida f(x) por funciones polinómicas, los únicos puntos en los que puede ser discontinua o no derivable será en los puntos de encuentro. Así:

Eu $x = -1$:

$f(-1) = 2$

$$\lim_{x \to -1} f(x) \rightarrow \left\{ \begin{array}{l} \lim\limits_{x \to -1^-} -2x = 2 \\ \lim\limits_{x \to -1^+} 2 = 2 \end{array} \right\} \Rightarrow \lim_{x \to -1} f(x) = 2 \Rightarrow \text{Continua en } x = -1$$

Eu $x = 1$:

$f(1) = 2$

$$\lim_{x \to 1} f(x) \rightarrow \left\{ \begin{array}{l} \lim\limits_{x \to 1^-} 2 = 2 \\ \lim\limits_{x \to 1^+} 2x = 2 \end{array} \right\} \Rightarrow \lim_{x \to 1} f(x) = 2 \Rightarrow \text{Continua en } x = 1$$

Veamos la derivabilidad:

$$f'(x) = \begin{cases} -2 & \text{si } x < -1 \\ 0 & \text{si } -1 < x < 1 \\ 2 & \text{si } x > 1 \end{cases}$$

Donde es fácil ver que:

$f'(-1)^- = -2 \neq f'(-1)^+ = 0 \Rightarrow$ No derivable en $x = -1$

$f'(1)^- = 0 \neq f'(1)^+ = 2 \Rightarrow$ No derivable en $x = 1$

En resumen:

$f(x) = |x+1| + |x-1| \rightarrow$ Continua en $\mathbb{R}$

$\hookrightarrow$ Derivable $\forall x \in \,]-\infty, -1[\, \cup \underbrace{]-1, 1[}_{\text{El intervalo pedido!}} \cup\,]1, +\infty[$ ✓

$$\left.\begin{array}{l} f(-1) = 2 \\ f(1) = 2 \end{array}\right\} \exists c \in \,]-1, 1[\, / \, f'(c) = 0$$

$f'(x) = 0 \quad \forall x \in \,]-1, 1[$

La derivada se anula en todos los puntos $x \in \,]-1, 1[$

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e) $f(x) = x - \frac{|x|}{x}$ $\Rightarrow$ Es obvio que $\nexists f(x)$ en $x=0$, y por tanto no se pueden satisfacer las hipótesis del teorema de Rolle en $[-1, 1]$

f) $f(x) = x^3+2x^2-2x$ $\rightarrow$ Continua y derivable en $\mathbb{R}$.

$\left.\begin{matrix} f(-3) = -3 \\ f(-2) = 4 \end{matrix}\right\}$ $f(-3) \neq f(-2)$ $\Rightarrow$ No se cumplen las condiciones del teorema

g) $f(x) = \cos x$ $\rightarrow$ $Dom(f(x)) = \mathbb{R}$ $\rightarrow$ Continua en $\mathbb{R}$

$f'(x) = -\operatorname{sen} x$ $\rightarrow$ $Dom(f'(x)) = \mathbb{R}$ $\rightarrow$ Derivable en $\mathbb{R}$

$\left.\begin{matrix} f(0) = \cos 0 = 1 \\ f(2\pi) = \cos(2\pi) = 1 \end{matrix}\right\}$ $\exists c \in]0, 2\pi[\;/\; f'(c) = 0$

$f'(x) = -\operatorname{sen} x$

$f'(x) = 0 \Rightarrow -\operatorname{sen} x = 0 \begin{cases} \rightarrow x = 0 \\ \rightarrow x = \pi \rightarrow x \in]0, 2\pi[\end{cases}$

h) $f(x) = \sqrt{1+\operatorname{sen} x}$

$(\operatorname{sen} x) \in [-1, 1] \Rightarrow (1+\operatorname{sen} x) \in [0, 2] \Rightarrow$

$\Rightarrow \exists \sqrt{1+\operatorname{sen} x} \;\; \forall x \in \mathbb{R}$ $\rightarrow$ Continua en $\mathbb{R}$

$$f'(x) = \frac{\cos x}{2\sqrt{1+\operatorname{sen} x}} \qquad 1+\operatorname{sen} x = 0$$

$$\operatorname{sen} x = -1 \to x = -\pi/2$$

$\Rightarrow \nexists\, f'(x)$ en $x = -\frac{\pi}{2} \Rightarrow$ No es derivable para $x \in]-\pi, 0[$

$\Rightarrow$ No se cumplen las condiciones del teorema

i) $f(x) = -x^4 + x^2 \to$ Continua y derivable en $\mathbb{R}$

$$\left.\begin{array}{l} f(0) = 0 \\ f(1) = 0 \end{array}\right\} \exists\, c \in]0,1[\,/\, f'(c) = 0$$

$$f'(x) = -4x^3 + 2x$$

$$f'(x) = 0 \to -4x^3 + 2x = 0 \Rightarrow 2x(-2x^2+1) = 0 \begin{cases} x = 0 \\ -2x^2+1 = 0 \begin{cases} x = 0'71 \\ x = -0'71 \end{cases} \end{cases}$$

$\Rightarrow x = 0'71 \in]0,1[$

1.3) a) $f(x) = x^3 - x \to$ Continua y Derivable en $\mathbb{R}$

$$\Rightarrow \exists\, c \in]-1,2[\,/\, f'(c) = \frac{f(2) - f(-1)}{2-(-1)}$$

$$f'(x) = 3x^2 - 1 \quad ; \quad \frac{f(2) - f(-1)}{2-(-1)} = \frac{6-0}{3} = 2$$

$$\Rightarrow 3x^2 - 1 = 2 \Rightarrow 3x^2 = 3 \begin{cases} x = -1 \\ x = +1 \to x \in]-1,2[\end{cases}$$

b) $f(x) = x^3 - 3x + 5 \rightarrow$ continua y derivable en $\mathbb{R}$.

$$\Rightarrow \exists\, c \in \left]-2,0\right[\;/\; f'(c) = \frac{f(0)-f(-2)}{0-(-2)}$$

$$f'(x) = 3x^2 - 3 \;;\; \frac{f(0)-f(-2)}{0-(-2)} = \frac{5-3}{2} = 1$$

$$\Rightarrow 3x^2 - 3 = 1 \Rightarrow 3x^2 = 4 \begin{cases} x = 1'1547 \\ x = -1'1547 \rightarrow x \in \left]-2,0\right[\end{cases}$$

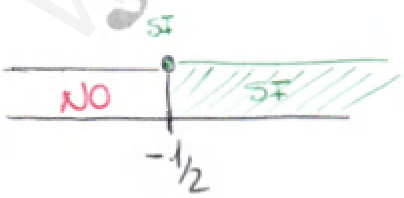
c) $f(x) = (x-2)^3 \longrightarrow$ Continua y derivable en $\mathbb{R}$.

$$\Rightarrow \exists\, c \in \left]0,3\right[\;/\; f'(c) = \frac{f(3)-f(0)}{3-0}$$

$$f'(x) = 3(x-2)^2 \;;\; \frac{f(3)-f(0)}{3-0} = \frac{1-(-8)}{3} = \frac{9}{3} = 3$$

$$\Rightarrow 3(x-2)^2 = 3 \Rightarrow (x-2)^2 = 1 \begin{cases} x-2 = 1 \Rightarrow x = 3 \\ x-2 = -1 \Rightarrow x = 1 \rightarrow x \in \left]0,3\right[\end{cases}$$

d) $f(x) = \sqrt{1+2x}$; $1+2x \geq 0 \Rightarrow 1+2x = 0 \Rightarrow x = -1/2$



$\Rightarrow \text{Dom}(f(x)) = \left[-1/2, +\infty\right[$

$\Rightarrow$ Continua en $\left[\frac{3}{2}, 4\right]$

$$f'(x) = \frac{1}{\sqrt{1+2x}}$$

$\Rightarrow \text{Dom}(f'(x)) = \left]-1/2, +\infty\right[$

$\Rightarrow$ Derivable en $\left]\frac{3}{2}, 4\right[$

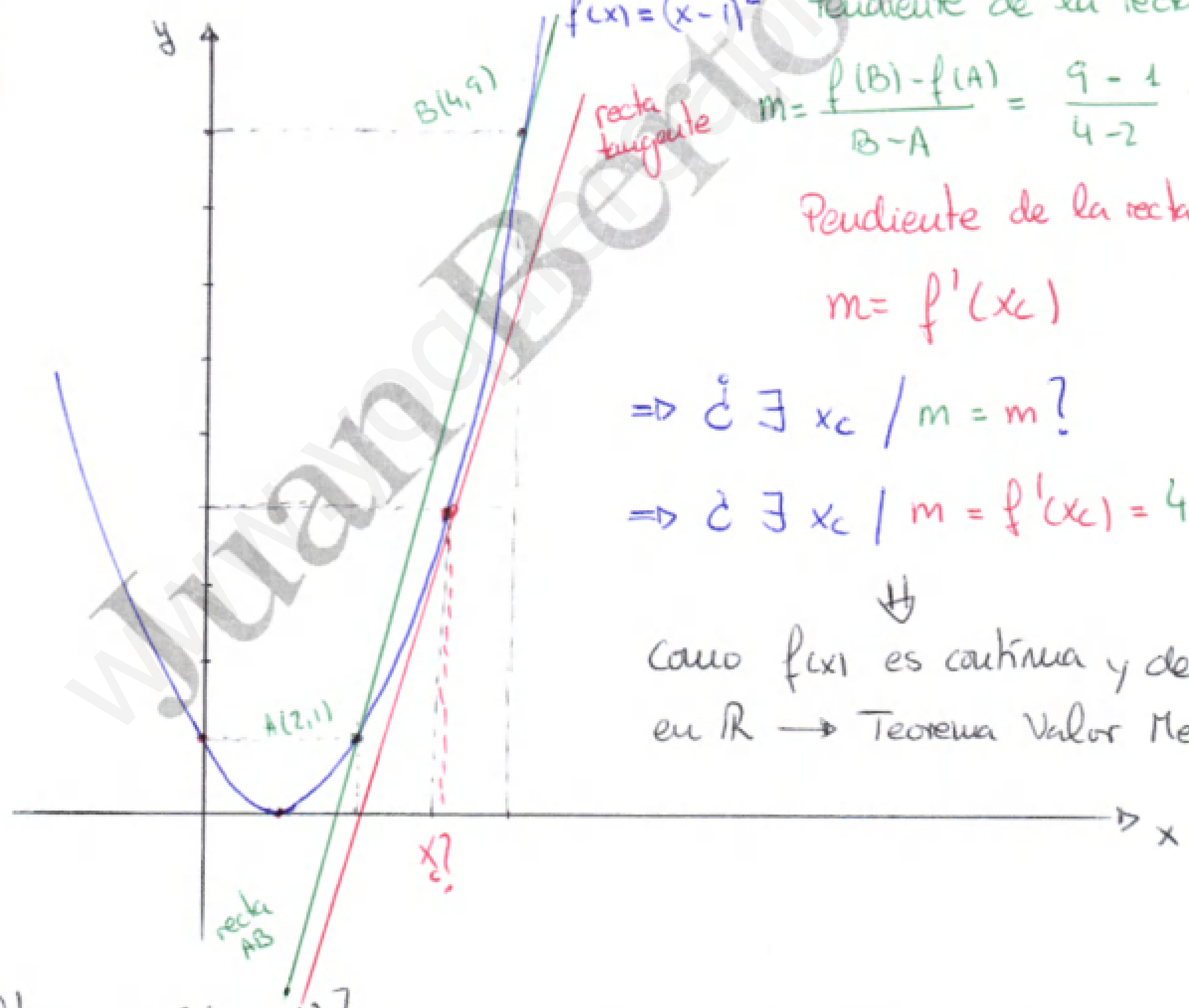
$$\Rightarrow \exists c \in \left]\tfrac{3}{2}, 4\right[\;/\; f'(c) = \frac{f(4) - f(3/2)}{4 - \frac{3}{2}}$$

$$f'(x) = \frac{1}{\sqrt{1+2x}} \quad ; \quad \frac{f(4) - f(3/2)}{4 - 3/2} = \frac{3-2}{5/2} = \frac{1}{5/2} = \frac{2}{5}$$

$$\Rightarrow \frac{1}{\sqrt{1+2x}} = \frac{2}{5} \Rightarrow \sqrt{1+2x} = \frac{5}{2} \Rightarrow 1+2x = \frac{25}{4} \Rightarrow$$

$$\Rightarrow x = \frac{21}{8} \in \left]\tfrac{3}{2}, 4\right[$$

1.4)



Pendiente de la recta AB

$$m = \frac{f(B) - f(A)}{B - A} = \frac{9-1}{4-2} = 4$$

Pendiente de la recta tangente

$$m = f'(x_c)$$

$$\Rightarrow ¿\exists x_c \,/\, m = m?$$

$$\Rightarrow ¿\exists x_c \,/\, m = f'(x_c) = 4?$$

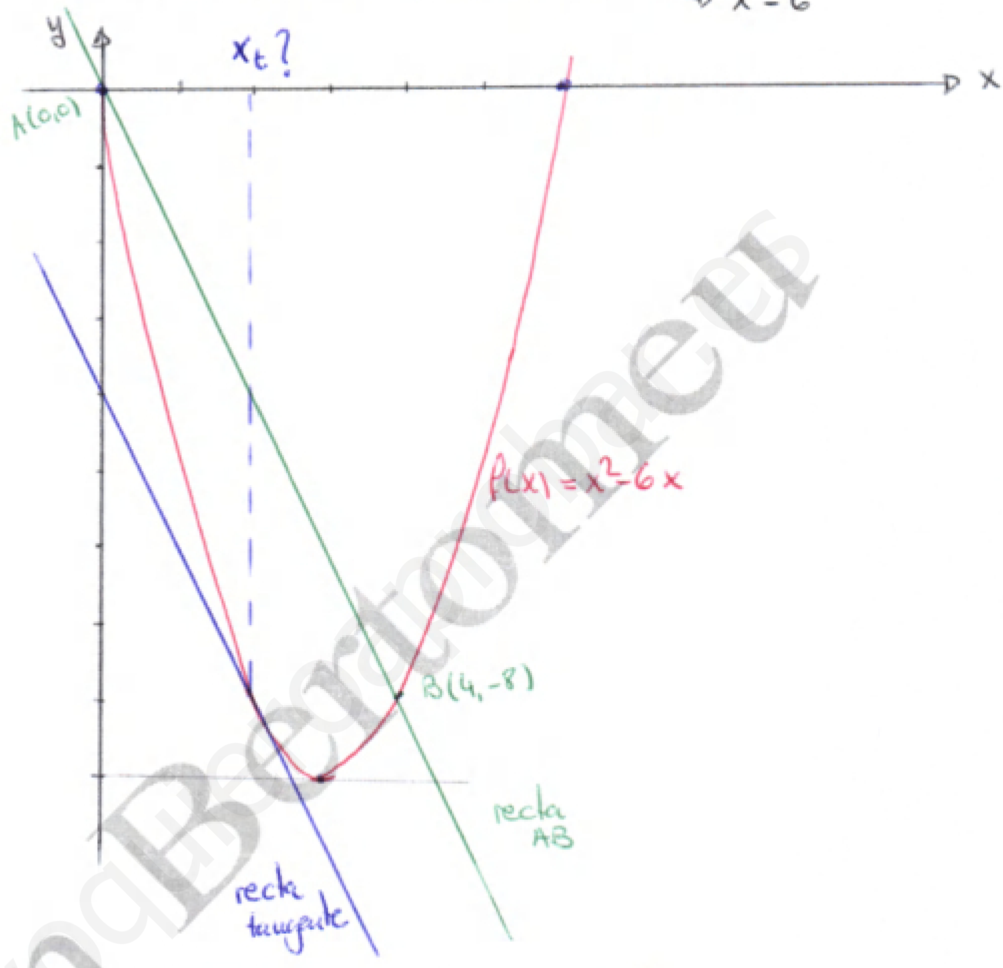
$\Downarrow$

Como $f(x)$ es continua y derivable en $\mathbb{R}$ $\longrightarrow$ Teorema Valor Medio $[2,4]$

$$\left.\begin{array}{l} f'(x) = 2(x-1) \\ m = 4 \end{array}\right\} \; 2(x-1) = 4 \Rightarrow x = 3 \in \left]2, 4\right[$$

1.5) $f(x) = x^2 - 6x$ → Parábola Cóncava
→ $X_{vértice} = 3$
→ Puntos de corte $\Rightarrow x^2 - 6x = 0$ → $x = 0$, $x = 6$

x	$y = x^2 - 6x$
0	0
3	-9
4	-8
6	0



Pendiente recta AB $\rightarrow m = \frac{f(B) - f(A)}{B - A} = \frac{-8 - 0}{4 - 0} = -2$

Pendiente recta tangente $\rightarrow m = f'(x_t)$

$\Big\}\ f'(x_t) = -2$

$$\left.\begin{array}{l} f'(x) = 2x - 6 \\ f'(x_t) = -2 \end{array}\right\} \quad 2x - 6 = -2 \Rightarrow 2x = 4 \Rightarrow x_t = 2 \in \,]0,4[$$

1.6) Tenemos que probar que $f'(x)=0$ para cualquier intervalo real. Tenemos que saber:

$$y = \sec x \longrightarrow y' = \sec x \cdot tg\, x$$

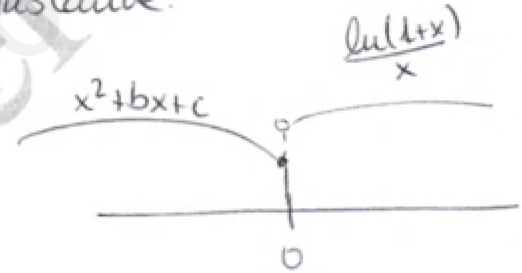
$$y = tg\, x \longrightarrow y' = \frac{1}{\cos^2 x}$$

Así, como $f(x) = \sec^2 x - tg^2 x$, se tendrá:

$$f'(x) = 2\sec x \cdot \sec x \cdot tg\, x - 2\, tg\, x \cdot \frac{1}{\cos^2 x} =$$

$$= 2\sec^2 x \cdot tg\, x - 2\sec^2 x \cdot tg\, x = 0 \quad \left(\sec x = \frac{1}{\cos x} \right)$$

$\Rightarrow$ Como $f'(x)=0 \Rightarrow f(x)$ es constante.



1.7) $f(x) = \begin{cases} x^2+bx+c & \text{si } x \le 0 \\ \dfrac{\ln(1+x)}{x} & \text{si } x > 0 \end{cases}$

Si $x<0 \rightarrow f(x) = x^2+bx+c \rightarrow$ Continua y Derivable en $\mathbb{R}$

Si $x>0 \rightarrow f(x) = \dfrac{\ln(1+x)}{x} \rightarrow \begin{cases} 1+x>0 \rightarrow x>-1 \\ x \neq 0 \end{cases} \Rightarrow$ Continua $]0, +\infty[$

$$\rightarrow f'(x) = \frac{\frac{1}{1+x} \cdot x - \ln(1+x)}{x^2} = \frac{x - (1+x)\cdot \ln(1+x)}{x^2(1+x)}$$

$$\begin{cases} 1+x>0 \rightarrow x>-1 \\ x^2(1+x) \neq 0 \begin{array}{l} \rightarrow x \neq 0 \\ \rightarrow x \neq -1 \end{array} \end{cases} \Rightarrow \text{Derivable }]0, +\infty[$$

Si $x = 0$: Continuidad

$$f(0) = c$$

$$\lim_{x \to 0} f(x) \longrightarrow \begin{cases} \lim_{x \to 0^-} x^2 + bx + c = c \\ \lim_{x \to 0^+} \dfrac{\ln(1+x)}{x} = \left[\dfrac{0}{0}\right] \overset{L'H}{=} \lim_{x \to 0^+} \dfrac{1}{1+x} = 1 \end{cases}$$

$$\Rightarrow \boxed{\text{Si } c = 1} \Rightarrow \lim_{x \to 0} f(x) = 1 = f(0) \Rightarrow \text{Continua en } x = 0$$

Derivabilidad:

$$f'(x) = \begin{cases} 2x + b & \text{si } x \leq 0 \\ \dfrac{x - (1+x)\ln(1+x)}{x^2(1+x)} & \text{si } x > 0 \end{cases}$$

$$\left.\begin{aligned} f'(0)^- &= 2 \cdot 0 + b = b \\ f'(0)^+ &= \frac{0}{0} \end{aligned}\right\}$$

Como al calcular $f'(0)^+$ se presenta una indeterminación (obvio, pues ya habíamos razonado que $f(x) = \dfrac{\ln(1+x)}{x}$ no era derivable en $x=0$), tendremos que aplicar la definición de derivada lateral.

$$f'(0)^+ = \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^+} \frac{\frac{\ln(1+x)}{x} - 1}{x} = \lim_{x \to 0^+} \frac{\ln(1+x) - x}{x^2} = \left[\frac{0}{0}\right]$$

$$\overset{L'H}{=} \lim_{x \to 0^+} \frac{\frac{1}{1+x} - 1}{2x} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x \to 0^+} \frac{\frac{-1}{1+x^2}}{2} = -1/2$$

Por lo tanto, si $b = -1/2$ la función será derivable en $x=0$

c) Hemos visto que para $c = 1$, la función es continua en $x=0$, y por tanto continua en $\mathbb{R}$.

También hemos visto que la función será derivable en $]0, +\infty[$ independientemente del valor de b.

Por tanto, se satisfacen las hipótesis del teorema del Valor medio en $[0, e-1]$, y el teorema nos asegura que existe $c \in]0, e-1[\,/\, f'(c) = \dfrac{f(e-1) - f(0)}{e-1-0}$. Veamoslo:

$$f(e-1) = \frac{\ln(\cancel{1} + e \cancel{-1})}{e-1} = \frac{1}{e-1}$$

$$f(0) = c = 1$$

$$\Rightarrow f'(c) = \frac{\frac{1}{e-1} - 1}{e-1} = \frac{\frac{1-e+1}{e-1}}{e-1} = \frac{2-e}{(e-1)^2}$$, como queríamos demostrar

2) REGLA DE L'HOPITAL

2.1) 1) $\lim\limits_{x \to 0} \dfrac{e^x - 1}{x} = \left[\dfrac{0}{0}\right] \overset{L'H}{=} \lim\limits_{x \to 0} \dfrac{e^x}{1} = 1$

2) $\lim\limits_{x \to 0} \dfrac{5x - \operatorname{sen} x}{2x} = \left[\dfrac{0}{0}\right] \overset{L'H}{=} \lim\limits_{x \to 0} \dfrac{5 - \cos x}{2} = \dfrac{4}{2} = 2$

3) $\lim\limits_{x \to 0} \dfrac{\operatorname{sen} x}{x} = \left[\dfrac{0}{0}\right] \overset{L'H}{=} \lim\limits_{x \to 0} \dfrac{\cos x}{1} = 1$

4) $$\lim_{x\to 0} \frac{1-\cos x}{x} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x\to 0} \frac{\operatorname{sen} x}{1} = 0$$

5) $$\lim_{x\to 1} \frac{\ln x}{x^2-5x+4} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x\to 1} \frac{1/x}{2x-5} = -1/3$$

6) $$\lim_{x\to 0} \frac{x-\operatorname{sen} x}{\operatorname{tg} x - x} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x\to 0} \frac{1-\cos x}{\cancel{1}+\operatorname{tg}^2 x \cancel{-1}} = \left[\frac{0}{0}\right] \overset{L'H}{=}$$

$$= \lim_{x\to 0} \frac{\operatorname{sen} x}{2\operatorname{tg} x\,(1+\operatorname{tg}^2 x)} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{\cancel{\operatorname{sen} x}}{2\cdot \frac{\cancel{\operatorname{sen} x}}{\cos x}(1+\operatorname{tg}^2 x)} =$$

$$= \lim_{x\to 0} \frac{\cos x}{2(1+\operatorname{tg}^2 x)} = \frac{1}{2(1+0)} = \frac{1}{2}$$

7) $$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x\to 0} \frac{\operatorname{sen} x}{2x} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x\to 0} \frac{\cos x}{2} = \frac{1}{2}$$

8) $$\lim_{x\to \infty} \frac{\sqrt{1+x}}{x} = 0,$$ porque el "∞" del denominador es de mayor orden. No obstante, también se puede aplicar L'Hôpital.

$$\lim_{x\to \infty} \frac{\sqrt{1+x}}{x} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty} \frac{\frac{1}{2\sqrt{1+x}}}{1} = \frac{0}{1} = 0$$

9) $$\lim_{x\to\infty} \frac{\ln x}{x^2} = 0,$$ porque el "∞" del denominador es de mayor orden. No obstante se puede ver con L'Hôpital.

$$\lim_{x\to\infty} \frac{\ln x}{x^2} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty} \frac{1/x}{2x} = \lim_{x\to\infty} \frac{1}{2x^2} = 0$$

10) $\lim\limits_{x \to \infty} \dfrac{e^x}{x^3} = \infty$, porque el "$\infty$" del numerador es de mayor orden. No obstante, apliquemos L'Hôpital.

$$\hookrightarrow \lim_{x \to \infty} \frac{e^x}{x^3} = \left[\frac{\infty}{\infty}\right] \overset{L'H}{=} \lim_{x \to \infty} \frac{e^x}{3x^2} = \left[\frac{\infty}{\infty}\right] \overset{L'H}{=} \lim_{x \to \infty} \frac{e^x}{6x} = \left[\frac{\infty}{\infty}\right] =$$

$$\overset{L'H}{=} \lim_{x \to \infty} \frac{e^x}{6} = \infty$$

11) $\lim\limits_{x \to \infty} \dfrac{x^3}{e^{3x}} = 0$, porque el "$\infty$" del denominador es de mayor orden. No obstante, apliquemos L'Hôpital.

$$\hookrightarrow \lim_{x \to \infty} \frac{x^3}{e^{3x}} = \left[\frac{\infty}{\infty}\right] \overset{L'H}{=} \lim_{x \to \infty} \frac{3x^2}{3e^{3x}} = \left[\frac{\infty}{\infty}\right] \overset{L'H}{=} \lim_{x \to \infty} \frac{6x}{9e^{3x}} = \left[\frac{\infty}{\infty}\right] =$$

$$\overset{L'H}{=} \lim_{x \to \infty} \frac{6}{27e^{3x}} = 0$$

12) $\lim\limits_{x \to \infty} \dfrac{3x^2-6x}{x^2-x-2} = \lim\limits_{x \to \infty} \dfrac{3\cancel{x^2}}{\cancel{x^2}} = 3$

$$\hookrightarrow \lim_{x \to \infty} \frac{3x^2-6x}{x^2-x-2} = \left[\frac{\infty}{\infty}\right] \overset{L'H}{=} \lim_{x \to \infty} \frac{6x-6}{2x-1} = \left[\frac{\infty}{\infty}\right] \overset{L'H}{=} \lim_{x \to \infty} \frac{6}{2} = 3$$

13) $\lim\limits_{x \to \infty} \dfrac{\ln x}{x} = 0$, por orden de magnitud de los infinitos.

$$\hookrightarrow \lim_{x \to \infty} \frac{\ln x}{x} = \left[\frac{\infty}{\infty}\right] \overset{L'H}{=} \lim_{x \to \infty} \frac{1/x}{1} = \frac{0}{1} = 0$$

14) $\lim\limits_{x \to 0^+} x \cdot \ln x = [0 \cdot (-\infty)] = \lim\limits_{x \to 0^+} \dfrac{\ln x}{1/x} = \left[\dfrac{\infty}{\infty}\right] \overset{L'H}{=} \lim\limits_{x \to 0^+} \dfrac{1/x}{-1/x^2} =$

$$= \lim_{x \to 0^+} -x = 0$$

15) $$\lim_{x\to 0}\left(\frac{1}{\operatorname{sen} x}-\frac{1}{x}\right)=[\infty-\infty]=\lim_{x\to 0}\frac{x-\operatorname{sen} x}{x\operatorname{sen} x}=\left[\frac{0}{0}\right]=$$

$$\stackrel{L'H}{=}\lim_{x\to 0}\frac{1-\cos x}{1\operatorname{sen} x+x\cos x}=\left[\frac{0}{0}\right]\stackrel{L'H}{=}\lim_{x\to 0}\frac{\operatorname{sen} x}{\cos x+\cos x+1\cdot(-\operatorname{sen} x)}=\frac{0}{2}=0$$

16) $$\lim_{x\to\infty}\left(e^{-x}\cdot \ln^2 x\right)=\lim_{x\to\infty}\frac{\ln^2 x}{e^x}=0,$$ por orden de magnitud de los infinitos.

$$\hookrightarrow \lim_{x\to\infty}\left(e^{-x}\cdot\ln^2 x\right)=[0\cdot\infty]=\lim_{x\to\infty}\frac{\ln^2 x}{e^x}=\left[\frac{\infty}{\infty}\right]\stackrel{L'H}{=}$$

$$=\lim_{x\to\infty}\frac{2\ln x\cdot 1/x}{e^x}=\lim_{x\to\infty}\frac{2\ln x}{x\cdot e^x}=\left[\frac{\infty}{\infty}\right]\stackrel{L'H}{=}\lim_{x\to\infty}\frac{2\,1/x}{e^x+xe^x}=$$

$$=\lim_{x\to\infty}\frac{2}{xe^x+x^2e^x}=\left[\frac{2}{\infty}\right]=0$$

17) $$\lim_{x\to\infty}\left(e^{-x}\cdot\sqrt{x}\right)=\lim_{x\to\infty}\frac{\sqrt{x}}{e^x}=0,$$ por orden de magnitud de los infinitos.

$$\hookrightarrow \lim_{x\to\infty}\left(e^{-x}\cdot\sqrt{x}\right)=[0\cdot\infty]=\lim_{x\to\infty}\frac{\sqrt{x}}{e^x}=\left[\frac{\infty}{\infty}\right]\stackrel{L'H}{=}\lim_{x\to\infty}\frac{1/2\sqrt{x}}{e^x}=$$

$$=\lim_{x\to\infty}\frac{1}{2\sqrt{x}\cdot e^x}=\left[\frac{1}{\infty}\right]=0$$

18) $$\lim_{x\to\infty}\left(e^x\cdot\frac{1}{x^2}\right)=\lim_{x\to\infty}\frac{e^x}{x^2}=\infty,$$ por orden de magnitud de los infinitos.

$$\hookrightarrow \lim_{x\to\infty}\frac{e^x}{x^2}=\left[\frac{\infty}{\infty}\right]\stackrel{L'H}{=}\lim_{x\to\infty}\frac{e^x}{2x}=\left[\frac{\infty}{\infty}\right]\stackrel{L'H}{=}\lim_{x\to\infty}\frac{e^x}{2}=\infty$$

19) $\lim\limits_{x\to\infty} x\cdot\left(e^{1/x}-1\right) = [\infty\cdot 0] = \lim\limits_{x\to\infty} \dfrac{e^{1/x}-1}{1/x} = \left[\dfrac{0}{0}\right] \overset{L'H}{=}$

$= \lim\limits_{x\to\infty} \dfrac{e^{1/x}\cdot\cancel{(-1/x^2)}}{\cancel{(-1/x^2)}} = \lim\limits_{x\to\infty} e^{1/x} = e^0 = 1$

20) $\lim\limits_{x\to\frac{\pi}{4}} (1-\operatorname{tg} x)\sec(2x) = [0\cdot\infty] = \lim\limits_{x\to\frac{\pi}{4}} \dfrac{1-\operatorname{tg} x}{\cos(2x)} = \left[\dfrac{0}{0}\right] \overset{L'H}{=}$

$\sec x = \dfrac{1}{\cos x}$

$= \lim\limits_{x\to\frac{\pi}{4}} \dfrac{+(1+\operatorname{tg}^2 x)}{+\operatorname{sen}(2x)\cdot 2} = \dfrac{2}{2} = 1$

21) $\lim\limits_{x\to 0^+} x^{\operatorname{sen} x} = [0^0] \Rightarrow A = \lim\limits_{x\to 0^+} x^{\operatorname{sen} x}$

$\ln A = \ln \lim\limits_{x\to 0^+} x^{\operatorname{sen} x} = \lim\limits_{x\to 0^+} \ln x^{\operatorname{sen} x} = \lim\limits_{x\to 0^+} \operatorname{sen} x\cdot\ln x$

$\lim\limits_{x\to 0^+} \operatorname{sen} x\cdot\ln x = [0\cdot\infty] = \lim\limits_{x\to 0^+} \dfrac{\ln x}{1/\operatorname{sen} x} = \left[\dfrac{-\infty}{\infty}\right] \overset{L'H}{=}$

$= \lim\limits_{x\to 0^+} \dfrac{1/x}{\frac{-\cos x}{\operatorname{sen}^2 x}} = \lim\limits_{x\to 0^+} \dfrac{\operatorname{sen}^2 x}{-x\cdot\cos x} = \left[\dfrac{0}{0}\right] \overset{L'H}{=} \lim\limits_{x\to 0^+} \dfrac{2\operatorname{sen} x\cos x}{-1\cos x + x\operatorname{sen} x} = \dfrac{0}{-1} = 0$

$\Rightarrow \ln A = 0 \Rightarrow A = \lim\limits_{x\to 0^+} x^{\operatorname{sen} x} = e^0 = 1$

22) $\lim_{x\to 0^+} x^x = [0^0] \Rightarrow A = \lim_{x\to 0^+} x^x$

$\ln A = \ln \lim_{x\to 0^+} x^x = \lim_{x\to 0^+} \ln x^x = \lim_{x\to 0^+} x\cdot \ln x$

$\lim_{x\to 0^+} x\cdot \ln x = [0\cdot(-\infty)] = \lim_{x\to 0^+} \frac{\ln x}{1/x} = \left[\frac{-\infty}{\infty}\right] \overset{L'H}{=} \lim_{x\to 0^+} \frac{1/x}{-1/x^2} =$

$= \lim_{x\to 0^+} -x = 0$

$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x\to 0^+} x^x = e^0 = 1$

23) $\lim_{x\to \infty} (1+x)^{1/x} = [\infty^0] \Rightarrow A = \lim_{x\to\infty} (1+x)^{1/x}$

$\ln A = \ln \lim_{x\to\infty} (1+x)^{1/x} = \lim_{x\to\infty} \ln(1+x)^{1/x} = \lim_{x\to\infty} \frac{1}{x}\cdot \ln(1+x)$

$\lim_{x\to\infty} \frac{1}{x}\cdot \ln(1+x) = [0\cdot\infty] = \lim_{x\to\infty} \frac{\ln(1+x)}{x} = \left[\frac{\infty}{\infty}\right] \overset{L'H}{=} \lim_{x\to\infty} \frac{1}{1+x} = 0$

$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x\to\infty} (1+x)^{1/x} = e^0 = 1$

24) $\lim_{x\to\infty} x^{1/x} = [\infty^0] \Rightarrow A = \lim_{x\to\infty} x^{1/x}$

$\ln A = \ln \lim_{x\to\infty} x^{1/x} = \lim_{x\to\infty} \ln x^{1/x} = \lim_{x\to\infty} \frac{1}{x} \ln x$

$\lim_{x\to\infty} \frac{1}{x}\cdot \ln x = \lim_{x\to\infty} \frac{\ln x}{x} = \left[\frac{\infty}{\infty}\right] \overset{L'H}{=} \lim_{x\to\infty} \frac{1}{x} = 0$

$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x\to\infty} x^{1/x} = e^0 = 1$

25) $\lim\limits_{x\to 0} (\cos x)^{1/x} = [1^{\infty}] \Rightarrow A = \lim\limits_{x\to 0} (\cos x)^{1/x}$

$\ln A = \ln \lim\limits_{x\to 0} (\cos x)^{1/x} = \lim\limits_{x\to 0} \ln(\cos x)^{1/x} = \lim\limits_{x\to 0} \frac{1}{x}\cdot \ln(\cos x)$

$\lim\limits_{x\to 0} \frac{1}{x}\cdot \ln(\cos x) = [\infty \cdot 0] = \lim\limits_{x\to 0} \frac{\ln(\cos x)}{x} = \left[\frac{0}{0}\right] \overset{L'H}{=}$

$= \lim\limits_{x\to 0} \frac{\frac{1}{\cos x}(-\operatorname{sen} x)}{1} = \lim\limits_{x\to 0} -\frac{\operatorname{sen} x}{\cos x} = 0$

$\Rightarrow \ln A = 0 \Rightarrow A = \lim\limits_{x\to 0} (\cos x)^{1/x} = e^0 = 1$

26) $\lim\limits_{x\to 0^+} \left(\frac{1}{x}\right)^{tg\, x} = [\infty^0] \Rightarrow A = \lim\limits_{x\to 0^+} \left(\frac{1}{x}\right)^{tg\, x}$

$\ln A = \ln \lim\limits_{x\to 0^+} \left(\frac{1}{x}\right)^{tg\, x} = \lim\limits_{x\to 0^+} \ln\left(\frac{1}{x}\right)^{tg\, x} = \lim\limits_{x\to 0^+} tg\, x \cdot \ln\left(\frac{1}{x}\right)$

$\lim\limits_{x\to 0^+} tg\, x\cdot \ln\left(\frac{1}{x}\right) = [0\cdot\infty] = \lim\limits_{x\to 0^+} \frac{\ln(1/x)}{\frac{1}{tg\, x}} = \left[\frac{\infty}{\infty}\right] \overset{L'H}{=}$

$= \lim\limits_{x\to 0^+} \frac{\frac{1}{\cancel{1/x}}\cdot\frac{+1}{x^{\cancel{2}}}}{\frac{+(1+tg^2 x)}{tg^2 x}} = \lim\limits_{x\to 0^+} \frac{tg^2 x}{x(1+tg^2 x)} = \lim\limits_{x\to 0^+} \frac{\frac{\operatorname{sen}^2 x}{\cos^2 x}}{x\cdot\left(1+\frac{\operatorname{sen}^2 x}{\cos^2 x}\right)} =$

$= \lim\limits_{x\to 0^+} \frac{\frac{\operatorname{sen}^2 x}{\cancel{\cos^2 x}}}{x\cdot\left(\frac{\cos^2 x+\operatorname{sen}^2 x}{\cancel{\cos^2 x}}\right)} = \lim\limits_{x\to 0^+} \frac{\operatorname{sen}^2 x}{x} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim\limits_{x\to 0^+} 2\operatorname{sen} x\cos x = 0$

$\cos^2 x+\operatorname{sen}^2 x = 1$

$\Rightarrow \ln A = 0 \Rightarrow A = \lim\limits_{x\to 0^+} \left(\frac{1}{x}\right)^{tg\, x} = e^0 = 1$

27) $$\lim_{x\to 0} \frac{x\cos x - \text{sen}\,x}{x^3} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x\to 0} \frac{\cancel{\cos x} - x\,\text{sen}\,x \cancel{- \cos x}}{3x^2} =$$

$$= \lim_{x\to 0} -\frac{\text{sen}\,x}{3x} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x\to 0} \frac{-\cos x}{3} = -\frac{1}{3}$$

28) $$\lim_{x\to 0} \frac{e^x - e^{-x} - 4x}{x - \text{sen}\,x} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x\to 0} \frac{e^x + e^{-x} - 4}{1 - \cos x} = \left[\frac{-2}{0}\right] = -\infty$$

$$\Rightarrow \begin{cases} \lim\limits_{x\to 0^-} f(x) = \dfrac{-}{+} = -\infty \\ \lim\limits_{x\to 0^+} f(x) = \dfrac{-}{+} = -\infty \end{cases} \Rightarrow \lim_{x\to 0} \frac{e^x - e^{-x} - 4x}{x - \text{sen}\,x} = -\infty$$

29) $$\lim_{x\to \frac{\pi}{2}} (1+2\cos x)^{1/\cos x} = [1^\infty] \Rightarrow A = \lim_{x\to \frac{\pi}{2}} (1+2\cos x)^{1/\cos x}$$

$$\ln A = \ln \lim_{x\to \frac{\pi}{2}} (1+2\cos x)^{1/\cos x} = \lim_{x\to \frac{\pi}{2}} \ln (1+2\cos x)^{1/\cos x} = \lim_{x\to \frac{\pi}{2}} \frac{1}{\cos x} \cdot \ln(1+2\cos x)$$

$$\lim_{x\to \frac{\pi}{2}} \frac{\ln(1+2\cos x)}{\cos x} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x\to \frac{\pi}{2}} \frac{\frac{1}{1+2\cos x}\cdot(-2\,\cancel{\text{sen}\,x})}{-\cancel{\text{sen}\,x}} =$$

$$= \lim_{x\to \frac{\pi}{2}} \frac{2}{1+2\cos x} = 2$$

$$\Rightarrow \ln A = 2 \Rightarrow A = \lim_{x\to \frac{\pi}{2}} (1+2\cos x)^{1/\cos x} = e^2$$

30) $$\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\ln(1+x)}\right)=[\infty-\infty]=\lim_{x\to 0}\frac{\ln(1+x)-x}{x\cdot\ln(1+x)}=\left[\frac{0}{0}\right]\overset{L'H}{=}$$

$$=\lim_{x\to 0}\frac{\frac{1}{1+x}-1}{\ln(1+x)+\frac{x}{1+x}}=\lim_{x\to 0}\frac{\frac{\cancel{1}-\cancel{1}-x}{\cancel{1+x}}}{\frac{(1+x)\ln(1+x)+x}{\cancel{1+x}}}=\lim_{x\to 0}\frac{-x}{(1+x)(\ln(1+x))+x}=$$

$$=\left[\frac{0}{0}\right]\overset{L'H}{=}\lim_{x\to 0}\frac{-1}{\ln(1+x)+\underbrace{\frac{(1+x)}{(1+x)}}_{1}+1}=-\frac{1}{2}$$

31) $$\lim_{x\to 0}\frac{2x\,\text{sen}\,x}{1-\cos x}=\left[\frac{0}{0}\right]\overset{L'H}{=}\lim_{x\to 0}\frac{2\,\text{sen}\,x+2x\cos x}{\text{sen}\,x}=\left[\frac{0}{0}\right]\overset{L'H}{=}$$

$$=\lim_{x\to 0}\frac{2\cos x+2\cos x-2x\,\text{sen}\,x}{\cos x}=\frac{4}{1}=4$$

32) $$\lim_{x\to 0}\frac{\ln(\cos x)-1+\cos x}{x^2}=\left[\frac{0}{0}\right]\overset{L'H}{=}\lim_{x\to 0}\frac{\frac{1}{\cos x}\cdot(-\text{sen}\,x)-\text{sen}\,x}{2x}=$$

$$=\lim_{x\to 0}\frac{\text{sen}\,x+\text{sen}\,x\cos x}{-2x\cos x}=\left[\frac{0}{0}\right]\overset{L'H}{=}\lim_{x\to 0}\frac{\cos x+\cos^2 x-\text{sen}^2 x}{-2\cos x+2x\,\text{sen}\,x}=\frac{2}{-2}=-1$$

33) $$\lim_{x\to 0}\frac{\ln(\cos 3x)}{\ln(\cos 2x)}=\left[\frac{0}{0}\right]\overset{L'H}{=}\lim_{x\to 0}\frac{\frac{1}{\cos 3x}\cdot -\text{sen}\,3x\cdot 3}{\frac{1}{\cos 2x}\cdot -\text{sen}\,2x\cdot 2}=$$

$$=\lim_{x\to 0}\frac{3}{2}\cdot\frac{\cos 2x}{\cos 3x}\cdot\frac{\text{sen}\,3x}{\text{sen}\,2x}=\frac{3}{2}\cdot 1\cdot\underbrace{\lim_{x\to 0}\frac{\text{sen}\,3x}{\text{sen}\,2x}}_{\frac{0}{0}}\overset{L'H}{=}$$

$$=\frac{3}{2}\cdot 1\cdot\lim_{x\to 0}\frac{\cos 3x\cdot 3}{\cos 2x\cdot 2}=\frac{9}{4}$$

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34) $$\lim_{x\to 0} \frac{e^x - x - \cos x}{\operatorname{sen}^2 x} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x\to 0} \frac{e^x - 1 + \operatorname{sen} x}{2 \operatorname{sen} x \cos x} = \left[\frac{0}{0}\right] =$$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{e^x + \cos x}{2\left[\cos^2 x - \operatorname{sen}^2 x\right]} = \frac{2}{2} = 1$$

35) $$\lim_{x\to 0} \frac{(e^x - e^{-x})^2}{x^2} = \left[\frac{0}{0}\right] \underset{\substack{\downarrow \\ \text{solo} \\ \text{operaciones}}}{=} \lim_{x\to 0} \frac{e^{2x} - 2e^x \cdot e^{-x} + e^{-2x}}{x^2} =$$

$$= \lim_{x\to 0} \frac{e^{2x} + e^{-2x} - 2}{x^2} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x\to 0} \frac{\cancel{2}e^{2x} - \cancel{2}e^{-2x}}{\cancel{2}x} = \lim_{x\to 0} \frac{e^{2x} - e^{-2x}}{x} =$$

$$= \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x\to 0} 2e^{2x} + 2e^{-2x} = 2 + 2 = 4$$

36) $$\lim_{x\to 0} \frac{x \operatorname{sen} x}{1 - \cos x} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x\to 0} \frac{\operatorname{sen} x + x\cos x}{+\operatorname{sen} x} = \left[\frac{0}{0}\right] \overset{L'H}{=}$$

$$= \lim_{x\to 0} \frac{\cos x + \cos x - x \operatorname{sen} x}{\cos x} = \frac{2}{1} = 2$$

37) $$\lim_{x\to 0} \frac{\operatorname{sen} x - x}{\operatorname{tg} x - x} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x\to 0} \frac{\cos x - 1}{\cancel{1} + \operatorname{tg}^2 x - \cancel{1}} = \left[\frac{0}{0}\right] \overset{L'H}{=}$$

$$= \lim_{x\to 0} \frac{-\operatorname{sen} x}{2\operatorname{tg} x (1 + \operatorname{tg}^2 x)} = \lim_{x\to 0} \frac{-\cancel{\operatorname{sen} x}}{2 \cdot \frac{\cancel{\operatorname{sen} x}}{\cos x} (1 + \operatorname{tg}^2 x)} =$$

$$= \lim_{x\to 0} \frac{-\cos x}{2(1 + \operatorname{tg}^2 x)} = \frac{-1}{2}$$

38) $\lim_{x \to \infty} \left[x \cdot \left(\operatorname{arctg} e^x - \frac{\pi}{2} \right) \right] = [\infty \cdot 0] = \lim_{x \to \infty} \frac{\operatorname{arctg} e^x - \frac{\pi}{2}}{1/x} = \left[\frac{0}{0}\right]$

$\overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{1+(e^x)^2} \cdot e^x}{-1/x^2} = \lim_{x \to \infty} \frac{-x^2 \cdot e^x}{1+e^{2x}} = \left[\frac{\infty}{\infty}\right] \overset{L'H}{=}$

↳ Este límite es 0 por orden de magnitud!

$= \lim_{x \to \infty} -\frac{(2x \cdot e^x + x^2 e^x)}{2e^{2x}} = \lim_{x \to \infty} \frac{\cancel{e^x}(-2x - x^2)}{2(e^x)^{\cancel{2}}} = \left[\frac{\infty}{\infty}\right] \overset{L'H}{=}$

$= \lim_{x \to \infty} \frac{-\cancel{2} - \cancel{2}x}{\cancel{2}e^x} = \lim_{x \to \infty} \frac{-1-x}{e^x} = \left[\frac{\infty}{\infty}\right] \overset{L'H}{=} \lim_{x \to \infty} \frac{-1}{e^x} = 0$

39) $\lim_{x \to \infty} \left(\sqrt{x + \sqrt{x}} - \sqrt{x} \right) = [\infty - \infty] =$

$= \lim_{x \to \infty} \frac{\left(\sqrt{x+\sqrt{x}} - \sqrt{x}\right)\left(\sqrt{x+\sqrt{x}} + \sqrt{x}\right)}{\sqrt{x+\sqrt{x}} + \sqrt{x}} = \lim_{x \to \infty} \frac{\left(\cancel{\sqrt{x+\sqrt{x}}}\right)^{\cancel{2}} - \left(\cancel{\sqrt{x}}\right)^{\cancel{2}}}{\sqrt{x+\sqrt{x}} + \sqrt{x}} =$

$= \underbrace{\lim_{x \to \infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x}} + \sqrt{x}}}_{\left[\frac{\infty}{\infty}\right]} = \lim_{x \to \infty} \frac{\sqrt{x}}{2\sqrt{x}} = \frac{1}{2}$ Cogiendo los infinitos más altos.

$\left[\frac{\infty}{\infty}\right] \to$ Por L'Hôpital saldría demasiado complejo al ser las derivadas muy largas. No lo hago!!

40) $\lim_{x \to 0} \left(\frac{x^2+2}{2-2\operatorname{sen} x} \right)^{2/x} = [1^\infty] \Rightarrow A = \lim_{x \to 0} \left(\frac{x^2+2}{2-2\operatorname{sen} x} \right)^{2/x}$

$\ln A = \ln \lim_{x \to 0} \left(\frac{x^2+2}{2-2\operatorname{sen} x} \right)^{2/x} = \lim_{x \to 0} \ln \left(\frac{x^2+2}{2-2\operatorname{sen} x} \right)^{2/x} = \lim_{x \to 0} \frac{2}{x} \ln \left(\frac{x^2+2}{2-2\operatorname{sen} x} \right)$

$$\lim_{x\to 0} \frac{2}{x}\ln\left(\frac{x^2+2}{2-2\operatorname{sen} x}\right) = [\infty\cdot 0] = \lim_{x\to 0}\frac{2\cdot\ln\left(\frac{x^2+2}{2-2\operatorname{sen} x}\right)}{x} = \left[\frac{0}{0}\right]$$

$$\overset{L'H}{=} \lim_{x\to 0} 2\cdot \underbrace{\frac{\cancel{2-2\operatorname{sen} x}}{x^2+2}}_{\frac{2}{2}=1}\cdot \underbrace{\frac{2x(2-2\operatorname{sen} x)-(x^2+2)\cdot(-2\cos x)}{(2-2\operatorname{sen} x)^{\cancel{2}}}}_{\frac{(-2)\cdot(-2)}{2}=2} = 1\cdot 2 = 2$$

$$\Rightarrow \ln A = 2 \Rightarrow A = \lim_{x\to 0}\left(\frac{x^2+2}{2-2\operatorname{sen} x}\right)^{2/x} = e^2$$

41) $\lim_{x\to\infty}\left(\frac{3x+1}{3x-1}\right)^x = [1^\infty] \Rightarrow A = \lim_{x\to\infty}\left(\frac{3x+1}{3x-1}\right)^x$

$$\ln A = \ln \lim_{x\to\infty}\left(\frac{3x+1}{3x-1}\right)^x = \lim_{x\to\infty}\ln\left(\frac{3x+1}{3x-1}\right)^x = \lim_{x\to\infty} x\cdot\ln\left(\frac{3x+1}{3x-1}\right)$$

$$\lim_{x\to\infty} x\cdot\ln\left(\frac{3x+1}{3x-1}\right) = [\infty\cdot 0] = \lim_{x\to\infty}\frac{\ln\left(\frac{3x+1}{3x-1}\right)}{1/x} = \left[\frac{0}{0}\right] \overset{L'H}{=}$$

$$= \lim_{x\to\infty}\frac{\cancel{\frac{3x-1}{3x+1}}\cdot\frac{3(\cancel{3x}-1)-3(\cancel{3x}+1)}{(3x-1)^{\cancel{2}}}}{-1/x^2} = \lim_{x\to\infty}\frac{6x^2}{9x^2-1} = \frac{6}{9} = \frac{2}{3}$$

$$\Rightarrow \ln A = \frac{2}{3} \Rightarrow A = \lim_{x\to\infty}\left(\frac{3x+1}{3x-1}\right)^x = e^{2/3}$$

42) $\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{x}{x-1}\right) = [\infty-\infty] = \lim_{x\to 1}\frac{x-1-x\ln x}{(x-1)\ln x} =$

$$= \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x\to 1}\frac{\cancel{1}-(\ln x+\cancel{x\cdot\tfrac{1}{x}})}{\ln x+\frac{x-1}{x}} = \lim_{x\to 1}\frac{-\ln x}{\ln x+\frac{x-1}{x}} = \left[\frac{0}{0}\right]$$

$$\overset{L'H}{=} \lim_{x\to 1} \frac{-1/x}{1/x + \frac{\cancel{x}+1\cancel{-x}}{x^2}} = \frac{-1}{1+1} = \frac{-1}{2}$$

43) $$\lim_{x\to \frac{\pi}{2}^-} (tg\, x)^{\cos x} = [\infty^0] \Rightarrow A = \lim_{x\to \frac{\pi}{2}^-} (tg\, x)^{\cos x}$$

$$\ln A = \ln \lim_{x\to \frac{\pi}{2}^-} (tg\, x)^{\cos x} = \lim_{x\to \frac{\pi}{2}^-} \ln (tg\, x)^{\cos x} = \lim_{x\to \frac{\pi}{2}^-} \cos x \cdot \ln (tg x)$$

$$\lim_{x\to \frac{\pi}{2}^-} \cos x \cdot \ln(tg x) = [0\cdot\infty] = \lim_{x\to \frac{\pi}{2}^-} \frac{\ln(tg x)}{1/\cos x} = \left[\frac{\infty}{\infty}\right] \overset{L'H}{=}$$

$$= \lim_{x\to \frac{\pi}{2}^-} \frac{\frac{\cos x}{\operatorname{sen} x}\cdot(1+tg^2 x)}{\frac{\operatorname{sen} x}{\cos^2 x}} = \lim_{x\to \frac{\pi}{2}^-} \frac{\frac{\cos x}{\operatorname{sen} x}\cdot \frac{\overbrace{(\cos^2 x + \operatorname{sen}^2 x)}^{\to 1}}{\cancel{\cos^2 x}}}{\frac{\operatorname{sen} x}{\cancel{\cos^2 x}}} =$$

$$= \lim_{x\to \frac{\pi}{2}^-} \frac{\cos x}{\operatorname{sen}^2 x} = \frac{0}{1} = 0$$

$$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x\to \frac{\pi}{2}^-} (tg x)^{\cos x} = e^0 = 1$$

44) $$\lim_{x\to 0} (\cos(2x))^{1/\operatorname{sen} x^2} = [1^\infty] \Rightarrow A = \lim_{x\to 0} (\cos(2x))^{\frac{1}{\operatorname{sen} x^2}}$$

$$\ln A = \ln \lim_{x\to 0} (\cos(2x))^{\frac{1}{\operatorname{sen} x^2}} = \lim_{x\to 0} \ln (\cos(2x))^{\frac{1}{\operatorname{sen} x^2}} = \lim_{x\to 0} \frac{1}{\operatorname{sen} x^2}\cdot \ln(\cos 2x$$

$$\lim_{x\to 0} \frac{1}{\operatorname{sen} x^2}\cdot \ln(\cos(2x)) = [\infty\cdot 0] = \lim_{x\to 0} \frac{\ln(\cos 2x)}{\operatorname{sen} x^2} = \left[\frac{0}{0}\right] \overset{L'H}{=}$$

$$= \lim_{x \to 0} \frac{\frac{1}{\cos(2x)} \cdot (-\operatorname{sen}(2x)) \cdot \cancel{2}}{\cos x^2 \cdot \cancel{2}x} = \lim_{x \to 0} \frac{-\operatorname{tg}(2x)}{x \cdot \cos x^2} = \left[\frac{0}{0}\right] \overset{L'H}{=}$$

$$= \lim_{x \to 0} \frac{-(1+\operatorname{tg}^2(2x)) \cdot 2}{\cos x^2 - 2x^2 \cdot \operatorname{sen} x^2} = \frac{-2}{1} = -2$$

$$\Rightarrow \ln A = -2 \Rightarrow A = \lim_{x \to 0} [\cos(2x)]^{1/\operatorname{sen} x^2} = e^{-2} = 1/e^2$$

45) $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\operatorname{sen} x} \right) = [\infty - \infty] = \lim_{x \to 0} \frac{\operatorname{sen} x - x}{x \cdot \operatorname{sen} x} = \left[\frac{0}{0}\right] \overset{L'H}{=}$

$$= \lim_{x \to 0} \frac{\cos x - 1}{\operatorname{sen} x + x \cos x} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x \to 0} \frac{-\operatorname{sen} x}{\cos x + \cos x - x \operatorname{sen} x} = \frac{0}{2} = 0$$

46) $\lim_{x \to 0^+} x^{\operatorname{tg} x} = [0^0] \Rightarrow A = \lim_{x \to 0^+} x^{\operatorname{tg} x}$

$$\ln A = \ln \lim_{x \to 0^+} x^{\operatorname{tg} x} = \lim_{x \to 0^+} \ln x^{\operatorname{tg} x} = \lim_{x \to 0^+} \operatorname{tg} x \cdot \ln x$$

$$\lim_{x \to 0^+} \operatorname{tg} x \cdot \ln x = [0 (-\infty)] = \lim_{x \to 0^+} \frac{\ln x}{1/\operatorname{tg} x} = \left[\frac{-\infty}{\infty}\right] \overset{L'H}{=}$$

$$= \lim_{x \to 0^+} \frac{1/x}{-\frac{(1+\operatorname{tg}^2 x)}{\operatorname{tg}^2 x}} = \lim_{x \to 0^+} \frac{-\operatorname{tg}^2 x}{x(1+\operatorname{tg}^2 x)} = \left[\frac{0}{0}\right] \overset{L'H}{=}$$

$$= \lim_{x \to 0^+} \frac{-2\operatorname{tg} x (1+\operatorname{tg}^2 x)}{(1+\operatorname{tg}^2 x) + x \cdot 2\operatorname{tg} x (1+\operatorname{tg}^2 x)} = \frac{0}{1+0} = 0$$

$$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x \to 0^+} x^{\operatorname{tg} x} = e^0 = 1$$

47) $\lim_{x\to\infty} \left(\frac{1}{2x}\right)^{1/x} = [0^0] \Rightarrow A = \lim_{x\to\infty} \left(\frac{1}{2x}\right)^{1/x}$

$\ln A = \ln \lim_{x\to\infty} \left(\frac{1}{2x}\right)^{1/x} = \lim_{x\to\infty} \ln \left(\frac{1}{2x}\right)^{1/x} = \lim_{x\to\infty} \frac{1}{x} \cdot \ln\left(\frac{1}{2x}\right)$

$\lim_{x\to\infty} \frac{1}{x} \cdot \ln\left(\frac{1}{2x}\right) = [0 \cdot (-\infty)] = \lim_{x\to\infty} \frac{\ln(1/2x)}{x} = \left[\frac{-\infty}{\infty}\right] \overset{L'H}{=}$

$= \lim_{x\to\infty} \frac{1}{x} = 0$

$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x\to\infty} \left(\frac{1}{2x}\right)^{1/x} = e^0 = 1$

48) $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{\operatorname{arctg} x}\right) = [\infty - \infty] = \lim_{x\to 0^+} \frac{\operatorname{arctg} x - x}{x \operatorname{arctg} x} = \left[\frac{0}{0}\right]$

$\overset{L'H}{=} \lim_{x\to 0^+} \frac{\frac{1}{1+x^2} - 1}{\operatorname{arctg} x + \frac{x}{1+x^2}} = \lim_{x\to 0^+} \frac{\frac{-x^2}{\cancel{1+x^2}}}{\frac{(1+x^2)\cdot\operatorname{arctg} x + x}{\cancel{1+x^2}}}$

$= \lim_{x\to 0^+} \frac{-x^2}{(1+x^2)\operatorname{arctg} x + x} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x\to 0^+} \frac{-2x}{2x\operatorname{arctg} x + 1 + 1} = \frac{0}{2} = 0$

49) $\lim_{x\to 1^-} \left[(x^2-1)\cdot \operatorname{tg}\left(\frac{\pi x}{2}\right)\right] = [0 \cdot \infty] = \lim_{x\to 1^-} \frac{(x^2-1)\cdot \operatorname{sen}\left(\frac{\pi x}{2}\right)}{\cos\left(\frac{\pi x}{2}\right)} = \left[\frac{0}{0}\right]$

$\overset{L'H}{=} \lim_{x\to 1^-} \frac{2x \operatorname{sen}\left(\frac{\pi}{2} x\right) + (x^2-1)\cdot \cos\left(\frac{\pi}{2} x\right)\cdot\frac{\pi}{2}}{-\operatorname{sen}\left(\frac{\pi}{2}\cdot x\right)\cdot \frac{\pi}{2}} = \frac{2}{-\pi/2} = -\frac{4}{\pi}$

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50) $\lim_{x \to \infty} \left(x \cdot \ln\left(\frac{x+1}{x}\right)\right) = [\infty \cdot 0] = \lim_{x \to \infty} \frac{\ln\left(\frac{x+1}{x}\right)}{1/x} = \left[\frac{0}{0}\right] =$

$\overset{L'H}{=} \lim_{x \to \infty} \frac{\frac{\cancel{x}}{x+1} \cdot \frac{\cancel{x} - \cancel{x} + 1}{x^{\cancel{2}}}}{+1/x^2} = \lim_{x \to \infty} \frac{x^{\cancel{2}}}{\cancel{x}(x+1)} = 1$

51) $\lim_{x \to 0} \frac{\operatorname{sen} x}{4x} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x \to 0} \frac{\cos x}{4} = \frac{1}{4}$

52) $\lim_{x \to 1} \frac{x^2 - 1}{\ln x} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x \to 1} \frac{2x}{1/x} = 2$

53) $\lim_{x \to 0} \frac{x^3 - x}{\sqrt{x+9} - 3} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x \to 0} \frac{3x^2 - 1}{\frac{1}{2\sqrt{x+9}}} = \frac{-1}{1/6} = -6$

54) $\lim_{x \to 0} \frac{\operatorname{tg} x}{x} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x \to 0} \frac{1 + \operatorname{tg}^2 x}{1} = 1$

55) $\lim_{x \to 1} \frac{4x^5 - 4x^2}{-x^2 + 1} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x \to 1} \frac{20x^4 - 8x}{-2x} = \frac{12}{-2} = -6$

56) $\lim_{x \to 0} \frac{x}{\operatorname{sen} x} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x \to 0} \frac{1}{\cos x} = 1$

57) $\lim_{x \to 0} \frac{e^{-x} - e^{x}}{3 \operatorname{sen} x} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x \to 0} \frac{-e^{-x} - e^{x}}{3\cos x} = \frac{-2}{3}$

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58) $$\lim_{x\to 7} \frac{x^2-9x+14}{\ln(x^2-6x-6)} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x\to 7} \frac{2x-9}{\frac{2x-6}{x^2-6x-6}} =$$

$$= \lim_{x\to 7} \frac{(2x-9)(x^2-6x-6)}{2x-6} = \frac{5}{8}$$

59) $$\lim_{x\to 0} \left(\frac{1}{4x} - \frac{2}{x\cdot e^x}\right) = [\infty - \infty] = \lim_{x\to 0} \frac{e^x-8}{4xe^x} =$$

$$= \left[\frac{-7}{0}\right] \longrightarrow \begin{cases} \lim\limits_{x\to 0^-} \dfrac{e^x-8}{4x\cdot e^x} = +\infty \\ \lim\limits_{x\to 0^+} \dfrac{e^x-8}{4xe^x} = -\infty \end{cases} \Rightarrow \nexists \lim_{x\to 0}\left(\frac{1}{4x} - \frac{2}{x\cdot e^x}\right)$$

60) $$\lim_{x\to 0^+} (\operatorname{tg} x)^x = [0^0] \Rightarrow A = \lim_{x\to 0^+} (\operatorname{tg} x)^x$$

$$\ln A = \ln \lim_{x\to 0^+} (\operatorname{tg} x)^x = \lim_{x\to 0^+} \ln(\operatorname{tg} x)^x = \lim_{x\to 0^+} x\cdot \ln(\operatorname{tg}(x))$$

$$\lim_{x\to 0^+} x\cdot\ln(\operatorname{tg}(x)) = [0\cdot(-\infty)] = \lim_{x\to 0^+} \frac{\ln(\operatorname{tg}(x))}{1/x} = \left[\frac{-\infty}{\infty}\right] \overset{L'H}{=}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{\operatorname{tg}(x)}\cdot(1+\operatorname{tg}^2 x)}{-1/x^2} = \lim_{x\to 0^+} \frac{-x^2\cdot(1+\operatorname{tg}^2 x)}{\operatorname{tg} x} = \left[\frac{0}{0}\right]$$

$$\overset{L'H}{=} \lim_{x\to 0^+} \frac{-2x(1+\operatorname{tg}^2 x) - x^2\cdot 2\operatorname{tg}(x)\cdot(1+\operatorname{tg}^2 x)}{1+\operatorname{tg}^2 x} = \frac{0}{1} = 0$$

$$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x\to 0^+} (\operatorname{tg} x)^x = e^0 = 1$$

61) $$\lim_{x\to\infty} \sqrt[x-2]{\frac{x^2+1}{x+3}} = \lim_{x\to\infty} \left(\frac{x^2+1}{x+3}\right)^{\frac{1}{x-2}} = [\infty^0]$$

$$A = \lim_{x\to\infty} \left(\frac{x^2+1}{x+3}\right)^{\frac{1}{x-2}}$$

$$\ln A = \ln \lim_{x\to\infty} \left(\frac{x^2+1}{x+3}\right)^{1/x-2} = \lim_{x\to\infty} \frac{1}{x-2}\cdot \ln\left(\frac{x^2+1}{x+3}\right) = [0\cdot\infty]$$

$$\lim_{x\to\infty} \frac{\ln\left(\frac{x^2+1}{x+3}\right)}{x-2} = \left[\frac{\infty}{\infty}\right] \overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{\cancel{x+3}}{x^2+1}\cdot\frac{2x(x+3)-(x^2+1)}{(x+3)^{\cancel{2}}}}{1} =$$

$$= \lim_{x\to\infty} \frac{x^2+6x-1}{(x^2+1)(x+3)} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty}\frac{x^2}{x^3} = 0$$

$$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x\to\infty} \left(\frac{x^2+1}{x+3}\right)^{1/x-2} = e^0 = 1$$

62) $$\lim_{x\to 0} \frac{\operatorname{sen} x}{1-\cos x} = \left[\frac{0}{0}\right] \overset{L'H}{=} \lim_{x\to 0} \frac{\cos x}{\operatorname{sen} x} = \left[\frac{1}{0}\right]$$

$$\rightarrow \begin{cases} \lim_{x\to 0^-} \frac{\cos x}{\operatorname{sen} x} = -\infty \\ \lim_{x\to 0^+} \frac{\cos x}{\operatorname{sen} x} = +\infty \end{cases} \Rightarrow \nexists \lim_{x\to 0} \frac{\operatorname{sen} x}{1-\cos x}$$

63) $\lim\limits_{x \to 0} \dfrac{2\operatorname{sen} x}{x^2-3x} = \left[\dfrac{0}{0}\right] \overset{L'H}{=} \lim\limits_{x \to 0} \dfrac{2\cos x}{2x-3} = \dfrac{2}{-3} = -\dfrac{2}{3}$

64) $\lim\limits_{x \to 0} \dfrac{x-\operatorname{sen} x}{x\cdot \operatorname{sen} x} = \left[\dfrac{0}{0}\right] \overset{L'H}{=} \lim\limits_{x \to 0} \dfrac{1-\cos x}{\operatorname{sen} x + x\cdot\cos x} = \left[\dfrac{0}{0}\right]$

$\overset{L'H}{=} \lim\limits_{x \to 0} \dfrac{\operatorname{sen} x}{\cos x + \cos x - x\operatorname{sen} x} = \dfrac{0}{2} = 0$

65) $\lim\limits_{x \to 0} \dfrac{x^4 - \left(\frac{1}{3}\right)x^3}{x - \operatorname{tg} x} = \left[\dfrac{0}{0}\right] \overset{L'H}{=} \lim\limits_{x \to 0} \dfrac{4x^3 - x^2}{\cancel{1} - \cancel{1} - \operatorname{tg}^2 x} =$

$= \left[\dfrac{0}{0}\right] \overset{L'H}{=} \lim\limits_{x \to 0} \dfrac{12x^2-2x}{-2\operatorname{tg} x(1+\operatorname{tg}^2 x)} = \lim\limits_{x \to 0} \dfrac{x-6x^2}{\operatorname{tg} x(1+\operatorname{tg}^2 x)} =$

$= \left[\dfrac{0}{0}\right] \overset{L'H}{=} \lim\limits_{x \to 0} \dfrac{1-12x}{(1+\operatorname{tg}^2 x)^2 + 2\operatorname{tg}^2 x(1+\operatorname{tg}^2 x)} = 1$

66) $\lim\limits_{x \to 0} \dfrac{x-2\operatorname{sen} x}{\operatorname{tg} x - 2\operatorname{sen} x} = \left[\dfrac{0}{0}\right] \overset{L'H}{=} \lim\limits_{x \to 0} \dfrac{1-2\cos x}{1+\operatorname{tg}^2 x - 2\cos x} = 1$

67) $\lim\limits_{x \to 0} \dfrac{\ln(e^x + x^3)}{x} = \left[\dfrac{0}{0}\right] \overset{L'H}{=} \lim\limits_{x \to 0} \dfrac{e^x + 3x^2}{e^x + x^3} = 1$

68) $\lim_{x\to 0^+} \frac{\ln(1+x)}{\sqrt[4]{x^3}} = \left[\frac{0}{0}\right] \stackrel{L'H}{=} \lim_{x\to 0^+} \frac{\frac{1}{1+x}}{\frac{1}{4\cdot\sqrt[4]{x^9}}\cdot 3x^2} =$

$= \lim_{x\to 0^+} \frac{\frac{1}{1+x}}{\frac{3}{4}\cdot\frac{1}{\sqrt[4]{x}}} = \lim_{x\to 0^+} \frac{4\sqrt[4]{x}}{3(1+x)} = \frac{0}{3} = 0$

69) $\lim_{x\to 0} \frac{\ln(\cos(3x))}{x^2} = \left[\frac{0}{0}\right] \stackrel{L'H}{=} \lim_{x\to 0} \frac{tg(3x)}{2x} = \left[\frac{0}{0}\right] \stackrel{L'H}{=}$

$= \lim_{x\to 0} \frac{[1+tg^2(3x)]\,3}{2} = \frac{3}{2}$

70) $\lim_{x\to 0} \frac{e^x - e^{\operatorname{sen} x}}{1-\cos x} = \left[\frac{0}{0}\right] \stackrel{L'H}{=} \lim_{x\to 0} \frac{e^x - e^{\operatorname{sen} x}\cdot\cos x}{\operatorname{sen} x} = \left[\frac{0}{0}\right] =$

$\stackrel{L'H}{=} \lim_{x\to 0} \frac{e^x - (e^{\operatorname{sen} x}\cdot\cos^2 x - \operatorname{sen} x\cdot e^{\operatorname{sen} x})}{\cos x} = \frac{0}{1} = 0$

71) $\lim_{x\to 0} \frac{1+x-e^x}{\operatorname{sen}^2 x} = \left[\frac{0}{0}\right] \stackrel{L'H}{=} \lim_{x\to 0} \frac{1-e^x}{2\operatorname{sen} x\cos x} = \left[\frac{0}{0}\right] \stackrel{L'H}{=}$

$= \lim_{x\to 0} \frac{-e^x}{2\cos^2 x - 2\operatorname{sen}^2 x} = -\frac{1}{2}$

72) $\lim\limits_{x\to 2} \dfrac{2^x - x^2}{9-3^x} = \left[\dfrac{0}{0}\right] \overset{L'H}{=} \lim\limits_{x\to 2}\dfrac{2^x \ln 2 - 2x}{9 - 3^x\cdot \ln 3} =$

$= \dfrac{4(\ln 2 - 1)}{9(1-\ln 3)} \approx 1'383$

73) $\lim\limits_{x\to 0}(e^x-x)^{1/x} = [1^\infty]$; $A = \lim\limits_{x\to 0}(e^x-x)^{1/x}$

$\ln A = \ln \lim\limits_{x\to 0}(e^x-x)^{1/x} = \lim\limits_{x\to 0}\ln(e^x-x)^{1/x} = \lim\limits_{x\to 0}\dfrac{1}{x}\ln(e^x-x)$

$\lim\limits_{x\to 0}\dfrac{\ln(e^x-x)}{x} = \left[\dfrac{0}{0}\right] \overset{L'H}{=} \lim\limits_{x\to 0}\dfrac{e^x-1}{e^x-x} = \dfrac{0}{1}=0$

$\Rightarrow \ln A = 0 \Rightarrow A = \lim\limits_{x\to 0}(e^x-x)^{1/x} = e^0 = 1$

74) $\lim\limits_{x\to 0}(\operatorname{cotg} x)^{\operatorname{sen} x} = [\infty^0]$; $A = \lim\limits_{x\to 0}(\operatorname{cotg} x)^{\operatorname{sen} x}$

$\ln A = \ln \lim\limits_{x\to 0}(\operatorname{cotg} x)^{\operatorname{sen} x} = \lim\limits_{x\to 0}\ln(\operatorname{cotg} x)^{\operatorname{sen} x} = \lim\limits_{x\to 0}\operatorname{sen} x\cdot\ln(\operatorname{cotg}$

$\lim\limits_{x\to 0}\operatorname{sen} x\cdot\ln(\operatorname{cotg} x) = [0\cdot\infty] = \lim\limits_{x\to 0}\dfrac{\ln(\operatorname{cotg} x)}{1/\operatorname{sen} x} = \left[\dfrac{\infty}{\infty}\right] \overset{L'H}{=}$

$= \lim\limits_{x\to 0}\dfrac{\cancel{\operatorname{tg}(x)}\cdot\left(\dfrac{+(1+\operatorname{tg}^2 x)}{\operatorname{tg}^{\cancel{2}} x}\right)}{\dfrac{+\cos x}{\operatorname{sen}^2 x}} = \lim\limits_{x\to 0}\dfrac{\operatorname{sen}^{\cancel{2}} x\cdot(1+\operatorname{tg}^2 x)}{\cancel{\cos x}\cdot\cancel{\dfrac{\operatorname{sen} x}{\cos x}}} =$

$$= \lim_{x\to 0} \operatorname{sen}|x| \cdot \left(1+tg^2 x\right) = 0 \cdot 1 = 0$$

$$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x\to 0} \left[\operatorname{cotg}|x|\right]^{\operatorname{sen}|x|} = e^0 = 1$$

75) $$\lim_{x\to 0} \left(\frac{2}{\ln(1+x)} - \frac{2}{x}\right) = [\infty - \infty] =$$

$$= \lim_{x\to 0} \frac{2x - 2\ln(1+x)}{x\cdot \ln(1+x)} = \left[\frac{0}{0}\right] \overset{L'H}{=}$$

$$= \lim_{x\to 0} \frac{2 - \frac{2}{1+x}}{\ln(1+x) + \frac{x}{1+x}} = \lim_{x\to 0} \frac{\frac{\cancel{2} + x \cancel{- 2}}{\cancel{1+x}}}{\frac{(1+x)\ln(1+x) + x}{\cancel{1+x}}} =$$

$$= \lim_{x\to 0} \frac{x}{(1+x)\ln(1+x) + x} = \left[\frac{0}{0}\right] \overset{L'H}{=}$$

$$= \lim_{x\to 0} \frac{1}{\ln(1+x) + 1 + 1} = \frac{1}{2}$$

76) $$\lim_{x\to \pi/2} (x-\pi/2)\cdot tg\, x = [0\cdot\infty] = \lim_{x\to \pi/2} \frac{(x-\pi/2)\cdot \operatorname{sen} x}{\cos x} = \left[\frac{0}{0}\right] =$$

$$\overset{L'H}{=} \lim_{x\to \pi/2} \frac{\operatorname{sen} x + (x-\pi/2)\cdot\cos x}{-\operatorname{sen} x} = -1$$

77) $\lim_{x \to \infty} x \cdot e^{2/x} = \infty$

78) $\lim_{x \to 0} (\text{arcsen}\, x \cdot \text{cotg}\, x) = [0 \cdot \infty] = \lim_{x \to 0} \frac{\text{arcsen}(x)}{tg(x)} = \left[\frac{0}{0}\right]$

$\stackrel{L'H}{=} \lim_{x \to 0} \frac{\frac{1}{1+x^2}}{1+tg^2 x} = \frac{1}{1} = 1$

79) $\lim_{x \to \infty} x \cdot \text{arctg}\left(\frac{2}{x}\right) = [\infty \cdot 0] = \lim_{x \to \infty} \frac{\text{arctg}\left(\frac{2}{x}\right)}{1/x} = \left[\frac{0}{0}\right]$

$\stackrel{L'H}{=} \lim_{x \to \infty} \frac{\frac{1}{1+(2/x)^2} \cdot \left(\cancel{+\frac{2}{x^2}}\right)}{\cancel{+1/x^2}} = \lim_{x \to \infty} \frac{2}{1+\left(\frac{2}{x}\right)^2} = 2$

80) $\lim_{x \to \infty} (\ln x)^{e^{-x}} = [\infty^0]$; $A = \lim_{x \to \infty} (\ln x)^{e^{-x}}$

$\ln A = \ln \lim_{x \to \infty} (\ln x)^{e^{-x}} = \lim_{x \to \infty} \ln(\ln(x))^{e^{-x}} = \lim_{x \to \infty} e^{-x} \cdot \ln(\ln(x))$

$\lim_{x \to \infty} e^{-x} \cdot \ln(\ln(x)) = [0 \cdot \infty] = \lim_{x \to \infty} \frac{\ln(\ln(x))}{e^x} = \left[\frac{\infty}{\infty}\right] \stackrel{L'H}{=}$

$\stackrel{||}{0}$ (orden magnitud)

$= \lim_{x \to \infty} \frac{\frac{1}{\ln(x)} \cdot \frac{1}{x}}{e^x} = \lim_{x \to \infty} \frac{1}{x \cdot e^x \cdot \ln x} = \frac{1}{\infty} = 0$

$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x \to \infty} (\ln x)^{e^{-x}} = e^0 = 1$